# Derived Willingness-To-Pay For Water: 

# Effects Of Probabilistic Rationing And Price 

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#### Abstract

A two stage linear programming approach is used to estimate the willingness to pay (WTP) of individual households or groups of households for changes in a combination of probabilistic water supply reliability and retail price of water. By modeling the costs (financial and 'perceived') of implementing long and short-term conservation measures and assuming rational (expected value cost minimizing behavior), demand curves for water and expected water use curves can be estimated. Derived demand curves for conservation measures can also be calculated. Next Monte Carlo simulation techniques are used to represent household variability in the model parameters and derive estimates of aggregate WTP for water supply reliability, water demand curves and demand curves for conservation measures. Several examples are provided to illustrate the approach.


## Introduction

The goal of water managers is to deliver a reliable water supply at a "reasonable" cost. Indeed, any increase in reliability involves a cost that has to be balanced against the benefits associated with the resulting reduction in frequency of water scarcity. The economically optimal water supply reliability will be such that the marginal cost of increased reliability equals the marginal cost of increased shortage (Howe and Smith 1994, Hoagland 1998). Since early modern engineering and planning studies in the water resources, a key question has been whether the users would be willing to pay the costs of increasing supply capacity (Dupuit, 1844).

Providing decision makers with a probabilistic supply valuation based on willingness-to-pay (WTP) becomes a key tool in reliability planning. If the cost of a reliability enhancement project (water recycling, extra capacity, water transfers etc) is below the consumer's WTP, the project is economically viable (Abrahams et al, 2000). Conversely, in highly reliable systems consumers might be willing to accept a greater frequency of shortages in exchange for reduced water bills (Howe and Smith, 1994). The extra water made available could be marketed by the water agency and transferred to other sectors (Hoagland, 1998).

On a regional perspective, where multiple water uses compete for constrained resources, increasing water service reliability to urban areas is likely to shift the risk of shortage (and the cost) to other sectors (Howe and Smith, 1994, Griffin and Mjelde 2000). Traditionally the cost of shortages has been borne by in-stream users, however this is becoming less likely (Griffin and Mjelde 2000) and the focus is slowly shifting to agricultural users (often with compensation from urban users). Since the 1991 drought in California and the resulting severe water rationing for urban users, cities have started to argue that the costs of water shortages to the urban/residential sector should be reevaluated upwards and that the value of service reliability to urban users out-weights agricultural losses. In California, in light of the increasing population pressure, the allocation of water to environmental uses, more stringent regulatory requirements and increasing marginal costs of new projects water, shortage management will become more important (Wilchfort and Lund 1997).

However, as Lund (1995) noted, little effort has been devoted to valuing urban water supply reliability. Most of the approaches have been empirical; either using price elasticity (Howe 1967 and 1982, Greene et al. 1998) or contingent valuation techniques (Carson and Mitchell 1987, CUWA 1994, Howe and Smith,

1994, Griffin and Mjelde 2000). These empirical studies typically ignore much of the interaction between long-term and short-term conservation measures and look only at a single shortage event, defined by a given level of shortage with a certain frequency, i.e., a $10 \%$ shortage once every 5 years (Lund 1995). Yet there is a necessity for estimating the shortage losses over the whole range of possible shortages (Howe and Smith 1994). Indeed, investments in water supply reliability enhancement alter the frequency of all shortage levels so estimating the value of an entire probability distribution of shortages is desirable. Several studies (Griffin and Mjelde 2000, CUWA 1994) have shown that consumers have difficulty interpreting probabilistic information which leads to inconsistent results. The CUWA (1994) study concluded that these limitations make it difficult to apply CV data to " a real world hydrology that produces a mix of shortages ". Thus, traditional empirical methods used for valuing the benefits of urban water supply reliability are in some ways ill-suited for probabilistic settings.

The approach developed by Lund (1995) and Wilchfort and Lund (1997) addresses some of these limitations. These authors proposed a two-stage optimization model to estimate WTP to avoid shortages that considers the user economic response to an entire shortage probability distribution (different levels and frequencies). This study extends their approach to include the retail price of water and allows for household variability in the model parameters. This allows derivation of demand curves for water and for conservation measures as well as probabilistic estimates of WTP for water supply reliability.

The paper begins with a simple analytical treatment of long- and short-term conservation options in the context of a probability distribution of water rationing levels and varying retail prices for water. A twostage linear program is then formulated to numerically estimate the WTP of a single household to avoid a probability distribution of shortages for different retail water prices. This two-stage optimization program is applied to derive individual consumer water demand curves with and without shortage. Input demand curves for conservation measures are also presented. The method is then extended to develop aggregate demand curves for both water and conservation measures for a group of households.

## General Formulation for Individual Consumers

The formulation of Lund (1995) can be extended to include the price of water for each shortage event $\mathrm{k}\left(p_{Q k}\right)$. The household's objective is to minimize the expected value of total annualized costs necessary to meet each level of probabilistic shortage subject to household water rationing and management constraints. This expected cost minimization assumption appears reasonable since water costs are usually a small part of most total household expenditures. This is formulated as a mathematical program:

$$
\begin{equation*}
\text { Minimize } \quad Z=f\left(\underline{X}_{1}\right)+\sum_{k=1}^{n} P_{k}\left[g_{k}\left(\underline{X}_{1}, \underline{X}_{2 k}\right)+p_{Q k} Q_{k}\right] \tag{1}
\end{equation*}
$$

Subject to:

$$
\begin{align*}
& h_{k}\left(\underline{X}_{1}, \underline{X}_{2 k}\right)=Q_{k}, \forall k  \tag{2}\\
& Q_{k} \leq r_{k}, \forall k \tag{3}
\end{align*}
$$

Where:
$\underline{X}_{1}=$ long-term conservation measures available,
$\underline{X}_{2 \mathrm{k}}=$ short-term conservation measures available for each shortage event k ,
$\mathrm{f}\left(\underline{X}_{1}\right)=$ cost of implementing permanent water conservation efforts $\underline{X}_{1}$,
$g_{k}\left(\underline{X}_{1}, \underline{X}_{2 k}\right)=$ cost of implementing short-term conservation efforts $\underline{X}_{2 k}$,
$h_{k}\left(\underline{X}_{1}, \underline{X}_{2 k}\right)=$ water use in event $k$ for implementation of short and long-term conservation options,
$\mathrm{p}_{\mathrm{Qk}}=$ retail price of water for each shortage event k ,
$\mathrm{Q}_{\mathrm{k}}=$ water use for each shortage event k ,
$\mathrm{P}_{\mathrm{k}}=$ probability of occurrence of each shortage event k , $r_{k}=$ ration amount for event $k$,

Here Z is the expected value of total household water costs, with component costs for long and short-term water conservation efforts and the purchase cost of water for each rationing event (including no rationing). The latter two terms are weighted by the probability of each rationing event to account for the hydrologic uncertainties in water supply (see Lund (1995) or Wilchfort and Lund (1997) for details). Constraints (2) and (3) state that water use is a function of the permanent and short-term reduction efforts by function $h_{k}()$ and that the total water use must be less than the ration amount $r_{k}$ for each rationing event.

This problem can be examined analytically using the method of Lagrange multipliers. Substituting Equation 2 into Equations 1 and 3, the Lagrangian function is:

$$
\begin{equation*}
L=f\left(\underline{X}_{1}\right)+\sum_{k=1}^{n} P_{k}\left[g_{k}\left(\underline{X}_{1}, \underline{X}_{2 k}\right)+p_{Q k} h_{k}\left(\underline{X}_{1}, \underline{X}_{2 k}\right)\right]-\sum_{k=1}^{n} \lambda_{k}\left(h_{k}\left(\underline{X}_{1}, \underline{X}_{2 k}\right)-r_{k}\right) \tag{4}
\end{equation*}
$$

The resulting first-order conditions are:

$$
\begin{align*}
& \frac{\partial L}{\partial X_{1 i}}=0=\frac{\partial f\left(\underline{X}_{1}\right)}{\partial X_{1 i}}+\sum_{k=1}^{n} P_{k}\left[\frac{\partial g_{k}\left(\underline{X}_{1}, \underline{X_{2 k}}\right)}{\partial X_{1 i}}+\left(p_{Q k}-\frac{\lambda_{k}}{p_{k}}\right) \frac{\partial h_{k}\left(\underline{X}_{1}, \underline{X}_{2 k}\right)}{\partial X_{1 i}}\right], \forall i  \tag{5}\\
& \frac{\partial L}{\partial X_{2 j}}=0=P_{k}\left[\frac{\partial g_{k}\left(\underline{X}_{1}, \underline{X}_{2 k}\right)}{\partial X_{2 j}}+\left(p_{Q k}-\frac{\lambda_{k}}{P_{k}}\right) \frac{\partial h_{k}\left(\underline{X}_{1}, \underline{X}_{2 k}\right)}{\partial X_{2 j}}\right], \forall j, k
\end{align*}
$$

Rearranging Equation 5 results in Equation 7,

$$
\begin{equation*}
\frac{\partial f\left(\underline{X}_{1}\right)}{\partial X_{1 i}}=\sum_{k=1}^{n} P_{k}\left[\left(\frac{\lambda_{k}}{P_{k}}-p_{Q k}\right) \frac{\partial h_{k}\left(\underline{X}_{1}, \underline{X}_{2 k}\right)}{\partial X_{1 i}}-\frac{\partial g_{k}\left(\underline{X}_{1}, \underline{X}_{2 k}\right)}{\partial X_{1 i}}\right], \forall i \tag{7}
\end{equation*}
$$

This result shows the effect of price on the optimal marginal cost of implementing any permanent conservation option $i$. The implementation of the permanent conservation options is encouraged to the extent of the expected value of decreased household water expenditures over the n possible shortage events (the entire shortage probability distribution). Thus, price effects on conservation implementation are proportional to the effectiveness of the conservation option. If price does not vary between shortage events ( $p_{Q k}=p_{Q}$ ), then the price effect is directly proportional to the expected value of water savings from a given conservation option.

The condition in Equation 6 is easily rearranged to:

$$
\begin{equation*}
\frac{\frac{\partial g_{k}\left(\underline{X}_{1}, \underline{X}_{2 k}\right)}{\partial X_{2 j}}}{\frac{\partial h_{k}\left(\underline{X}_{1}, \underline{X}_{2 k}\right)}{\partial X_{2 j}}}=\left(\frac{\lambda_{k}}{P_{k}}-p_{Q k}\right), \forall j, k \tag{8}
\end{equation*}
$$

This condition holds that the marginal implementation costs-effectiveness should be equal across shortterm conservation actions for each different event. This result is very similar to Lund (1995), except that considering price lowers the marginal level of implementation for short-term measures. Higher water prices make it optimal to use short-term conservation measures that are less implementation-cost-effective (for water use reduction).

Putting Equation 8 into Equation 5 yields a modified first-order condition for permanent conservation options. This first-order condition no longer explicitly includes the retail water price; price is implicitly included through Equation 8 on the optimal magnitude of short-term implementation-cost/effectiveness.

$$
0=\frac{\partial f\left(\underline{X}_{1}\right)}{\partial X_{1 i}}+\sum_{k=1}^{n} P_{k}\left[\frac{\partial g_{k}\left(\underline{X}_{1}, \underline{X}_{2 k}\right)}{\partial X_{2 j}}-\left(\frac{\frac{\partial g_{k}\left(\underline{X}_{1}, \underline{X}_{2 k}\right)}{\partial X_{2 j}}}{\frac{\partial h_{k}\left(\underline{X}_{1}, \underline{X}_{2 k}\right)}{\partial X_{2 j}}}\right) \frac{\partial k_{k}\left(\underline{X}_{1}, \underline{X}_{2 k}\right)}{\partial X_{1 i}}\right], \forall i
$$

If implementation of long term (permanent) conservation measures does not affect event-specific (short term) implementation costs, $\delta \mathrm{g}_{\mathrm{k}}() / \delta \mathrm{X}_{1 \mathrm{i}}=0$, and Equation 9 becomes:

$$
\begin{equation*}
\frac{\partial f\left(\underline{X}_{1}\right)}{\partial X_{1 i}}=\sum_{k=1}^{n} P_{k}\left[\left(\frac{\frac{\partial g_{k}\left(\underline{X}_{1}, \underline{X}_{2 k}\right)}{\partial X_{2 j}}}{\frac{\partial h_{k}\left(\underline{X}_{1}, \underline{X}_{2 k}\right)}{\partial X_{2 j}}}\right) \frac{\partial k_{k}\left(\underline{X_{1}}, \underline{X}_{2 k}\right)}{\partial X_{1 i}}\right], \forall i \tag{10}
\end{equation*}
$$

Further, if the implementation of permanent options has the same water-conservation effectiveness for each rationing event, then $\delta h_{k}() / \delta X_{1 i}=$ constant not varying with k . In this case, Equation 10 becomes:

$$
\begin{equation*}
\frac{\frac{\partial f\left(\underline{X}_{1}\right)}{\partial X_{1 i}}}{\frac{\partial h_{k}\left(\underline{X}_{1}, \underline{X}_{2 k}\right)}{\partial X_{1 i}}}=\sum_{k=1}^{n} P_{k}\left(\frac{\frac{\partial g_{k}\left(\underline{X}_{1}, \underline{X}_{2 k}\right)}{\partial X_{2 j}}}{\frac{\partial h_{k}\left(\underline{X}_{1}, \underline{X}_{2 k}\right)}{\partial X_{2 j}}}\right), \forall i \tag{11}
\end{equation*}
$$

Where these conditions hold, the marginal implementation-cost-effectiveness should be equal across all permanent conservation options implemented and should also equal the expected value of the marginal implementation-cost-effectiveness for each implemented short-term conservation option.
However this simplified result does not consider that implementing long-term conservation measures often reduces the effectiveness of short-term options (demand hardening). Therefore $\delta \mathrm{X}_{2 \mathrm{j}} / \delta \mathrm{X}_{1 \mathrm{i}}>0$ and the RHS will have to be adjusted to account for this, making the implementation of permanent conservation options less likely.

While these analytical solutions provide some insight, in practice more specific numerical formulations are more useful. This problem can be nicely formulated into linear programs for several cases.

## Linear Program Formulation for a Single Household

For an individual household, undertaking a given water conservation measure $(X)$ implies a unit cost to the household of $(c)$. Economic motivations for water conservation measures are the retail price of water and limits placed on water availability due to shortage event $k$ (rationing levels or outages). The overall impact of water shortages depends on the probability of occurrence of each event as specified by a shortage probability distribution similar to those generated from water resource models (Hirsch, 1978). This means that once a long-term conservation measure is implemented the household bears its full cost $\left(\mathrm{c}_{1 \mathrm{i}}\right)$ whereas the $\operatorname{cost}\left(\mathrm{c}_{2 \mathrm{jk}}\right)$ of short-term water conservation options has to be weighted by the probability
of each shortage event $P_{k}$. The same applies to the water bill $\mathrm{p}_{\mathrm{Qk}} \mathrm{Q}_{\mathrm{k}}$, allowing price to vary with shortage event $k$.
This stochastic optimization problem can be represented as a two-stage linear program (Wagner 1975) as shown below. The linear program extends earlier formulations (Lund 1995; Wilchfort and Lund 1997) by including the retail price of water. The first stage decisions concern long-term conservation measures, which must be implemented before the shortage and normally have a long life span and fixed annualized costs. The second stage decisions concern the implementation of short-term conservation measures to reduce demand/water use for each shortage event $k$.

$$
\begin{equation*}
\operatorname{MinZ}=\sum_{i=1}^{n_{1}} c_{1 i} X_{1 i}+\sum_{k=1}^{m} P_{k}\left[\sum_{j=1}^{n_{2}} c_{2 j k} X_{2 j k}+p_{Q k} Q_{k}\right] \tag{12}
\end{equation*}
$$

Subject to:

$$
\begin{gather*}
\mathrm{X}_{1 \mathrm{i}} \leq \mathrm{u}_{1 \mathrm{i}} \forall \mathrm{i}  \tag{13}\\
\mathrm{X}_{2 \mathrm{jk}} \leq \mathrm{u}_{2 \mathrm{jk}} \forall \mathrm{j}, \mathrm{k}  \tag{14}\\
d_{k}-\left[\sum_{i=1}^{n_{1}} q_{1 i} X_{1 i}+\sum_{j=1}^{n_{2}} q_{2 j k} X_{2 j k}\right]=Q_{k} \quad \forall k  \tag{15}\\
Q_{k} \leq r_{k} \quad \forall k  \tag{16}\\
\mathrm{X}_{1 \mathrm{i}}, \mathrm{X}_{2 \mathrm{jk}} \text { o } 0 \forall \mathrm{i} . \mathrm{j}, \mathrm{k} \tag{17}
\end{gather*}
$$

Where:
$\mathrm{X}_{1 \mathrm{i}}=$ level of implementation of long-term measure i ,
$X_{2 j k}=$ level of implementation of short-term measure $j$ in shortage event $k$,
$c_{1 i}=$ cost of long-term measure i (annualized),
$\mathrm{c}_{2 \mathrm{jk}}=$ annual cost of short-term mesure j in event k ,
$q_{1 i}=$ unit annual water saved by long-term a measure $i$,
$\mathrm{q}_{2 \mathrm{jk}}=$ unit annual water saved by short-term measure j during shortage k ,
$\mathrm{u}_{1 \mathrm{i}}=$ upper limit of long-term conservation measure i ,
$\mathrm{u}_{2 \mathrm{jk}}=$ upper limit of short-term conservation measure j under event k ,
$\mathrm{p}_{\mathrm{Q} \mathrm{k}}=$ retail price of water for each shortage event k ,
$\mathrm{Q}_{\mathrm{k}}=$ water use for each shortage event k ,
$P_{k}=$ probability of occurrence of shortage event $k$,
$r_{k}=$ ration amount for event $k$,
$d_{k}=$ full service demand/water use for event $k$,
$\mathrm{m}=$ number of shortage levels (events),
$n_{l}=$ number of long-term measures,
$n_{2}=$ number of short-term measures,
The total expected value cost of household water management efforts is represented in Equation 12, including the costs for long-term water conservation measures ( $\mathrm{X}_{1 \mathrm{i}}$ ), the cost of short-term water conservation efforts ( $\mathrm{X}_{2 \mathrm{jk}}$ ) for each of the $m$ shortage events, and the payments for water use (the water bill) for each event $k$, allowing the price of water $p_{Q k}$ to vary with shortage event $k$.

Each long and short term water conservation measure has a maximum level of implementation as shown in Equations 13 and 14. The lower limit of application (no application at all) is represented by the nonnegativity constraint in Equation 17. If more detailed water use data is available this limits could also be a function of the water use affected by each conservation measure (i.e. $10 \%$ of water used for toilet flushing). More elaborate equations representing the upper bounds of conservation options exist where long-term
conservation options (such as xeriscaping) affect the effectiveness of short-term conservation (such as restricting lawn watering) and viceversa (Lund 1995; Wilchfort and Lund 1997).

Equation 15 calculates the quantity of water purchased for each event $k$ (full service water use $\mathrm{d}_{\mathrm{k}}$ minus the effectiveness of water conservation efforts). Several authors (Dziegielewski et al 1993, Vickers 2001) have suggested that considering effectiveness as a percentage reduction of the water use relevant to each option might be more appropriate. Indeed if a household uses twice as much water as another one for clothes washing, it is likely that a more water-efficient clothes-washer will conserve more water in the first case. Equation 16 defines the water availability for each event k and requires that total water use not exceed a given water ration $r_{k}$.

Often it is difficult to reduce the water bill to a unit cost of water times the quantity consumed $\left(\mathrm{p}_{\mathrm{Qk}} \mathrm{Q}_{\mathrm{k}}\right)$ as required in this formulation. Many utilities charge sewer rates and other charges based on water use so the model's price parameter should be adjusted accordingly. The fixed part of the bill that utilities commonly charge can be ignored. If the consumer conserves water he will benefit by paying less for water use and consequent wastewater charges, but will still have to pay the fixed part of the water bill.
Specific rate structures can be accommodated in the model. If consumers are operating under an increasing (convex) block rate structure this can be accommodated by separation of $\mathrm{Q}_{\mathrm{k}}$ into pieces for each block. A decreasing (concave) block rate structure can similarly be accommodated, with the addition of suitable binary integer variables as an integer-linear program.

If we are concerned about the effect of water conservation on water utility revenues, a constraint can be set on net utility revenues by making the price parameter $p_{Q k}$ into a decision variable $\mathbf{p}_{Q k}$ and adding a constraint on revenue, $\sum_{k=1}^{m} P_{k} \mathrm{pQk} Q_{k}=$ desired revenue. Variable drought pricing can be examined by replacing the price parameter $\mathbf{p}_{Q k}$ with a decision variable and a revenue constraint and even a constraint limiting revenue variability. In both these cases, the linear program above becomes non-linear. The same approach applies to a block rate structure.

A more difficult aspect to integrate in the model is to know exactly what price variable consumers respond to (if any at all). An abundant (and controversial) body of literature has considered how consumers respond to the price of water (Howe and Linaweaver 1967, Wong 1972, Young 1973, Nordin 1976, Billings and Agthe 1980, Howe 1982, Nieswiadomy and Molina 1991). Yet, price and technology (conservation options) are only some of a complex set of variables that affect the consumer's demand for water. Household's characteristics, location or income are some others. One might want to capture these effects by restating the problem using different coefficients for different locations and classes of household. This possibility will be discussed at length later in this paper.

In any case, the cost coefficients must be annualized and include both financial and perceived costs. For toilet retrofitting it is likely that provided that low flush toilets work correctly the consumer will see no inconvenience in the change. However some consumers might prefer to use a standard rather than a low-flow showerhead and therefore implementation of such and option will entail both financial and inconvenience cost. It is therefore assumed that the model's cost coefficients are estimates of consumer's willingness to pay to avoid implementing specific water conservation measures (as determined by financial and contingent valuation or other valuation techniques).

The approach and method is illustrated by a series of examples. Integer constraints are neglected in this problem, since many of the measures could be partially implemented or implemented only by a fraction of the households (Lund 1995). For simplicity the retail price of water and the cost and effectiveness of conservation measures will be fixed over the range of shortage events for these examples.

A single-family household is assumed to have the following long and short term conservation options available : long-term : (1) retrofiting toilet from 3.5 gallons ( 13.25 liters) per flush (gpf) to 1.6 gpf (6.1lpf)- TR-, (2) Xeriscape 1, implement xeriscaping in a part of their garden-X1-, (3) Xeriscape 2, implement xeriscaping in all of their garden area -X1-. Short-term options: (1) TD installing a
displacement device in the toilet, (2) DR stop watering the lawn but not the shrubs and trees, (3) DS stop watering the shrubs and trees in the garden. The costs and water conservation effectiveness for each water conservation measure are given in Table 1 and have values representative of the literature (Schulman and Berk, 1994).

## Formulation for a Single Household Without Rationing

We first examine the effects of price on a single household's water use in the absence of rationing. This can be simulated by setting $\mathrm{m}=1$ in the problem formulation (only one event) and eliminating constraint 16 (the only rationing event has no rationing). The linear program is then solved for different price levels. As the $\mathrm{p}_{\mathrm{Qk}}$ coefficient increases in equation (12) the optimal mix of conservation measures will change and reduce the quantity of water used $\left(\mathrm{Q}_{\mathrm{k}}\right)$. Results are represented in Figure 1 as a thick solid line.

As illustrated in Figure 1, the common sense result is that as the retail price of water increases, households face an incentive to implement more conservation measures and reduce their water use. When faced with an increase in water price the household has to decide whether to continue consuming as much water as before (and pay a higher water bill) or implement conservation measures to reduce their water use at some cost.
When water is inexpensive, the household has little incentive to incur a cost (in money, time etc) to reduce its water use. At some point the price of water gets high enough that a given conservation option becomes cost-effective and is implemented; then the reduction in the water bill from conservation exceeds the costsfinancial and perceived- of conservation. For each of the six conservation measures, these points are given by steps in the water demand curve. At those turning points the price of water equals the cost per unit of water saved by implementing a given conservation measure (in $\$ / 1000$ gallons). Conservation measures will be implemented in order of their relative cost-effectiveness. For example installing a toilet dam or displacement device (bricks) can save a reasonable amount of water, has virtually no financial cost, provides the same flushing performance as before and does not take a lot of time (perceived costs). On the other hand to stop watering garden shrubs can entail great financial expenses for replacement and may have large perceived cost in lost aesthetic benefits. Yet water might be so expensive that having a garden is an unaffordable luxury (i.e., in some arid countries).

A household's demand for conservation measures can be derived indirectly from the solution to this LP problem. As price increases so does use of conservation measures. In this LP formulation, once the price hits the level where a specific conservation measure becomes cost-effective, it is fully implemented. In a way we could say that the household is substituting water use for conservation measures. This tradeoff between conservation costs and savings on the water bill is represented in the objective function. Therefore the objective function provides the least costly way of covering the household's water needs for a given retail price of water.

Under the assumption that the model's cost coefficients are estimates of household's willingness-to-pay to avoid implementing specific water conservation measures and that they capture the household's utility preferences, this means that the objective function minimizes the loss of consumer surplus (CS) (well-being or welfare) due to an increase in the retail price of water (where free water serves as the baseline). A price increase will lead to some loss in consumer surplus by: 1) paying more for the water the household uses and 2) paying to reduce water use by implementing conservation measures. The optimization program minimizes the sum of those losses, and hence minimizes the loss of consumer surplus.

Graphically, the CS is given by the area between the demand curve and a horizontal line drawn at a given price level. This area is maximal when price is zero and will diminish as price increases. The change (loss) in CS due to an increase in price from zero to ' $p$ ' is therefore given by the shaded area in Figure 2. This figure also shows what the demand curve for water would look like if no conservation measures where implemented. In this case the loss of CS due to an increase in price would be higher, confirming that implementing conservation measures, though costly, contributes to minimizing the loss in CS. Table 2 compares the loss in CS calculated using the objective function and using the graphical method. A conversion factor has to be applied to the area calculations to translate them to the same units as the objective function (\$/year).The results show the correspondence between the objective function and the
loss of CS. The slight discrepancies that arise are due to discretization errors that influence the calculation of the areas under the demand curve.

## Formulation for a Class of Consumers Without Rationing

Even within a class of customers, responding to the same prices, individual users will have varying perceived conservation implementation costs (c), effectiveness of water conservation options (q) and normal levels of water use (d) (Lund 1995). The types of conservation measures available to each household also may vary.

Variability in water demand is widely recognized even for households with similar characteristics. Household occupancy rates can vary, but more importantly, lifestyle characteristics and water use patterns vary between households in ways that are not well understood. For example, studies have shown that toilet flushing by males and females have different patterns and there is a widespread variability in the duration of showers (Vickers, 2001). Such factors are likely to affect the household's total water use and can enhance or reduce the effectiveness of (lets say) a low flow showerhead.

The cost of implementing a given conservation measure can be decomposed into a financial expenditure that accounts for the cost of materials and installation of conservation devices and a 'perceived'cost accounting for all other non market values such as the consumer's preference for one or another conservation measure, the value of the time the consumer will spend implementing a conservation option, resistance to bother with conserving water, inconveniences of taking shorter showers or flushing the toilet less often etc.

Financial cost may seem easier to estimate; yet the choice of conservation devices for a specific water use is rapidly increasing and costs vary widely. Vickers (2001) reports ranges of 75-125 to $650 \$$ for low flush toilets. The perceived costs associated with the implementation of a specific conservation measure cannot be easily ascertained. The CUWA (1994) contingent valuation study reported that the major cost of conservation was having to spend time and effort to monitor water use. It should be expected that the range of variability in perceived costs would be larger than for financial costs. Contingent valuation studies of willingness-to-pay to avoid implementation of a specific conservation measure would be required to gain insight on this matter. Such specific contingent valuation studies should represent an improvement over the more general and vague direct contingent valuation studies of willingness-to-pay to avoid probabilistic shortages since they would only solicit evaluations of direct water conserving actions that consumers might take in response to shortages (Lund 1995). To the author's knowledge no such study has been undertaken up to date. As noted by Lund (1995) a lower bound for the cost coefficients could be estimated by considering only financial costs.

Though Lund 1995 focused his discussion on the difficulty of estimating actual and 'perceived'costs of conservation measures, great variability also exists in the effectiveness of conservation measures. In their manual for evaluating urban water conservation programs Dziegielewski et al. (1993) note: "An underlying shortfall involving the implementation of water conservation as a demand management tool is the lack of reliable knowledge of actual water savings, market penetration and interaction effects between conservation measures". Further: "For many conservation measures data are non existent or non reliable" (Dziegielewski et al. 1993). Indeed a review of the abundant studies on this subject shows that for the same type of conservation measure reported, effectiveness values can range from simple to double or triple (Dziegielewski et al. 1993, Vickers 2001, CUWCC 2000, Maddaus 1987). All these sources agree that the effectiveness of a conservation measure will depend on the appliances being replaced, on the new ones installed (i.e., from a 5 gpflush toilet to a 1.6 or a 1 gpf ), on technical considerations such as the water pressure of the service area and on behavioral aspects of water use patterns.

Variability in the effectiveness of water conservation measures should be considered because of its effect on the relative cost effectiveness of the different conservation options (the ratio of cost to conservation; $\$ /$ cost per unit water use reduction). Some models that calculate the effectiveness of conservation measures exist (Walski 1984) but they are data intensive and account only for variability in engineering parameters. This makes them of limited usefulness.

To give some intuition about the effects of parameter variability in the models results, the no rationing case discussed above is solved for different sets of parameter values (costs, effectiveness, full service demand) and results are presented in Figure 1. These demand curves are not only shifted along the X axis due to changes in full service demand, but the steps that mark implementation of a particular conservation measure also vary widely. The average curve that results from solving the LP using the average of the parameters of the other runs features a full service demand similar to the original case yet the pattern of implementation of conservation measures is very different.

A more rigorous approach would be to consider the model's parameters probabilistic and to assign them a probability distribution based on the best available knowledge of household variability in model parameters. Monte Carlo simulations can then be used to examine the importance of this variability within a class of customers. By solving the LP model for a set of random parameters using Monte Carlo simulations, confidence intervals can be calculated for the loss of consumer surplus and water use.

The Monte Carlo approach has the advantage of accounting for variability in the model's parameters with minimum information requirements. Using maximum entropy (ME) estimation we can assign a probability distribution to a specific parameter with as little as one observation (truncated exponential distribution). Generally more information is likely to be available from the literature or can be generated relatively easily. Water utilities can provide extensive information about demand/water use levels that can be used to fit a probability distribution. Information about lower and upper bounds for the financial costs can be drawn from the literature. Information about perceived costs can be obtained by conducting contingent valuation studies and should generate enough information (mean, standard deviation) to estimate probability distributions. New developments in measuring and modeling end uses of water and determining the effectiveness of conservation efforts using flow trace analysis techniques make it now possible to gather empirical evidence of the effectiveness of specific conservation measures at the local level quickly and at a relatively low expense (Weber 1993, Mayer, DeOreo et al 1999, Trace Wizard ${ }^{\ominus}$ ). This technique also can help in studying the interaction between different conservation measures. If we wished to account for such interactions the same Monte Carlo approach would apply.

For this work we assumed the model parameters to be normally distributed with known mean and standard deviation as shown in Table 1.

Figure 3 shows the results for the 'No ration' case for different numbers of Monte Carlo iterations. By accounting for variability in parameters, the points at which different conservation measures become cost effective shift to give a smooth aggregate demand curve. The water use curve stabilizes after a hundred iterations. This aggregate curve can be used to estimate the price elasticity of water within different price ranges as shown in Table 3. These elasticity values should not be taken literally because the model's parameters used for their estimation are not empirically based, yet their relative values and variation along the demand curve are consistent with what the theory of residential water use predicts. The almost perfectly inelastic response that we observe in the low ranges of water price $(0-6 \$ / 1000 \mathrm{~g})$ can be attributed to the limited range of low cost conservation options considered for this model. It can also be argued that when the price of water is low, the effects of a price increase on the household's income will be small and lead to very little change in water use. For such low price levels, the effort of bothering about water use monitoring esceeds any foreseeable benefit. As the price of water gets higher ( $6-23 \$ / 1000 \mathrm{~g}$ ) water use becomes more responsive to changes in price water until water becomes essential (water used for drinking, cooking), so water use cannot be curtailed any further and barely responds to price increases.

## Formulation for a Single Household With Rationing

Water demands in arid and semi-arid regions are significantly affected by perceptions of water availability. Households expecting frequent episodic reductions in water availability (water rationing) are likely to change their water-related investments in landscaping and plumbing to reduce normal water use and allow them to more easily reduce water demands/use during periods of shortage.

The response of a single household in the presence of water rationing can be studied by solving the LP problem for several shortage probability distributions $\left(\mathrm{P}_{\mathrm{k}}\right)$. The solution output provides the mix of conservation measures that minimizes total cost to the consumer while ensuring that water use in each of the shortage events $\left(\mathrm{Q}_{\mathrm{k}}\right)$ is within the ration for that event $\left(\mathrm{r}_{\mathrm{k}}\right)$.

To illustrate the most relevant aspects of the rationing case let's focus on a very simple case with a $50 \%$ probability of no shortage -event1- and a $50 \%$ probability of a $10 \%$ shortage -event 2 - (a mandatory reduction of $10 \%$ compared to the full service water use). The relevant results for this case appear in Table 4.

Focusing on water use for event 2 we can see that for low price levels, demand is driven by the water ration. Because rationing is mandatory there is a need to implement conservation measures at price levels where these are not cost-effective. For those prices no conservation would have been implemented in the absence of rationing. Within this range of prices, rationing imposes higher total costs on the household (column V on Table 4). However, as price increases the effect of the ration is diluted because the implementation of conservation measures becomes cost-effective even in the absence of rationing. Conserving water is now (at those water prices) economically interesting for the expected value costminimizing household; the effect of the ration is no longer felt and the water use curves for all events converge to the 'no ration' water use curve (see Figure 5).

At each price level the higher costs that rationing imposes on the household result in an additional loss of consumer surplus (compared with the 'no rationing' case) that is given by the difference in the value of the objective function. This 'extra loss' can alternatively be calculated as the area between the demand curves with and without rationing. Note that for the shortage probability distribution we are considering the water use curve for Event 1 (no shortage) is superimposed to the original 'no ration' curve. Since the level of water use specified by Event 2 only occurs with a certain probability ( 0.5 ), this factor has to be included in the area calculation (column IX in table 4 ) ${ }^{1}$. Table 4 shows that the two approaches are equivalent and clearly establishes discretization as the source of error. The divergence observed between columns VII and XI can be traced to the points where there is a shift on the demand curve. The same result can be obtained using the expected value curve for this shortage probability distribution (the weighted average of water use for each event).

This 'extra loss' provides an estimate of the consumer's maximum willingness to pay (WTP) to avoid a particular shortage distribution hence indirectly establishing the value of water supply reliability for the household. If by paying $\$ Y$ those shortages could be avoided, the household should be willing to incur that cost as long as it is less than the loss of CS imposed by a particular shortage distribution ( $\mathrm{P}_{\mathrm{k}}$ ). The household's WTP to avoid a specific shortage probability distribution decreases as the retail price of water increases (column VII in Table 4) because higher price levels provide an economic incentive to implement conservation measures and reduce water use voluntarily, thus neutralizing the effect of mandatory rationing. When water is very expensive, households would rather reduce their water use than keep their current consumption level and pay extremely high water bills. These results are consistent with findings from contingent valuation studies (Griffin and Mjelde, 2000).

In practice there is some level of unreliability associated with every water supply system, and therefore in most cases, the relevant analysis would be for the incremental value of moving from one distribution of unreliability to another (CUWA, 1994). This can be simulated in this model by comparing two probability distributions that entail different levels of unreliability (where not all shortages are avoided).

For the analysis of WTP to avoid shortage, two aspects of the reliability of a water supply system are relevant. The reliability of a system is characterized by 1) the probability of having no shortages (the higher the better) and by 2) the levels and probabilities of shortage associated with given levels of unreliability. A system that has $70 \%$ chance of having no shortages and $30 \%$ probability of a $10 \%$ shortage is potentially

[^0]less damaging than one that has the same probability of no shortages but where there is a $30 \%$ probability of a $20 \%$ shortage and should therefore be preferred.

This is illustrated by comparing the consumer's WTP to avoid shortage probability distributions A, B and C described in Table 5. Though distributions B and C appear to be more reliable than A, if we consider not only the probability of experiencing no shortage but the whole probability distribution, they turn out to be probabilistically equivalent. The three of them have an expected value shortage of 8 gallons per day (gpd). However, these shortage probability distributions impose widely different costs on the consumer, as is shown by the different WTP to avoid them (Figure 5). More severe shortages encourage more conservation efforts (and costs), so the consumer is willing to pay more to avoid these events. To cope with the shortages imposed by distribution C, long-term (LT) conservation options have to be implemented, boosting the consumer's WTP to avoid this situation. The cost of LT options is somewhat a fixed cost as opposed to the short-term options that are only paid for when there is actually a shortage. Therefore the optimal solution would often be to implement short-term (ST) options first and turn to LT options only if the former do not achieve the levels of water conservation needed. This very simple example provides grounding to the commonly-accepted notion that frequent small shortages should be preferred to big but infrequent shortages and is consistent with results reported by other authors (CUWA 1994, Koss and Khawaja 2001). This notion is the main justification for hedging in reservoir management.

As discussed before, WTP to avoid rationing decreases as the price of water increases and the expected value (EV) demand curves converge with the 'no rationing' case (see Figure 6). Note also that the EV demand corresponding to distributions B and C are not perfect step functions as one would expect in a linear optimization model. This is the result of a shift from short-term conservation to long-term conservation in the most extreme event as price rises. When price reaches a certain level (in this case around $11 \$ / 1000$ gallons) those ST measures that where used because of the water availability constraint but where not cost-effective (Dry Shrubs) are replaced by permanent measures (Toilet Retrofitting or Xeriscape II). The benefits of paying the high cost of Dry Shrubs only during shortages are less than the benefits of extra water conservation made possible by LT options (reducing both the water bill and the need for the Dry Shrubs measure).

This example illustrates the relevance of considering the whole probability distribution of shortages in estimating the costs imposed by shortage and hence the value placed in reliability. The distribution of unreliability between different shortage levels is an aspect of the consumer's WTP that contingent valuation studies of reliability cannot fully grasp because they only consider one shortage level at a time.

To further illustrate the interaction between the shortage probability distribution and the costumer's WTP to avoid those shortages lets analyze the examples provided in Tables 6,7 and 8.

Table 6 shows a series of shortage probability distributions where the frequency of small shortages gradually increases making each distribution less reliable than the previous. As seen from Figure 7, the WTP to avoid shortages increases linearly with the EV of shortage and decreases (also linearly) with price. Therefore, for small shortages, the cost of moving from no shortage to $1 \%$ shortage is the same as the cost of moving from $4 \%$ shortage to $5 \%$ shortage. In many cases small shortages can be handled by implementing only short-term conservation measures, so increasingly frequent shortages will impose costs that are directly proportional to the frequency of occurrence.

In contrast, for very severe shortages (Table 7) WTP does not respond linearly to increases in the level of shortage (Figure 8). In this case the cost (and hence the consumer's WTP) of moving from no shortage to $10 \%$ shortage is bigger than cost of going from 20 to $30 \%$ shortage. This non-linear response is triggered by the need to implement LT conservation and incur fixed costs to cope with the shortage. When moving from a situation with no shortage to a distribution such as $I$, the household is forced to implement long-term conservation measures. After that, if the shortage becomes more frequent (distributions $J$ and $K$ ), LT options will already be in place, but because shortages are more frequent short-term conservation will be paid for more often. This however represents a relatively minor cost when compared to the fixed expenditure for long-term options. As price increases there is decay in this non-linear response because LT conservation becomes cost-effective and is implemented regardless of the level of rationing. For high levels
of shortage there is high value associated with avoiding severe events and there is comparatively less value from reducing the probability of those events. Severe events make it necessary to implement LT conservation that is paid for independently of the frequency of occurrence of the event. Figure 8 b shows the very drastic reduction in EV water use imposed by distribution $I$ (with respect to the non-ration case) compared to the more mild curtailment in demand needed to absorb the effects of increased frequency of shortage.

Yet to emphasize the complex relationship between the WTP and the shortage probability distribution the example in Table 8 illustrates that for a particular consumer there is no WTP to go from distribution $L$ to the more benign $M$, which represents a $1 \%$ reduction in the EV of shortage! This is because the possibility of having a very severe event ( $40 \%$ shortage) makes LT conservation measures necessary. For this particular set of parameters these LT conservation measures alone are more than enough to cope with small shortages of $10 \%$ and no extra short-term measures are required. Therefore changing the probability of having a small $10 \%$ shortage has no effect on costs. LT conservation will have to be paid for in any case and there would be no need to pay for short-term conservation more frequently.

## Formulation for a Class of Costumers with Rationing

The analysis presented above can be extended to a class of costumers. We illustrate the more relevant aspects for this formulation by studying distribution $M$ in the context of a class of costumers. Figure 9 presents the average demand curves for a class of customers confronted by shortage probability distribution $M$. The demand curve for event 1 (no shortage) clearly illustrates the permanent effects of long-term conservation (reducing water use for all events). The curves corresponding to the no ration case, with both deterministic and probabilistic parameters, are provided for comparison. By imposing the need for conservation measures this rationing scheme reduces the consumer's price-responsiveness for the intermediate price ranges.

Statistical analysis of the Monte Carlo simulation results of household response to rationing can provide useful information for the water utility and help gain understanding about the structure of demands for water and conservation measures within a particular class of customers. Figure 10 provides the derived demand for long-term conservation measures and shows that as the price of water increases so does the demand for conservation measures. Figure 11 shows how implementation of a specific conservation measure (installing a toilet dam) responds to price for each of the five shortage events.
Similar demand analyses can be done using the reduced costs of non-implemented conservation measures, which provides information about the reduction in cost necessary for a conservation measure to be implemented. Figure 12 shows the high degree of variability in the reduced cost of the Xeriscape II option from the Monte Carlo results. This figure can be used to answer questions such as how the cost of a given measure affects its market penetration in the user population.
Reduced cost of different conservation measures also can be used to compare the relative subsidies/refunds needed to make customers shift from one conservation measure to another to better accommodate to water supply conditions. Some combinations of short-term measures might be more beneficial than others from the water utility's standpoint. For example a water utility might want to encourage costumers to shift from Toilet Dams to Dry Lawn in order to reduce peak weekend demand on the system.
Curves such as Figures 10 to 12 can be generated for each conservation measure, providing the water utility with information for the design of a water conservation campaign.

Perhaps the most relevant information concerns the range of variability in willingness to pay to avoid shortage within a class of costumers. Figure 13 shows that for any given retail price of water $(\$ 6 / 1000 \mathrm{~g}$ in this case) some costumers are willing to pay more than $\$ 947$ per year to avoid the shortage probability distribution $M$ while others are only willing to pay $\$ 167$. These differences in WTP are derived from the structure of preferences of each costumer (costs, full service demand, effectiveness of the conservation options) and should be considered when planning to ask the costumers for funds to increase the reliability of the system.
The implications of variability in WTP are very interesting. First, it shows that many people would be willing to pay a premium each bill for a preferential level of reliability. This would be like "buying insurance for protection during shortage" (Flory and Panella 1994). The funds generated by the premium
rates to finance long term conservation or reliable supply options (desalination plants, dry year contracts with the agricultural sector etc). One could envision a system of contracts that offered increased (higher than standard) reliability for customers that are willing to pay for it and discount rates for lower reliability (i.e., interruptible service during drought events). Such "service options" or priority pricing is already in operation in the electric industry (Flory and Panella 1994). In the water industry such a pricing scheme would probably be easier to implement by integrating different water use sectors. Industrial users or some emergency service facilities could be offered higher reliability by establishing different levels of priority during drought but at a higher price.
Other possibility would be establishing a water market where water rations could be transferred from the low value users to those who value them the most. Shortages would be allocated to those customers with lower costs from rationing, creating some gains in economic efficiency. Yet, from an operational standpoint any of these systems might be difficult to handle and important social and equity issues would need to be studied.

## Formulation for a Water Service Area

Price and technology (conservation options) are only some of a complex set of variables that affect the consumer's demand for water. Household characteristics, location or income are some others (Howe 1967, Berk 1993, Dziegielewski et al. 1993, Griffin and Mjelde 2000). The approach can be extended to a whole water service area by restating the problem using different coefficients for different locations and classes of consumers. The range of conservation options available could also be adapted as needed. The service area can be discretized into groups as homogeneous as possible and the partial results can be aggregated to study the global effect of different parameters and shortage distributions on consumers' willingness-to-pay. Alternatively we could account for differences in the water service area by increasing the degree of variability in model parameters.

## $\underline{\text { Discussion }}$

The approach presented in this paper enables the derivation of water demand curves that are consistent with our current understanding about residential water use and management. Demand curves for different shortage events can be generated easily and used to study the effects of rationing on customer's water use. This approach should provide valuable information to understand and perhaps predict the effects of the retail water price and the interaction of long-term and short-term conservation measures in water demands. These interactions are important for water utilities because long-term conservation measures can severely affect water utility revenues and operation. Implementation of long-term conservation measures entails significant (and permanent) reductions in water use that reduce utility revenue. This would lead to price increases likely to make water conservation even more attractive to costumers. If appropriately planned, it also can lead to the implementation of emergency punitive water price, agreements to sell the surplus of saved water or to expand the utility's operations to new costumers.

The main contribution of this model is to provide estimates of the consumer's WTP for changes in reliability in ways consistent with economic theory and that eliminates the inconsistent results sometimes obtained in contingent valuation studies (CUWA, 1994). Another advantage over contingent valuation studies is its capability to examine a complete shortage probability distribution and the ability to account for price effects.

However, the mathematical formulation of the problem is unable to capture the empirically observed asymmetries in willingness to pay for increased reliability and willingness to accept for decreased reliability. As Griffin and Mjelde (2000) put it: "the change in value for an increase in reliability can be expected to be less, in absolute value, than the change in value for an equivalent reliability fall ". This asymmetry of increase and decrease in reliability may be due to the fixity of durables (Griffin and Mjelde 2000) not accounted for in the optimization model.

It is important to emphasize that "the willingness to pay interpretation of the results rests on the assumption that the model's costs coefficients are estimates of the willingness to pay of customers to avoid
implementing specific water conservation measures" Lund (1995) and that the household optimization process is costless.

The Monte Carlo probabilistic optimization approach presented has the advantage of providing information about consumer's variability in willingness to pay as well as some relevant information for exploring innovative management options in the residential sector such as 'priority pricing' or urban water markets. This approach could also contribute to the design of cost-effective water conservation programs by using the information provided by the model (market penetration estimates, implementations for different events and prices, reduced costs for each measure, sensitivity analysis) to adapt conservation programs to the characteristics of each group of customers. The model can be used, for example, to estimate the effects of a water conservation campaign launched by a local water agency. This can be done by fixing to the extent possible the material costs of a conservation measure and letting the variation involve only the " perceived or inconvenience costs" to people of implementing it. Inconvenience costs would basically involve time spent implementing the conservation measure, which can be reduced if the marketing strategy is appropriate. The resulting reduction in the model's cost coefficients is however very difficult to determine.

The approach should also provide a way of integrating retail water price into studies of the economic impact of shortage or water resources planning models that explicitly consider urban shortage management (Jenkins and Lund 2000, Hoagland 1998).

It could be argued that the model's parameters will vary for different events. Indeed it is likely that the effectiveness of short-term conservation measures will increase with the severity of the shortage event. Similarly the cost coefficients might be reduced in very extreme events because increased customer concern about drought is likely to drive down perceived costs associated with conservation. Appropriate long-term monitoring of conservation effectiveness and contingent valuation studies might provide estimates on this.

There is a long-standing literature regarding behavioral problems with cost-minimizing expected-value decision-making assumed by this method (Khanemann and Tversky, 1979). Problems with the linear programming formulation are discussed in Lund (1995).

Finally the formulation presented constrains the household to meet the ration level for all events independently of their probability of occurrence. This constraint results in an extremely risk-adverse behavior by the household because even a very small probability of occurrence (i.e. $0.1 \%$ ) of a shortage event will force the household to reduce water use and incur conservation costs. Further it assumes that households can perfectly monitor their water use and that the water agency can cut water supply to the household when it has consumed its ration. An alternative approach would be to introduce penalties for exceeding the ration level, perhaps as excess use prices.

The approach taken here relies on micro-scale modeling of consumer demand decisions. This requires a great deal of model calibration and computational effort for demand studies of classes of consumers and service area studies. An alternative approach would be to use a more semi-empirical approach such as positive mathematical programming (Howitt 1995; Paris and Howitt 1998). Here, the quadratic matrix in the objective function of a quadratic program or two-stage quadratic program might be calibrated based on aggregate consumer decisions and water uses, either by customer class or by water service area.

## Conclusions

The two-stage linear programming approach presented by Lund (1995) is extended to include the retail price of water and to allow for variability in the model's parameters for an urban population. This allows for an easy derivation of demand curves for water (with or without rationing) that are consistent with the current knowledge about residential water demands. Derived demands for conservation measures also can be obtained using this approach. The model provides information about the interaction between long and short-term conservation, the retail price of water and water use in the residential sector that should prove valuable for water agencies to design water conservation programs. Under the assumptions that the model's cost coefficients represent the consumer's willingness-to-pay to avoid implementation of specific
conservation options and that the customer behaves rationally (expected value cost minimization behavior), probabilistic estimates of WTP to avoid specific shortage probability distributions are obtained. These estimates are derived in a way that is consistent with economic theory.

## Tables:

Table 1: Parameter values

|  |  | Conservation <br> Effectiveness <br> (gpd) | Cost/Effectiveness <br> (\$/1000 gallons) |
| :--- | :---: | :---: | :---: |
| Long-term options | $150(30)$ | $30(6)$ | 13.70 |
| Toilet Retrofit | $500(100)$ | $100(20)$ | 13.70 |
| Xeriscape I | $1000(200)$ | $150(30)$ | 18.26 |
| Xeriscape II |  |  |  |
| Short-term options | $5(1)$ | $2(0.4)$ | 6.85 |
| Toilet dam | $400(80)$ | $100(20)$ | 10.96 |
| Dry Lawn | $1200(240)$ | $70(14)$ | 46.97 |
| Dry Shrubs |  |  |  |

The numbers in brackets represent the standard deviation used for the Monte Carlo simulations

Table 2:Calculation of the loss of Consumer Surplus

| I | II | III | IV | V | VI |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Price levels <br> $(\$ / 1000 \mathrm{~g})$ | Objective <br> Function value <br> $(\$ /$ year $)$ | Incremental area <br> under demand <br> curve (\$/year) | Cumulative area <br> under demand <br> curve | Difference <br> (II-IV) | Difference <br> in $\%$ |
| 0.00 | 0.0 |  |  |  |  |
| 2.00 | 584.0 | 584.0 | 584.0 | 0 | 0 |
| 2.50 | 730.0 | 146.0 | 730.0 | 0 | 0 |
| 3.00 | 876.0 | 146.0 | 876.0 | 0 | 0 |
| 3.25 | 949.0 | 73.0 | 949.0 | 0 | 0 |
| 3.50 | 1022.0 | 73.0 | 1022.0 | 0 | 0 |
| 3.75 | 1095.0 | 73.0 | 1095.0 | 0 | 0 |
| 4.00 | 1168.0 | 73.0 | 1168.0 | 0 | 0 |
| 6.84 | 1997.3 | 829.3 | 1997.3 | 0 | 0 |
| 6.85 | 2000.2 | 2.9 | 2000.2 | -0.014 | 0 |
| 10.95 | 3191.4 | 1191.2 | 3191.4 | -0.014 | 0 |
| 10.96 | 3194.3 | 2.5 | 3193.9 | -0.339 | -0.011 |
| 11.00 | 3204.4 | 10.2 | 3204.1 | -0.339 | -0.011 |
| 13.69 | 3887.8 | 683.4 | 3887.5 | -0.339 | -0.009 |
| 13.70 | 3890.3 | 2.0 | 3889.4 | -0.843 | -0.022 |
| 15.00 | 4144.6 | 254.3 | 4143.8 | -0.843 | -0.020 |
| 16.00 | 4340.2 | 195.6 | 4339.4 | -0.843 | -0.019 |
| 18.26 | 4782.4 | 442.1 | 4781.5 | -0.843 | -0.018 |
| 18.27 | 4784.1 | 1.4 | 4783.0 | -1.108 | -0.023 |
| 20.00 | 5027.8 | 243.7 | 5026.7 | -1.108 | -0.022 |
| 25.00 | 5732.3 | 704.5 | 5731.1 | -1.108 | -0.019 |
| 46.96 | 8826.2 | 3093.9 | 8825.1 | -1.108 | -0.013 |
| 46.97 | 8827.5 | 1.2 | 8826.2 | -1.280 | -0.014 |
| 72.00 | 11714.5 | 2887.0 | 11713.2 | -1.280 | -0.011 |

Table3: Price Elasticity Estimates

| Price Range <br> $(\$ / 1000 \mathrm{~g})$ | Price Elasticity <br> Estimate |
| :---: | :---: |
| $0-6$ | 0.0025 |
| $6-8$ | 0.0655 |
| $8-11$ | 0.3081 |
| $11-12.5$ | 0.600 |
| $12.5-16.5$ | 0.6083 |
| $16.5-23$ | 0.474 |
| $23-37$ | 0.1578 |
| $37-50$ | 0.2190 |
| $50-72$ | 0.1452 |

Table 4

| I | II | III | IV | V | VI | VII | VIII | IX | X | XI |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \text { Price } \\ (\$ / 1000 \mathrm{gal}) \end{gathered}$ | EV Water Use | Event 1 | Event 2 | $\begin{gathered} \text { LOCS (\$/year) } \\ \text { or ObjFunt } \\ \hline \end{gathered}$ | Obj function in NO Ration case | Dif in obj funct (WTP) | Cummulative area between curves at each price | Correct by probability of event | Difference | Differenc in \% |
| 0 | 760 | 800 | 720 | 157.0 | 0 | 157.0 | 314.0 | 157.0 | 0.0 | 0.0 |
| 2.00 | 760 | 800 | 720 | 711.8 | 584.0 | 127.8 | 255.6 | 127.8 | 0.0 | 0.0 |
| 2.50 | 760 | 800 | 720 | 850.5 | 730.0 | 120.5 | 241.0 | 120.5 | 0.0 | 0.0 |
| 3.00 | 760 | 800 | 720 | 989.2 | 876.0 | 113.2 | 226.4 | 113.2 | 0.0 | 0.0 |
| 3.25 | 760 | 800 | 720 | 1058.6 | 949.0 | 109.6 | 219.1 | 109.5 | 0.0 | 0.0 |
| 3.50 | 760 | 800 | 720 | 1127.9 | 1022.0 | 105.9 | 211.8 | 105.9 | 0.0 | 0.0 |
| 3.75 | 760 | 800 | 720 | 1197.3 | 1095.0 | 102.3 | 204.5 | 102.2 | 0.0 | 0.0 |
| 4.00 | 760 | 800 | 720 | 1266.6 | 1168.0 | 98.6 | 197.2 | 98.6 | 0.0 | 0.0 |
| 6.84 | 760 | 800 | 720 | 2054.4 | 1997.3 | 57.1 | 114.0 | 57.0 | 0.1 | 0.2 |
| 6.85 | 758 | 796 | 720 | 2057.2 | 2000.2 | 57.0 | 114.0 | 57.0 | 0.0 | 0.0 |
| 10.95 | 758 | 796 | 720 | 3191.5 | 3191.4 | 0.1 | 0.0 | 0.0 | 0.1 | 100.0 |
| 10.96 | 696 | 696 | 696 | 3194.3 | 3194.3 | 0.0 |  |  |  |  |
| 11.00 | 696 | 696 | 696 | 3204.4 | 3204.4 | 0.0 |  |  |  |  |
| 13.69 | 696 | 696 | 696 | 3887.8 | 3887.8 | 0.0 |  |  |  |  |
| 13.70 | 536 | 536 | 536 | 3890.3 | 3890.3 | 0.0 |  |  |  |  |
| 15.00 | 536 | 536 | 536 | 4144.6 | 4144.6 | 0.0 |  |  |  |  |
| 16.00 | 536 | 536 | 536 | 4340.2 | 4340.2 | 0.0 |  |  |  |  |
| 18.26 | 536 | 536 | 536 | 4782.4 | 4782.4 | 0.0 |  |  |  |  |
| 18.27 | 386 | 386 | 386 | 4784.1 | 4784.1 | 0.0 |  |  |  |  |
| 20.00 | 386 | 386 | 386 | 5027.8 | 5027.8 | 0.0 |  |  |  |  |
| 25.00 | 386 | 386 | 386 | 5732.3 | 5732.3 | 0.0 |  |  |  |  |
| 46.96 | 386 | 386 | 386 | 8826.2 | 8826.2 | 0.0 |  |  |  |  |
| 46.97 | 316 | 316 | 316 | 8827.5 | 8827.5 | 0.0 |  |  |  |  |
| 72.00 | 316 | 316 | 316 | 11714.5 | 11714.5 | 0.0 |  |  |  |  |

Table 5

|  | Shortage Level <br> (reduction in full service <br> demand) | P Dist A | P Dist B | P Dist C |
| :---: | :---: | :---: | :---: | :---: |
| Event 1 | $0 \%$ | 0.9 | 0.95 | 0.97 |
| Event 2 | $10 \%$ | 0.1 | 0 | 0 |
| Event 3 | $20 \%$ | 0 | 0.05 | 0 |
| Event 4 | $30 \%$ | 0 | 0 | 0.03 |
| Event 5 | $40 \%$ | 0 | 0 | 0 |
| EV shortage (gpd) |  | 8 | 8 | 8 |

Table 6

|  | Shortage Level <br> (reduction in full service <br> demand) | P Dist D | P Dist E | P Dist F | P Dist G | P Dist H |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Event 1 | $0 \%$ | 0.9 | 0.8 | 0.7 | 0.6 | 0.5 |
| Event 2 | $10 \%$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |
| EV shortage (gpd) |  | $8(1 \%)$ | $16(2 \%)$ | $24(3 \%)$ | $32(4 \%)$ | $40(5 \%)$ |


| Table 7 |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | Shortage Level <br> (reduction in full service <br> demand) | P Dist I | P Dist J | P Dist K |
| Event 1 | $0 \%$ | 0.75 | 0.5 | 0.25 |
| Event 2 | $10 \%$ | 0 | 0 | 0 |
| Event 3 | $20 \%$ | 0 | 0 | 0 |
| Event 4 | $30 \%$ | 0 | 0 | 0 |
| Event 5 | $40 \%$ | 0.25 | 0.5 | 0.75 |
| EV shortage (gpd) |  | $80(10 \%)$ | $160(20 \%)$ | $240(30 \%)$ |

Table 8

|  | Shortage Level <br> (reduction in full service <br> demand) | P Dist L | P Dist M |
| :--- | :---: | :---: | :---: |
| Event 1 | $0 \%$ | 0.5 | 0.6 |
| Event 2 | $10 \%$ | 0.2 | 0.1 |
| Event 3 | $20 \%$ | 0.1 | 0.1 |
| Event 4 | $30 \%$ | 0.1 | 0.1 |
| Event 5 | $40 \%$ | 0.1 | 0.1 |
| EV shortage (gpd) |  | $88(11 \%)$ | $80(10 \%)$ |
| LOCS $(\$ /$ year $)$ at $\mathrm{p}=0$ |  | 933 | 933 |

Figures:


Figure 2: Calculation of the Consumer Surplus



Figure 4: Single Household With Rationing $P\{0.5,0.5\}$


Figure 5: WTP to Avoid An Expected Shortage of 8gpd


Figure 6: Variability in Demand Curves for an Expected Shortage of 8gpd




Figure 9: Customer Class Demand Curve M \{0.6,0.1,0.1,0.1,0.1 $\}$




Figure 11: Implementation of Toilet Dams for each event


Figure 12: Variability in the Reduced Cost of Xeriscape II


Figure 13: Variability in WTP to avoid shortage at $p=6$


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[^0]:    ${ }^{1}$ Note that a unit conversion is necessary to translate the area under the curves (in $\$ /$ day) to the same units as the objective function (\$/year).

