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REPRESENTING GROUNDWATER IN WATER MANAGEMENT MODELS — APPLICATIONS IN CALIFORNIA

PIER FINAL PROJECT REPORT

Prepared For:
California Energy Commission
Public Interest Energy Research Program

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December 2008
CEC-500-2008-092-REV

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Commission Contract No. 500-02-004
Commission Work Authorization No: MR-019

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Acknowledgments

The authors wish to thank Thomas Harter, Steven Phillips, David Prudic, Stanley Leakey, and Sandy Williamson for providing assistance and CVRASA1 data and Guido Franco for support in this research. An anonymous reviewer provided comments that substantially improved the report.

This report is based on the first author's Ph.D. thesis of which the acknowledgment statement is reproduced below.

The author would like to thank his advisor Jay R. Lund for his constant encouragement, support and positive outlook over the last four years. Other professors to whom he would like to extend particular thanks for guidance and inspiration include Richard Howitt (committee member), Pete Loucks, Jery Stedinger, Wilfried Brutsaert, Tim Ginn, Miguel Mariño (committee member), Graham Fogg and Philippe Sonnet. He is also grateful for learning from scholars such as Thomas Harter and Marion Jenkins.

Colleagues and friends at UC Davis and beyond who provided intellectual nourishment, companionship and comic relief included Josue Medellin, Marcelo Olivares, David Rosenberg, Manuel Pulido, Guilherme Marques, Tingju Zhu, Ines Ferreira, Randi Field, Stacy Tanaka, Siwa Msangi, Kristiana Hansen, Sarah Null, Kaveh Madani, Sebastian Vicuña and Aaron Blake. The author thanks all students and visitors of their research group for supporting him in his research.

For their guidance and support the author is indebted to his colleagues at the Hydrologic Engineering Center (HEC) including Beth Faber, Christopher Dunn, Robert Carl, John Hickey and Fauwaz Hanbali among others.

The course that brought the author to this dissertation began long ago when mentors, teachers, and family members transmitted their love of learning and scholarship. He is especially grateful for the support of his parents and sisters, and in particular to Prof. P. Harou who successfully brainwashed him into becoming a lot like him. Finally, thank you to the author's inspirational wife Amaya and to their freshly baked Olivia.

Cite as:

Harou , Julien J. and Jay R. Lund. 2008. *Representing Groundwater in Water Management Models—Applications to California.* California Energy Commission, PIER Energy-Related Environmental Research Program. CEC-500-2008-092.

Preface

The Public Interest Energy Research (PIER) Program supports public interest energy research and development that will help improve the quality of life in California by bringing environmentally safe, affordable, and reliable energy services and products to the marketplace.

The PIER Program, managed by the California Energy Commission (Energy Commission), conducts public interest research, development, and demonstration (RD&D) projects to benefit California.

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Representing Groundwater in Water Management Models—Applications to California is the final report for the Adaptation to Climate Change for California's Water System: Societal Layers of Adaptations and Impacts project (Contract Number 500-02-004) conducted by the University of California, Davis. The information from this project contributes to PIER's Energy-Related Environmental Research Program.

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Abstract

This report describes, implements, and compares options for representing groundwater in regional water resource system management models. Different agencies in California use these models to develop water resource plans. This report investigates ways to effectively represent groundwater in such models because this task is generally recognized as in need of improvement. Water management models either simulate or optimize water resource systems. Simulation tools typically represent physical and regulatory constraints, while optimization models relax the regulatory constraints to search for improved management schemes. Three groundwater models are tested in simulation and optimization modes using a two-dimensional groundwater model of the Sacramento Valley (California). Two case studies in California show potential water management and economic benefits of managing surface water and groundwater as a single system (conjunctive use). Groundwater resources could be used to store water and ameliorate climate change impacts on water resources in California.

Keywords: Groundwater modeling, regional water systems, groundwater optimization

Executive Summary

Introduction

Management of water resources in California has the potential to benefit from improved computer modeling tools. Computer models help scientists, resource managers, and others to build integrated, sustainable, and efficient water management plans. Water resource systems models include information from a wide variety of sources, including hydrologic data, existing and potential infrastructure, operational and economic data, and regulatory and policy aspects of existing water management systems. Groundwater resources are often an essential part of hydrologic and water supply systems and are key to creating models that represent real systems. This is particularly true in California, where some areas manage surface water and groundwater together as a single system to provide reliable supplies and other benefits.

Purpose

This report focuses on ways to represent groundwater in integrated models of regional water resource systems. The report describes, applies, and compares three mathematical formulations representing groundwater resources in simulation and optimization modes. Simulation predicts a single system outcome and performance based on a particular set of conditions. Optimization searches through many alternatives to find the one that best satisfies a pre-defined objective. Most water management models maintained by public agencies in California are simulation models, but interest in optimization formulations is increasing, so they are investigated here. The two case studies featured in the appendices feature optimization applications.

Project Objectives

The contributions of this report are:

- Brief review of California water management context, groundwater data, and some existing regional management models (Chapter 1).
- Description and evaluation of options for representing groundwater within integrated regional water resource models (Chapter 2).
- Computationally compare different methods to simulate and optimize a simplified two-dimensional groundwater model of the Sacramento Valley (Chapter 3).
- Summary of findings and presentation of recommendations for representing groundwater in water resource management models in California (Chapter 4).
- Demonstration of the value of the modeling techniques using two case-studies of the Tulare Basin and Redding Basin (Appendices).

Three different groundwater models are tested in simulation and optimization modes. The first two "distributed" methods estimate groundwater levels at many locations. The third "lumped" method estimates a single representative groundwater level over large areas. The three tested models are applied to a two-dimensional version of the legacy U.S. Geological Survey's Central

Valley Regional Aquifer System Analysis 1 (CVRASA1) groundwater model of the Sacramento Valley. In optimization mode, all models are embedded directly into the optimization as constraints. Response function methods, which use pre-established information on groundwater responses to pumping or recharge, are not investigated in this report as they are commonly used and have already been the focus of numerous reports within and outside of California.

Project Outcomes

Simulation results show the two-dimensional version of the Central Valley Regional Aquifer System Analysis 1 (CVRASA1) model is equivalent to representing only the topmost layer of the aquifer. This was deemed acceptable for the purpose of investigating management model formulations since a majority of pumping occurs from the top of the aquifer and because groundwater levels of lower layers in the three-dimensional model behave similarly to the topmost layer.

All models produced good results in simulation mode. The two "distributed" models showed some instability when included in an optimization program attempting to maximize the amount of groundwater pumped without lowering below historical groundwater levels. The "lumped" storage coefficient method was stable and fast in optimization mode. It was selected to be included in the optimization application discussed in Appendix B.

The case studies featured in the appendices demonstrate the value of representing groundwater in integrated water management optimization models. The model described in Appendix A applies part of the California Value Integrated Network (CALVIN) model to estimate the cost of ceasing groundwater overdraft in California's Tulare basin. The model evaluates the economic benefits generated by groundwater banking infrastructure. Appendix B describes a conjunctive use model of the Redding basin (northern Sacramento Valley) that includes groundwater pumping costs. The value of leaving water in reservoirs for future dry years is considered to enable a more realistic optimization model. This optimization model considers future conditions up to one year in advance, rather than being able to consider inflows of all future years.

Conclusions and Recommendations

In simulation mode all tested groundwater formulations performed well. Model choice should be made depending on the level of spatial detail required by a particular application. Using spatially distributed groundwater models may cause numerical instability in optimization models. Unless practitioners can use specialized techniques to make the groundwater flow equations more stable in optimization mode, the authors recommend using response function methods. These methods are well-understood and conceptually simple and have been programmed into several freely available software packages, although they do not work well for some complex systems. The hydro-economic applications featured in the appendices demonstrate that including even basic groundwater models in systems models leads to water management and policy insights.

1.0 California Groundwater Management, Data and Systems Models

1.1. Introduction

To develop and evaluate operational and institutional strategies for dealing with increased scarcity, water managers use mathematical models of their systems. Water resource system models represent hydrologic, infrastructure, operational, economic and/or institutional aspects of real systems. Integrated models combine multiple disciplines, for example hydro-economic models combine hydrology, water supply infrastructure, operations, and economics.

This report focuses on representing groundwater within integrated models of water resource systems. Both simulation and optimization methods are described and applied. Most water management models maintained by public agencies in California are simulation models. Because experience with groundwater optimization models is less widespread, and because of the authors' research interests, the two case studies featured in the appendices describe the groundwater representations of two optimization models. The optimization formulations described in this report are linear or non-linear multi-period deterministic optimization. Dynamic optimization (such as dynamic programming or optimal control) or global search techniques (e.g. evolutionary and other heuristic search algorithms) are not considered here because the focus is on large scale regional systems (with potentially too many state variables for standard dynamic optimization techniques and too many decision variables for search algorithms).

The content of this report is organized as follows: Chapter 1 briefly introduces water management in California, the availability of groundwater data in California and a selection of water management models used in the state. Chapter 2 reviews the different techniques available to model groundwater within water resource system models. Both simulation and optimization techniques are discussed and literature on theory and applications of each method is surveyed. Chapter 3 implements three different embedding methods to simulate and optimize groundwater levels in the Sacramento Valley of Northern California. Computational advantages and disadvantages of each technique are evaluated using a regional-scale 2D finite difference model based on the USGS's CVRASA1 model (Williamson et al. 1989). Chapter 4 summarizes benefits and limitations of the modeling approaches and offers preliminary recommendations for integrated water resource systems modeling in California. Appendix A features an application of the CALVIN hydro-economic model to estimate the cost of ceasing groundwater overdraft in California's Tulare basin. The model evaluates economic benefits of conjunctive use infrastructure for use in groundwater banking schemes. Appendix B describes a proof of concept conjunctive use hydro-economic model of the Redding basin (northern Sacramento Valley) which includes dynamic groundwater pumping costs and uses a limited foresight yearly optimization time-horizon. Both case-studies represent groundwater in coarse sub-basins and do not use fully spatially discretized groundwater flow equations.

The contributions of this report are:

- Briefly review California water management context, groundwater data and some existing regional models (chapter 1).
- Describe and evaluate the options for representing groundwater within integrated regional water resource models (chapter 2).
- Computationally compare different methods to simulate and optimize (using embedding methods only) an upscaled 2-D groundwater model of the Sacramento Valley, California (chapter 3).
- Summarize findings and present recommendations for integrated water resource management models in California.
- Demonstrate the value of the modeling techniques with two case-studies featuring hydro-economic optimization models of the Tulare Basin and Redding Basin in California (appendices).

1.2. California Water Management and Modeling

Increasing human population and economic development in California will increasingly strain its natural and engineered water resource system. Climate change will affect how the system is best operated and may further increase water scarcity (Vicuna and Dracup 2007). Awareness and appreciation of environmental and ecological quality will play an increasing role in water resource use and management. Favorable sites for surface water storage have already been developed making conventional supply expansion solutions less likely today and placing an emphasis on better system-wide management.

California is a test-case for how a developed economy can adapt to increasing water scarcity. The state has a tradition of implementing large-scale innovative water resources management solutions such as conjunctive use of surface and groundwater, water transfers, water markets, desalination, water conservation, and waste-water reuse. Appreciation and revenue derived from environmental quality in the state are such that new on-stream water storage has been out of the question for some time.

California possesses extensive water resource projects that store water and transfer flows over a wide geographic area and across drought periods of several years. The largest surface water projects are the State Water Project (SWP) and the federal Central Valley Project (CVP); they are complemented with scores of smaller locally managed projects. Groundwater is a major source, constituting 30% of California's supply during normal years and up to 60% in drought years (CDWR 2003a). California's economy is highly dependent on a reliable and sufficient water supply for domestic, industrial and irrigated agriculture uses as well as to support the environmental and recreation resources. This has led to a complex interconnected regulated system that is intensely managed to serve many different and growing demands. Chung and colleagues (2002) describe how by 2020 the deficit between water supply and demand will be between 2 to 6 million acre-feet per year (1 AF = 1233 m3; 1 million AF = 1233 Hm3 or 1.23 km3). Many studies, recently reviewed by Vicuna and Dracup (2007), evaluate the probable effects of climate change on California's future water supply. Recent data mapped in Figure 1 show that scarcity caused by lower than normal rainfall is already a reality in some years.

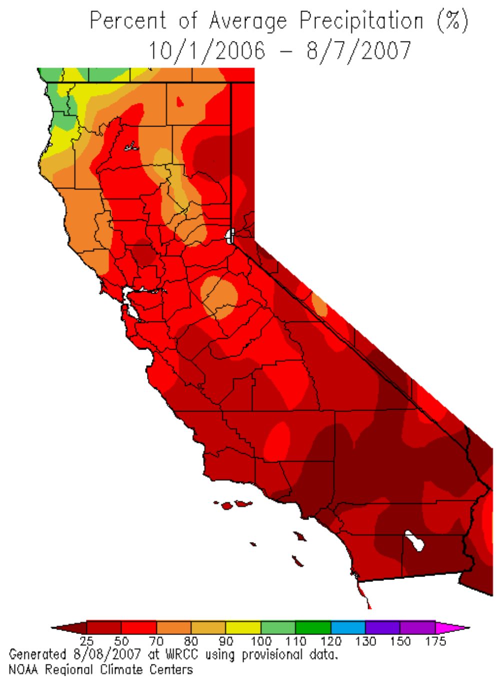

Figure 1: 2007 hydrologic year precipitation as percent of average historical precipitation

Source: CCDA 2007

Alternative sources and/or management strategies need to be developed for California's use of its water resources to be sustainable. If such sources or strategies are not developed, groundwater overdraft could worsen under the brunt of future scarcity caused by droughts and population growth. Overdraft, i.e., the extraction of groundwater beyond its long-term rate of replenishment, is estimated at between 1 and 2 million acre-feet per year statewide (CDWR 2003a). Expanded and better-managed conjunctive use schemes may help California avert a water crisis. To implement such conjunctive use schemes, groundwater resources must be more effectively represented in the mathematical models used to plan for California's long-term water future. Historically water models focused on surface water systems models, affected by California's "fragmented approach to groundwater management" (Chung et al. 2002). Because no single authority in California has the rights to prescribe groundwater management actions (Kretsinger Grabert and Narasimhan 2006), water resource system planning models focused on reservoir operations rather than on groundwater. Including groundwater in reservoir operation models, especially distributed groundwater models, increases model size and adds technical complexity. The trend of greater model complexity seems irreversible, however, due to sustained interest in analyzing local effects of regional policies and conjunctive use schemes and concerns for overdraft of aquifers.

1.3. California Groundwater Data

To include groundwater in large regional or state-wide models requires not only software and numerical methods, but also data that accurately describe hydrogeology. In the case of California, absence of a single groundwater management body means that no institution has gathered all the information needed to model groundwater basins throughout California. The California Department of Water Resources (CDWR) and the U.S. Geological Survey (USGS) have gathered much of the information that currently exists.

CDWR publishes Bulletin 118 that compiles and summarizes the current state of knowledge on groundwater resources in California. The Bulletin has been published twice since 1975 with the latest update in 2003. For this latest version no new information was gathered; "The information on groundwater basins presented in Bulletin 118 Update 2003 is mostly limited to the acquisition and compilation of existing data previously developed by federal, state, and local water agencies". Chapter 10 of Bulletin 118 provides descriptions of groundwater basins and sub-basins in California's 10 hydrologic regions. Also included are GIS maps of basin boundaries and summaries of available water use, hydrological budget and water quality data. Each groundwater basin subchapter also provides a description of local groundwater management arrangements and a list of the relevant local institutional players. References allow the user to find original sources of data and original hydrogeologic investigation reports. For example, the CDWR districts are a major source of local hydrogeologic data, including extensive records of well logs and ongoing groundwater level measurements. Much of this data is available online with CDWR, often in the form of maps or downloadable listings.

The Groundwater Atlas of the United States-Segment on California and Nevada is another important document concerning statewide groundwater (Planert and Williams 1995). The publication is didactic and contains many illustrative and informative maps and cross-sections of Californian groundwater basins. Maps of historic overdraft and land subsidence in the Central Valley are included.

The most readily useable data source from which to build groundwater representations in systems models are local or regional groundwater simulation models. Distributed groundwater models are ideal as an information source because they contain all relevant descriptions of aquifer geometry and composition which have been compiled from local hydrogeologic data. Regional management models typically use local simulation models as data sources in addition to empirical hydrogeologic data. For example in California's Central valley the C2VSim or CVRASA models are state-of the art sources of data. Some of these models are briefly reviewed in the following section.

1.4. Selected California Water Management Models

California has a long and great tradition of computer modeling for water resources management. This tradition arises from the existence of extensive water resource projects that store water and transfer flows over a wide geographic area and across drought periods of several years. System-wide water management models have been used in California since the 1960's (Draper et al. 2004). In this chapter the authors survey a selected group of representative models. The generic CALSIM model, maintained jointly by the California Department of Water Resources (CDWR) and the U.S. Bureau of Reclamation (USBR) was built to manage the state's major water storage and conveyance systems. CALSIM II and CALSIM III are applications of CALSIM to California. CALSIM II contains a rudimentary multi-cell groundwater model to simulate regional head levels and stream-aquifer interaction. In CALSIM III various methods are being investigated to incorporate a fully spatially distributed groundwater model. Another reservoir system simulation model, FREDSIM allocates water based on economic penalty functions for the Friant-Kern canal system. Both of these tools are simulation models that use an optimization algorithm to allocate water deliveries at each monthly time step. The Integrated Water Flow Model (IWFM; formerly IGSM) is an exception to the tendency of separating surface and groundwater models. Development since the mid 1970's focused on building an integrated model capable of analyzing the hydrology and water management of both resources within one model. Applications of IGSM and IWFM in California are numerous; examples are C2VSIM (Central Valley) and WestSim (Western San Joaquin Valley), both of which are briefly described below. The CV-RASA models comprise two generations of USGS Central Valley groundwater models. All models mentioned above are simulation models. Lastly we discuss CALVIN, a state-wide economic integrated water resource optimization model. CALVIN represents groundwater storage and use but due to its network flow algorithm, is not able to dynamically model groundwater heads or flows. Other optimization models of groundwater in California have been applied at smaller local scales (e.g. Nishikawa 1998).

1.4.1. CALSIM II and III

CALSIM II and CALSIM III are Californian applications of CALSIM – a general-purpose reservoir–river basin simulation model (Chung et al. 1989; Draper et al. 2004; Munevar and Chung 1999). CALSIM is a data-driven (i.e., generic) simulation model that uses optimization to distribute flows during each time step. A mixed integer programming (MIP) solver routes and allocates water throughout the system according to specified objectives and constraints for each week or month. Different objectives are introduced as weighted components of the objective function rather than using the conventional setup of a single-objective objective function and numerous constraints. A tailor-made high level computer language called Water Resources Engineering Simulation Language (WRESL), serves as the interface between the user, solver and databases. Because WRESL was developed with object oriented programming languages, new functionalities can be added with less effort (Munevar and Chung 1999). Individual models are self-documenting for those who can read WRESL code. CALSIM's GUI does not yet include a visual schematic of the system.

The groundwater representation in CALSIM II is a multi-cell model that covers the northern part of the Central Valley. Draper et al. (2004) describes the cells as "series of inter-connected lumped-parameter basins." Groundwater pumping, recharge from irrigation, stream-aquifer interaction and inter-basin flow are calculated dynamically within the model. This is accomplished by embedding the groundwater flow continuity equations as equality constraints into the monthly CALSIM MIP model. The multi-cell model contains 14 cells; 9 of which represent groundwater basins and 5 that are narrow strip aquifers underlying major rivers (CDWR 2003b). The geometry of the cells was dictated by the Depletion Study Areas (DSA) used for previous studies. These sub-basins (cells) correspond to C2VSim regions so that parameters from that detailed groundwater model could be aggregated and translated into the multi-cell model. The multi-cell model was also calibrated against results from C2VSim. The use of strip aquifers underlying rivers facilitates the stream-aquifer interaction modeling – which is the main goal of the CALSIM II multi-cell groundwater model. Indeed, CDWR (2003b) states: "In its present form it is not intended that CALSIM II be used to assess the groundwater impacts of various surface water management alternatives. The purpose of the multi-cell groundwater model is to better represent groundwater levels in the vicinity of the stream system in order to better estimate stream gains and losses to the aquifer."

Efforts are underway to integrate CALSIM and C2VSim into the new CALSIM III model. Several methods are being implemented to integrate the spatial groundwater model including response functions (discrete kernels), implemented by the Stockholm Environment Institute (SEI), and a linked simulation–optimization approach (both CALSIM and C2VSim are run at each monthly time step) implemented by CDWR. In theory, if stresses are considered in each cell of the groundwater model, the conventional sequential time marching simulation involves less floating point operations (FLOPS) than the response function method (Andreu and Sahuquillo 1987). This suggests that in principle the linked simulation-optimization approach, where CALSIM exchanges information on a time-step by time-step basis with C2VSIM, is more efficient. In practice however, the link between the models is a complex software project, the practical implementation of which will decide which methodology produces a faster running CALSIM. In addition, the assumptions and numerical techniques each effort will implement, for example with respect to non-linear processes such as stream-aquifer interaction, will also determine the speed at which CALSIM can integrate the spatial groundwater model.

1.4.2. FREDSIM

FREDSIM is an application of the MODSIM software to the Friant-Kern canal system (Leu 2001; Marques et al. 2006). FREDSIM stands for "Friant Economics-Driven SIMulation model, as the model is intended to simulate how economic water demands for agriculture would employ a portfolio of surface and groundwater resources. MODSIM (Labadie and Baldo 2000) is a reservoir system simulation model that optimizes on a daily to monthly time-step using a pure network flow solver. It can perform non-linear updating between optimization time-periods using the Perl scripting language. The FREDSIM regional model is built from irrigation district components and a network schematic including reservoirs, groundwater sub-basins, canals, cities and irrigation districts. The simulated water infrastructure network is underlain by a multi-cell grid of 12 cells of relatively homogenous specific yield (drainable porosity). The multi-cell representation allows the monthly simulation model to dynamically represent spatially varying pumping costs and flows between cells. Groundwater dynamics are calculated using Perl scripts which are activated between monthly optimizations. Semi-confined steady-state pumping drawdowns are calculated using the Theim equation and pumping costs are calculated based on energy consumption (Marques et al. 2006). FREDSIM uses economically based penalty functions to prescribe water allocation, developed from the Statewide Agricultural Production Model (SWAP).

1.4.3. C2VSim

C2VSim is an application of the Integrated Water Flow Model (IWFM) to the Central Valley. IWFM (CDWR 2007) is an integrated surface-groundwater simulation model that considers surface water hydrology, land-use dependent soil-water budgets, surface water - groundwater interaction and groundwater flow. A three-dimensional finite-element groundwater flow simulation model forms the core of the model. Hydrologic variables modeled by IWFM include soil-moisture accounting in the root zone, surface water runoff and infiltration, unsaturated flow between the root zone and the ground water table, and the routing of water in streams. IWFM considers key non-linear aspects of surface water, groundwater and stream-aquifer interaction. IWFM possesses a basic reservoir operations and water rights module that can model multiple reservoir operations and diversions based on a designated priority system (WRIME 2003). This module allows prioritizing allocations for downstream diversion requirements, downstream minimum flow requirements, trans-basin diversions, reservoir on-site storage requirements (such as recreational demands) and flood control requirements.

C2VSim groundwater flow is quasi-3D and uses a 3-layered 1400 element finite element grid that overlays the entire Central Valley. It is a monthly model. The model has undergone frequent updating and calibrating since its inception in 1990. Recent developments include implementing automatic calibration using the parameter estimation model WinPEST in a parallel processing environment as well as updating hydrologic input time-series.

1.4.4. WESTSIM

WESTSIM is an application of IWFM by the USBR to the west side of the San Joaquin Basin. It covers an area of 6000 km2 and possesses 63 regions discretized at the irrigation district scale. Daily to monthly time steps are used to model the 30-year period between 1970 and 2000. It considers land use, agricultural water use efficiency, river diversions, return flows and small watersheds. Eleven streams are modeled in the grid. The subsurface is modeled with three layers: root zone, shallow semi-confined and confined aquifer layers. An innovation is the use of IWFM to model seasonal wetland hydrology. WESTSIM has been used to study land retirement, conjunctive use planning, safe yield analysis and salinity management.

1.4.5. CV-RASA

CVRASA2 (Central-Valley Regional Aquifer System Analysis) is a groundwater-hydrologic model of the Central Valley that uses MODFLOW-2000 with the new Farm Package. It has a grid resolution of 1 by 1 mile and is the follow-up of the CVRASA1 model (Williamson et al. 1989) that had a 6 by 6 mile resolution. The yet unpublished CVRASA2 is maintained by the USGS. The new MODFLOW Farm Package dynamically integrates irrigation water demand, surface and ground-water supply, and return flow from excess irrigation (Schmid et al. 2006). CVRASA2 is in continued development and future versions will use the Streamflow Routing Package to simulate semi-routed deliveries via rivers and major canals (SAHRA 2007). Connections to other models are planned such as linkage to global climate models (GCMs), to the GWM Package (distributed optimization via the response function method) and to economic/biological/water-quality models. New developments will also aim to develop a better understanding of the internal architecture of the freshwater bearing deposits in the Central Valley and increased accuracy of the groundwater model at the local scale.

1.4.6. CALVIN

CALVIN (Draper et al. 2003) is an economic-engineering optimization model of the majority of California's water supply system. Its computational engine is the HEC-PRM software which uses a generalized network flow LP algorithm to perform multiperiod optimization. CALVIN optimizes water management over the 72-year hydrologic record (representing a wide range of hydrologic variability) for a particular level of infrastructure and land use development. The CALVIN model optimizes surface and groundwater operations as well as water use and water allocations to maximize statewide net economic benefits. Water demands are represented as economic penalty functions, representing each water user's economic willingness-to-pay for water deliveries. Operating costs for pumping and treatment also are represented.

As a large-scale regional economic engineering model, CALVIN does not provide a detailed representation of groundwater processes. Groundwater representation in CALVIN and its limitations are discussed in Appendix J of Jenkins et al. (2001) and Pulido-Velasquez et al. (2004). The model does not dynamically quantify groundwater flow within and between groundwater sub-basins. Instead, CALVIN uses fixed series of flows between sub-basins derived from historical levels used in C2VSim. Groundwater recharge time series also are taken from the C2VSim model. Pumping lifts and thus pumping costs are constant for each sub-basin and based on representative water levels during the mid 1990's. Overdraft quantities for each sub-basin are based on average volumes estimated during the 1990s (USBR 1997). These levels were extrapolated out for 72 years to derive final groundwater basin storage volumes. CALVIN's groundwater representation of static sub-basins resembles a multi-cell representation except interbasin flows are not dynamic. This is because the network flow algorithm restricts the optimization model constraint set to mass balance and capacity constraints. Finally, as in all deterministic multiperiod optimization, the drawdown-refill cycles suggested by various model runs benefit from perfect hydrologic foresight over the entire modeled time horizon. Although this representation of groundwater is very simplified it accomplished its task of long term tracking of state-wide groundwater use.

1.4.7. Discussion and Summary

Many options exist for representing groundwater in water resource system models and several of these have been applied in California. The different approaches are categorized in Table 1 based on whether they perform simulation or optimization, whether they use multi-cell or distributed groundwater models and whether the groundwater flow calculations are performed endogenously or are pre-processed (exogenous).

Table 1. Integration of groundwater in selected recent regional water resource system models in California

	Endogenous groundwater model		Exogenous groundwater model	
	Multi-cell	**Distributed**	**Multi-cell**	**Distributed**
Simulation	CALSIM II FREDSIM	CALSIM III (if linked to C2VSim) C2VSim CVRASA2 WESTSIM		CALSIM III (if response function scheme used)
Optimization	Chapter 3, Appendix B	Chapter 3	CALVIN	

CALSIM II and FREDSIM use a multi-cell groundwater model that endogenously simulates flow between sub-basins and estimates average head levels at the sub-basin scale. The strength of these models is the representation of complex surface water storage and conveyance networks while maintaining a coarse but physically sound accounting of groundwater storage, movement and basic interaction with surface water. Multi-cell groundwater models have been the norm in California system management models until recently. CALSIM III is being designed to incorporate results from the spatially distributed C2VSim groundwater model. CDWR, USBR and the Stockholm Environment Institute (SEI) are implementing the link in two ways: having C2VSim run in parallel to CALSIM at each time step (endogenously) or exogenously using preprocessed linear response coefficients ("discrete kernels"). Generic multireservoir simulation systems that already implement the link between a system network and distributed groundwater models include MODSIM-MODRSP (Fredericks et al. 1998) (response function method) and AQUATOOL (Andreu et al. 1996) (eigenvalue method).

Also in the distributed simulation category are integrated surface-groundwater models such as C2VSim, CVRASA2 and WESTSIM. These are 3-D groundwater simulation models that include surface water hydrology to better estimate surface fluxes (recharge, pumping, stream-aquifer interactions). Such tools are useful when local conditions and hydrologic interactions play a key role in water management decisions. These models can be implemented over large areas such as the Central Valley, but they are not designed to represent multiple distinct groundwater basins connected by a complex network of conveyance infrastructure and water demands. Such situations are more easily represented with operations and planning models such as CALSIM covered in the preceding paragraph.

Multi-period deterministic or implicit stochastic optimization models differ from simulation by providing results that maximize a mathematically stated objective. By providing the set of optimal operations given historical or synthetic time-series, they suggest efficient modes of operation not bound by current operating patterns. Unfortunately optimization models can be more difficult to solve than simulation; this is especially true for nonlinear models which may converge only slowly to the optimal solution, if at all. Optimization formulations can also create problems when they include distributed groundwater flow models that use discretized numerical approximations of partial differential equations. Numerical difficulties in optimizing distributed groundwater models are explored in chapter 3. In light of the challenges that may accompany optimizing spatially distributed models, use of lumped groundwater representations such as in CALVIN and in most economic optimization models will continue. Optimization models, especially when used in close collaboration with simulation models, can lead to new operating procedures and beneficial water resources management policies.

2.0 Representing Groundwater in Management Models

2.1. Introduction

When aquifers are an important source of water, representing groundwater is an essential part of water resource system management modeling. Groundwater management models and the use of systems analysis in groundwater engineering were reviewed by Gorelick (1983), Willis and Yeh (1987), Yeh (1992) and Ahlfeld and Heidari (1994). Integrating groundwater into systems models poses a challenge because groundwater flows and responses to stresses (e.g. pumping, recharge) depend on spatially distributed hydrogeologic properties and hydraulic conditions at aquifer boundaries. Nonlinear interactions between surface water bodies and aquifers and the nonlinearity introduced in optimization by groundwater pumping costs further complicate modeling.

Three levels of aquifer discretization can be distinguished: single cell, multi-cell and spatially distributed groundwater models. Simulation and optimization at each scale use various mathematical formulations that are described in this chapter. Distributed groundwater models are incorporated into optimization models using embedding or response function methods. Embedding methods incorporate into the mathematical program constraint set a set of algebraic equations established by a numerical approximation of the groundwater flow equation. The governing flow equation can be discretized in space and time (standard time-marching schemes) or only in space (continuous-time formulation). The eigenvalue method embeds an efficient version of the continuous-time formulation designed for use in conjunctive use models. The response function method uses an external groundwater simulation model as a pre-processor to build a database of the aquifer's linear responses to unit stresses.

In addition to hydrogeology and groundwater modeling, natural resource economics has significantly impacted the field groundwater management during the last 50 years. Economic perspectives and models were reviewed by Koundouri (2004a). A preoccupation of groundwater engineers, scientists and economists has been increasing the spatial resolution of management models. Groundwater is a spatially variable resource with local conditions determining flow direction, cost and availability of water. Spatially distributed numerical groundwater models best represent this reality. Such models are built by engineers and hydrogeologists to simulate (predict) the effects of changing conditions on groundwater flow, levels and quality, or to optimize groundwater system design. Most economic models however cannot focus on spatial groundwater behavior because they rely on closed form analytical solutions that require a single cell discretization. Recent work in groundwater economics has reiterated the importance of spatial groundwater economic effects (Koundouri 2004b; Brozovic et al. 2006) although the preoccupation was already present in early work (Bredehoeft and Young 1970).

2.1.1. Simulation and Optimization

Simulation and optimization models in water resource planning and management serve complementary roles (Loucks et al. 1981; Loucks et al. 1985; Rogers and Fiering 1986). Simulation models are predictive tools that answer "what if?" questions. A simulation model is a group of mathematical statements (equations) executed in a specific order which represent the response of a system to a particular set of environmental and management conditions. In the case of deterministic models, once boundary conditions, system stresses and internal parameters are set, the model produces a unique solution. Typically deterministic simulation models are calibrated with historical data, validated with a subset of historical data not used in the calibration, then used to predict effects of future conditions.

In the case of groundwater results are piezometric head (i.e., groundwater level) at discretized points throughout the modeled domain, boundary conditions refer to the hydraulic conditions at edges of the modeled domain, and system stresses are time series of inflows and/or outflows into each discretized volume. Possible hydraulic boundary conditions in hydrogeology include specified head (Dirichlet), specified flux (Neuman) or head-dependent flux (Cauchy).

An optimization model consists of an objective function subject to system of constraint equations. The constraint set is an underdetermined system (i.e., number of variables > number of equations) implying that a potentially infinite set of solutions can satisfy the equations. The role of the objective function is to select the best solution amongst the feasible set, effectively answering the question "what's best?"An optimization model differs from a simulation model in three aspects: a) solution-seeking purpose, b) a mathematical objective function which serves to evaluate competing solutions, and c) an optimization algorithm which systematically examines many solutions (values for a set of decision variables) until an "optimal" solution is found.

In groundwater simulation head levels are variables while system stresses are fixed input data. With optimization a subset of managed stresses (e.g. pumping or artificial recharge) and head levels at control locations are the decision variables. Groundwater management (optimization) formulations differ by the equations and numerical methods used to model groundwater flow and by the objective function. Hydraulic management models focus on hydraulic performance, e.g. maximizing pumping subject to head or flux constraints. Economic management models use cost minimization or net benefit maximization objective functions subject to institutional or environmental constraints.

2.1.2. Discretization

Groundwater models that simulate or optimize groundwater levels use different spatial discretization schemes. When integrated into regional system models, the groundwater representation should depend on the level of spatial detail best suited for a particular application. Three levels of spatial detail are available: single reservoir, multi-cell, and spatially distributed.

If basic water quantity accounting is the objective, a single reservoir model, so-called "single cell", "lumped" or "bucket" models, may be sufficient. This approach considers the water bearing formation(s) as a single underground reservoir possessing a single water table elevation (piezometric head). The lumped formulation tracks aquifer volume with a standard mass balance equation; groundwater levels can be grossly modeled using the storage coefficient (specific yield for unconfined aquifers) equation.

Slightly more complex models comprise two or more sub-regions or sub-basins with the possibility of exchanging flow between them. Examples of such multi-cell models are reviewed by Brozovic et al. (2006). Flow between cells is proportional to their differences in piezometric head (Darcy equation). Individual sub-regions are established based on institutional boundaries (e.g. irrigation district) or by lumping areas of relatively homogeneous physical parameters. Some multi-cell models incorporate basic stream-aquifer interaction (Draper et al. 2004; Pulido-Velazquez et al. 2005).

If the status of the aquifer (e.g. heads or fluxes) at specific locations and its spatial interaction with surface water bodies are important, then a fully spatially distributed approach is needed. Distributed groundwater simulation models describe the spatial head distribution of an aquifer over time given specified initial and boundary conditions and time series of external stresses. They quantify aquifer head and flow velocity at discrete nodes of a virtual grid overlaying the aquifer. The mesh can be one, two or three-dimensional.

Commonly economic models tend to use lumped formulations while engineering applications favor the distributed parameter approach. There are many exceptions however, e.g. CALSIM II (Draper et al. 2004), an engineering model used the multi-cell approach while optimal control economic models have integrated distributed models (Makinde-Odusola and Mariño 1989; Taghavi et al. 1994).

2.2. Groundwater Simulation

2.2.1. Single- and Multi-Cell Modeling

Different model formulations exist at each level of spatial discretization. For example single-cell models can model aquifer dynamics using storage volume or piezometric head. This section reviews equations used in single- and multi-cell groundwater model formulations: the storage coefficient equation and the Darcy equation.

The storage coefficient equation can model piezometric head levels in single- or multi-cell aquifer models. The storage coefficient relates the volume of water released (or absorbed) from (into) storage per unit surface area of aquifer per unit change in hydraulic (piezometric) head in a confined aquifer. For gravity drainage in unconfined aquifers its equivalent is the specific yield.

$$h_g^t = h_g^{t-1} + \frac{Q_g^t}{sc_g * area_g} \qquad \forall g, t \tag{1}$$

where Q_g^t = mean stress (net pumping, recharge term) in groundwater sub-basin *g* at time *t*

h^t_g = mean hydraulic head in groundwater sub-basin *g* at time *t*

scg = mean storage coefficient of sub-basin *g*

areag = surface area of sub-basin *g*

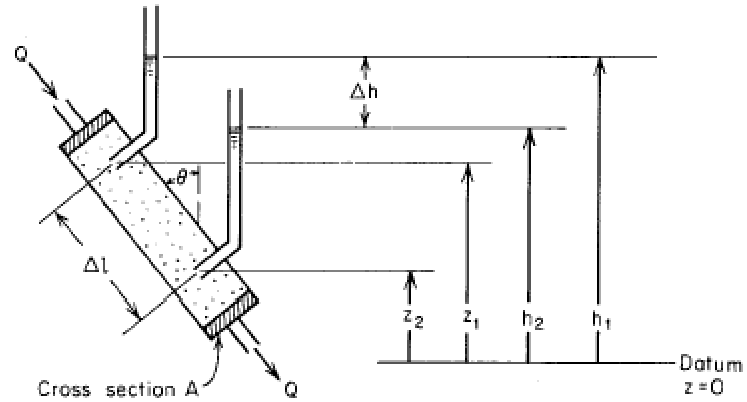

Figure 2. Experimental apparatus illustrating the Darcy equation.

Source: Adapted from Freeze and Cherry (1979, p. 15)

The Darcy equation relates flow through a porous media with the gradient in head that causes it. The two quantities are proportional and the linear constant that relates them is hydraulic conductivity, K. Darcy's law is an empirical expression of continuity of momentum; in one-dimensional form it can be written:

$$Q = KA\frac{h_1 - h_2}{l} = -KA\frac{h_2 - h_1}{l} = -KA\frac{\Delta h}{\Delta l} = -KA\frac{dh}{dl}$$

where Q is the flow rate, A is the cross-sectional area of flow and l or Δl is the distance separating the points where hydraulic head, h, is measured. The negative sign indicates that fluid moves in the direction of decreasing hydraulic head (the gradient is taken by definition in the direction of steepest ascent; so in the gradient dh is defined as $h_2 - h_1$ where $h_2 < h_1$). Dividing both sides by the area gives the 1-D equation for the Darcy flux (also called specific discharge), q:

$$q = -K\frac{dh}{dl} \qquad [q] = L/t$$

For three-dimensional flow through an anisotropic medium, Darcy's law becomes:

$$\vec{q} = -\overline{\overline{K}}\,\nabla h \qquad \Leftrightarrow \qquad \begin{pmatrix} q_x \\ q_y \\ q_z \end{pmatrix} = -\begin{bmatrix} K_{xx} & K_{xy} & K_{xz} \\ K_{yx} & K_{yy} & K_{yz} \\ K_{zx} & K_{zy} & K_{zz} \end{bmatrix} \cdot \begin{pmatrix} \frac{\partial h}{\partial x} \\ \frac{\partial h}{\partial y} \\ \frac{\partial h}{\partial z} \end{pmatrix}$$

where $\overline{\overline{K}}$ is a 3x3 tensor. If the coordinate system aligns with the principal axis (i.e. coordinates align to the principal directions of anisotropy (layering)), Darcy's law can be simplified to:

$$\vec{q} = -\vec{K}\,\nabla h \qquad or \qquad q_x = -K_x \frac{\partial h}{\partial x}, \qquad q_y = -K_y \frac{\partial h}{\partial y}, \qquad q_z = -K_z \frac{\partial h}{\partial z}$$

The Darcy equation is relevant to multi-cell models because it can estimate groundwater flow between adjacent basins given the surface area of the boundary between the regions and their difference in piezometric head. The Darcy equation also is relevant in distributed modeling, as reviewed in the next section.

2.2.2. Spatially Distributed Groundwater Modeling Framework

The governing groundwater flow equation is a linear second order partial differential analytical equation (PDE) built by combining an expression of conservation of mass (continuity equation) with an expression of conservation of momentum (Darcy equation). The continuity equation for a saturated confined porous volume takes the form: Inflow – Outflow = Change in storage, written as a partial differential equation (Freeze and Cherry 1979):

$$-\nabla \cdot \vec{q} + Q = S_s \frac{\partial h}{\partial t}$$

where $\vec{q}$ is a water flux vector, Q is a source/sink term, S_s is specific storage and h is hydraulic head. On the left hand side, $-\nabla \cdot \vec{q}$ is the rate of net water inflow and Q is a recharge function: $Q(x,y,z,t)$ ($Q > 0$ for recharge or injection, $Q < 0$ for extraction). After substituting in the Darcy equation and the transient groundwater flow equation through a heterogeneous anisotropic saturated porous medium becomes:

$$\frac{\partial}{\partial x}\left(K_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial h}{\partial z}\right) + Q = S_s \frac{\partial h}{\partial t}$$

Spatially distributed groundwater models are spatial numerical approximations of the governing groundwater flow equation. Given appropriate initial and boundary conditions, it can be solved numerically to obtain $h(x,y,z,t)$ – piezometric head as a function of space and time. Various numerical methods are available for solving PDEs in 1, 2 or 3 dimensions such as finite difference (FD) or finite element techniques. These different numerical approximation techniques discretize the domain with a grid and solve for $h(x,y,z,t)$ at all nodes. The grids have a different aspect depending on which numerical method is used (Figure 3).

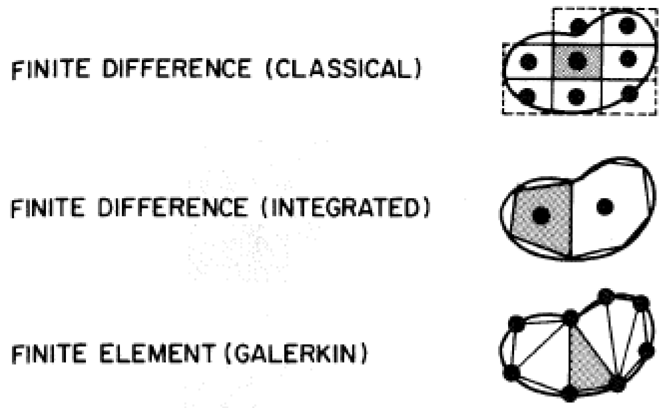

Figure 3. Diagrammatic representation of the most common numerical approximation methods

Source: Adapted from Pinder (1988)

Simple derivation of the finite difference (FD) formulation

The starting point for the FD equations is the governing groundwater flow PDE in 2-D:

$$\frac{\partial}{\partial x}\left(K_x\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_y\frac{\partial h}{\partial y}\right)+Q=S_s\frac{\partial h}{\partial t}$$

Using finite differences to replace the derivatives:

$$\frac{1}{\Delta x}\left(K_{r4}\frac{(h_4^t - h_r^t)}{\Delta x}-K_{r2}\frac{(h_r^t - h_2^t)}{\Delta x}\right)+\frac{1}{\Delta y}\left(K_{r1}\frac{(h_1^t - h_r^t)}{\Delta y}-K_{r3}\frac{(h_r^t - h_3^t)}{\Delta y}\right)+Q_r=S_s\frac{(h_r^{t+1}-h_r^t)}{\Delta t}$$

Alternatively, writing a water balance of cell r using the Darcy equation, we arrive at the same result:

$$Input - Output = \Delta Storage$$

$$Q_{r1}+Q_{r2}+Q_{r3}+Q_{r4}=\Delta Storage$$

$$K_{r1}\frac{(h_1 - h_r)}{\Delta y}\Delta x+K_{r2}\frac{(h_2 - h_r)}{\Delta x}\Delta y+K_{r3}\frac{(h_3 - h_r)}{\Delta y}\Delta x+K_{r4}\frac{(h_4 - h_r)}{\Delta x}\Delta y+Q_r=S_s\Delta x\Delta y\frac{(h_r^{t+1}-h_r^t)}{\Delta t}$$

(divide by Δx, Δy and rearrange and you get same equation as above)

Or alternatively written to closely resemble the Darcy equation:

$$-K_{r1}A\frac{(h_r - h_1)}{\Delta y}-K_{r2}A\frac{(h_r - h_2)}{\Delta x}-K_{r3}A\frac{(h_r - h_3)}{\Delta y}-K_{r4}A\frac{(h_r - h_4)}{\Delta x}+Q_r=S_s\Delta x\Delta y\frac{(h_r^{t+1}-h_r^t)}{\Delta t}$$

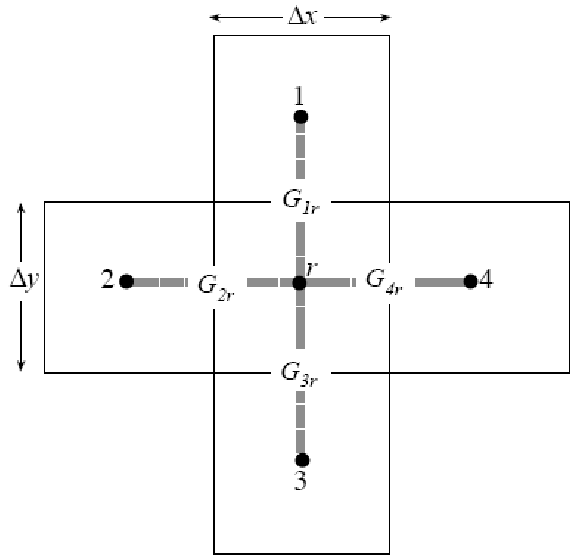

Figure 4: Groundwater model computational molecule—used to derive one equation of the system of ODEs constituting the finite difference (FD) groundwater model. Below the authors assume cells have unit-depth.

Source: Fogg (2004)

To simplify the expression, and in analogy to the flow of electricity, conductance and capacitance parameters are defined. Conductance, G, is the total ability of a connection to transmit water; it is the flow into the central node r assuming a unit gradient (dh/dl=1).

$$G_{ij} = -\frac{K_{ij} A_{ij}}{a_{ij}} \;\; or \;\; G_{ij} = -\frac{T_{ij} w_{ij}}{a_{ij}}$$

where G_{ij} = conductance between nodes *i* and *j*,

K_{ij} = hydraulic conductivity between the two cells (blocks),

A_{ij} = cross-sectional area between nodes *i* and *j* ($A = b*w$, = depth * width),

a_{ij} = distance between center points of both nodes (distance over which flow takes place)

The Darcy equation using conductance is written $Q = G * dh$.

Capacitance, *D*, is the total ability of a node to *store* water; it is the volume of water stored or released per change in head.

$$D_r = S_s * Vol_r$$

where D_r = capacitance of node *r*,

S_s = specific storage of node *r*,

Vol_r = volume of node *r*.

Using the definitions to simplify the equations we obtain (writing h for h^t to simplify notation):

$$G_{r1}(h_r - h_1) + G_{r2}(h_r - h_2) + G_{r3}(h_r - h_3) + G_{r4}(h_r - h_4) + Q_r = D_r \frac{(h_r^{t+1} - h_r^t)}{\Delta t}$$

$$(G_{r1} + G_{r2} + G_{r3} + G_{r4})h_r - (G_{r1}h_1 + G_{r2}h_2 + G_{r3}h_3 + G_{r4}h_4) + Q_r = D_r \frac{(h_r^{t+1} - h_r^t)}{\Delta t}$$

$$\sum_j G_{rj} h_r - \sum_j G_{rj} h_j + Q_r = D_r \frac{(h_r^{t+1} - h_r^t)}{\Delta t}$$

Defining the total conductance (total ability of a node to take on water), G_{rr} as:

$G_{rr} = -\sum_j G_{rj}$ and multiplying by both sides of the equation by -1 we arrive at:

$$G_{rr} h_r + \sum_j G_{rj} h_j + D_r \frac{(h_r^{t+1} - h_r^t)}{\Delta t} = Q_r \qquad (2)$$

The steady-state version of the equation is:

$$G_{rr} h_r + \sum_j G_{rj} h_j = Q_r$$

General Groundwater Model Matrix Equation

A finite difference groundwater model contains, at each time step *t, r* times equation 2 (one equation for each r^th cell of the model). In matrix notation the system of equation is written (Bear 1979, p. 500):

$$[G]\{h\} + [D]\left\{\frac{\partial h}{\partial t}\right\} = \{Q\} \qquad (3)$$

where [*G*] is the conductance matrix (square, banded & symmetric) depending on aquifer conductivities and cell configurations, [*D*] is the capacitance matrix (square, banded & symmetric in the case of FD) depending on storativity and cell configurations, {*h*} is the hydraulic head column matrix and {*Q*} is a column matrix of source/sink terms.

Equation 3, with time yet undiscretized, applies whichever numerical representation scheme is used (finite element or finite difference). To solve for *h(x,y,z)* in the steady-state case, this system of equations is solved directly as

$$[G]\{h\} = \{Q\} \qquad (4)$$

This equation can be passed on to a generic solver for system of equations since it has the generic form of [*A*]*{*x*}={*b*}, {*x*} being a vertical vector of *h(x,y,z)* at all nodes of the grid.

General Head Boundary Condition in FD Model

Boundary conditions can be applied anywhere in the modeled aquifer domain and are not limited to cells on the edge ("boundary") of the grid; internal boundary conditions can alternatively be called source/sink terms. One way to implement both Dirichlet and Cauchy boundary conditions (both head-dependent) in FD models is by using the general head boundary condition. The general expression for a general head boundary condition is:

$$Qbr = Gbr\ (hb - hr)$$

where *Qbr* = flow from fictitious "general head" node to variable head node affected by boundary condition,

Gbr = conductance between fictitious "general head" node to variable head node, Note: in this case there is no negative sign; *G* = *KA*/*b*.

hb = known head at the fictitious "general head" node,

hr = unknown head at the variable head node.

The use of the general head boundary condition for a Cauchy boundary is evident, since a Cauchy boundary is by definition a head dependent flux. The general head boundary condition can also be used to model Dirichlet (specified head) boundaries by specifying a very high conductance value and setting *hb* to be the specified head (imagine the high conductance as a water pipe equalizing levels between *h* and *hb*).

To show how a general head boundary condition is integrated into a FD cell water balance equation, we take the steady-state case of the general balance equation derived above and add a conductance from a specified head node, b, to the central variable-head cell (node) *r* (see Figure 5).

$$G_{br}(h_b - h_r) + G_{rr}h_r + \sum_j G_{rj}h_j = Q_r$$

$$-G_{br}h_r + G_{rr}h_r + \sum_j G_{rj}h_j = Q_r - G_{br}h_b$$

$$(G_{rr} - G_{br})h_r + \sum_j G_{rj}h_j = \mathrm{Q_r} - G_{br}h_b$$

This will build a system of equations in which the diagonal of the conductance matrix, [G], is negative and the bands are positive. Terms dependent on boundary conditions are added on the right hand side of the equation.

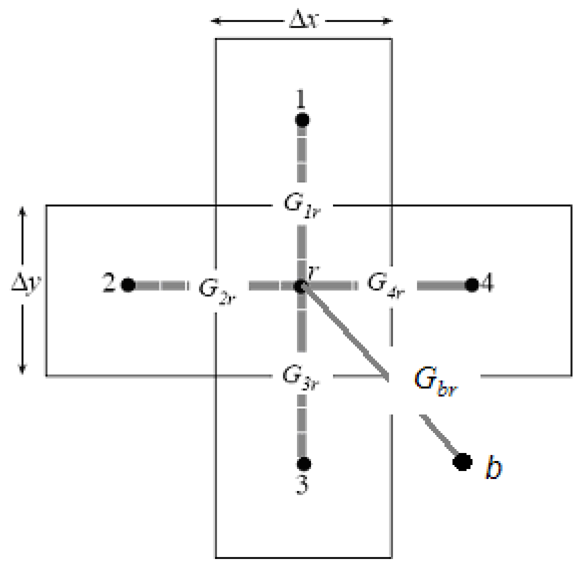

Figure 5. FD node with general head boundary condition link

Source: Adapted from Fogg (2004)

2.2.3. Transient Distributed Groundwater Simulation

Sequential Time-Marching Simulation

Sequential time-marching simulation uses the numerical approximation of the time derivative. The finite difference approximation of the time derivative for node r is:

$$\frac{\partial h}{\partial t} = \frac{h_r^{t+1} - h_r^t}{\Delta t}$$

Substituting the temporal finite difference into matrix equation 3 and multiplying out the temporal difference term, the following equation is derived (Wang and Anderson 1982, p. 161):

$$\left[\Delta t[G] + [D]\right] * \left\{h^{t+1}\right\} = \left\{[D] * \left\{h^t\right\} + \{Q\}\Delta t\right\} \qquad (4)$$

This equation has the form *[A]*{x}={b}* and allows solving for h^t at all nodal points using an implicit time marching scheme.

Continuous Time Simulation: Eigenvalue Methods

Continuous time simulation uses a spatially discretized but time-continuous formulation to solve the system of algebraic equations specifying the groundwater model (Kuiper 1973; Sahuquillo 1983a; Martinez Rodriguez 2002; Pulido-Velazquez et al. 2007). This spatially discretized form is contrasted to the fully discretized form (both in space and time) of the governing equation used in conventional groundwater simulation models. The time continuous form of the groundwater equation has been applied in groundwater simulation (Kuiper 1973;

Hwang et al. 1984; Umari and Gorelick 1986a; Umari and Gorelick 1986b; Andreu and Sahuquillo 1987). Andreu and Sahuquillo (1987) present a efficient eigenvalue method aimed at representing linear groundwater response (with time-invariant transmissivity) in conjunctive use simulation models. Pulido-Velazquez et al. (2007) generalized the approach so that it can be applied to unconfined aquifers. The continuous time formulation has also seen applications in the inverse problem (Ginn et al. 1990) and in stochastic analysis (Hantush and Marino 1995).

The authors begin with the vector differential equation established by applying finite difference or finite element numerical methods to the PDE governing confined groundwater flow (equation 3). Its solution (i.e. the vector of piezometric heads, {h(t)}) can be expressed as a continuous function of time by matrix exponentiation. First, we rewrite the system of algebraic equations (equation 3) in the form (Willis and Yeh 1987, p. 118):

$$\left\{\frac{\partial h}{\partial t}\right\} = [P]\{h\} + \{z\}, \qquad \{h(0)\} = \{h_0\}$$

where $[P] = -[D]^{-1}[G]$, and $\{z\} = [D]^{-1}\{Q\}$. Bellman's solution to this inhomogeneous equation is (Bellman 1960, p. 173):

$$\{h(t)\} = e^{Pt}\{h_0\} + \int_0^t e^{P(t-s)}\{z(s)\}\, ds$$

Moler and Van Loan (1978) present many ways to compute the exponential of a matrix; one of these is based on the diagonalization of the $[G]$ and $[D]^{-1}$ matrix and involves decomposition using eigenvalue and eigenvectors. Its use leads to the following equation (Sahuquillo 1983a; Andreu and Sahuquillo 1987):

$$\{h_t\} = [A]\cdot[E]\cdot[A]^T\cdot[D]\cdot\{h_0\} + [A]\cdot([I]-[E])\cdot[\alpha]^{-1}\cdot[A]^T\cdot\{Q_t\}$$

which is alternatively expressed as:

$$\{h_t\} = [M_1]\cdot\{h_0\} + [M_2]\cdot\{Q_t\}$$

$[M_1]$ and $[M_2]$ are the matrices obtained by diagonalization:

$$[M_1] = [A]\cdot[E]\cdot[A]^T\cdot[D], \qquad [M_2] = [A]\cdot([I]-[E])\cdot[\alpha]^{-1}\cdot[A]^T$$

where $[E]$ is a diagonal matrix whose elements are $e^{-\alpha_i \cdot \Delta_t}$. $[\alpha]$ is the diagonal matrix of eigenvalues and $[A]$ is the matrix of eigenvectors of the following eigenproblem:

$$[G]\cdot[A] = [D]\cdot[A]\cdot[\alpha]$$

In the case of a diagonal capacitance matrix (which occurs when mass lumping is used), $[B] = [D]^{1/2}$. The matrix of eigenvectors is found by writing $[A] = ([B]^T)^{-1}\{Y\}$. Sahuquillo (1983a) shows that the set of eigenvectors are real and orthonormal in the aquifer domain, i.e.

$$[A]^T[D][A]=[I]$$

If time is discretized in management periods $t_1, t_2, ..., t_n$ of equal duration Δt, the vector of piezometric head at the end of the t interval, $\{h_t\}$, can be expressed in the basis provided by the orthogonal eigenvectors as (Sahuquillo 1983a; Andreu and Sahuquillo 1987),

$$\{h_t\}=[A]\cdot\{L_t\}$$

where $\{L_t\}$ is a vector that contains the aquifer state in the eigenvector's basis during time period t,

$$\{L_t\}=[E(\Delta t)]\ \{L_{t-1}\}+[I-E(\Delta t)]\ [\alpha]^{-1}[A]^T\ \{Q_t\}$$

where $[I]$ is an identity matrix, $[E(\Delta t)]$ is a diagonal matrix whose elements are $e^{-\alpha_i \cdot \Delta t}$. $\{L_0\}$ is defined by initial conditions as:

$$\{L_0\}=\ [A]^T\cdot[D]\cdot\{H_0\}$$

Andreu and Sahuquillo (1987) recognize that only some stress and control variables are of interest in the water resource planning and management process. An important contribution of their paper is the introduction of an efficient continuous-time formulation using reduced sets of basic stresses and control variables.

Any management action can be expressed as a linear combination of a reduced set of e basic stresses. A basic stress can be uniform pumping or recharge at a single cell or a group of cells.

$$\{Q_t\}=[QB]\{QI_t\}$$

where $\{Q_t\}$ is the vector of stresses applied at the N cells of the groundwater model,

> $[QB]$ is the N by e matrix of basic stresses, built by assemblage of the e basic stress vectors,
>
> $\{QI_t\}$ is the vector of intensities of length e associated with each basic stress.

Instead of using the equation for $\{h_t\}$ to obtain heads in all cells of the groundwater model, it is possible to define $\{CV_t\}$, a vector of c control variables. Examples of control variable are head in an individual cell, average head or volume of water in a group of cells, gradients between cells, and flow through a boundary.

$$\{CV_t\}=[AR]\{L_t\} \qquad (1)$$

where $[AR]$ is a c by N matrix called the "reduced A matrix". It contains those rows of eigenvector matrix $[A]$ that are relevant as control variables in the reduced management problem. For example, if the head in cell x is a control variable, row x of $[A]$ will be inserted into $[AR]$. If a control variable can be expressed as a linear combination of head variables, the corresponding row in [*AR*] will be a linear combination of rows in [*A*]. This allows calculating the value of a control variable which depends on several piezometric head values, without explicitly computing head at each node. $\{L_t\}$ is an *N* by 1 state vector describing the aquifer during time period *t*, it is written as:

$$\{L_t\} = [E(\Delta t)]\{L_{t-1}\} + [F]\{QI_t\} \qquad (2)$$

where [*F*] is an *N* by *e* matrix of constant terms in the equation of $\{L_k\}$ which includes matrix [*QB*]. It is written:

$$[F] = [I - E(\Delta t)][\alpha]^{-1}[A]^T[QB]$$

where $[I - E(\Delta t)][\alpha]^{-1}[A]^T$ is an *N* by *N* matrix, and [*QB*] is a *N* by *e* matrix (*e* is the index of basic stresses), so [*F*] is an *N* by *e* matrix.

When, as above, [*F*] is multiplied by {*QI*} (an *e* by 1 vector), there results an *N* by 1 vector, which fits right into the dimensions of the equation for $\{L_t\}$.

The vector $\{L_0\}$ and matrices [*E*], [*F*] and [*AR*] are computed only once by a pre-processor outside of the management model. They incorporate information on initial conditions, model geometry, hydrogeologic parameters and boundary conditions of the aquifer system as well as information on the stresses and control variables relevant to the management model. Only equations for $\{CV_t\}$ and $\{L_t\}$ need to be included at each time step *t* of the management model.

Continuous-time simulation allows solving for *h(t)* explicitly over any time duration with constant stresses and boundary conditions. Time marching simulation most commonly used in groundwater modeling is not necessary. This produces more accurate results but comes at increasing computational cost for larger grids (Umari and Gorelick 1986a; Umari and Gorelick 1986b; Andreu and Sahuquillo 1987). By reducing the sets of control variables and by representing stresses as a linear combination of basic stresses, Andreu and Sahuquillo's eigenvalue technique is efficient at including distributed groundwater simulation models in general water systems models. Their method has been integrated successfully into the generalized AQUATOOL Decision Support System (Andreu et al. 1996). A generalization of the method (Pulido-Velazquez et al. 2007) to make it applicable to unconfined systems is not reviewed here.

Simulation Using Response Functions

In the response function method, an external analytical or numerical model is used to estimate and compile the aquifer's responses over time to unit stresses applied at pumping/recharge locations. These unit responses are stored in a database called a response matrix (hence the method's other name: response matrix method). Unit response functions can be incorporated into water resource system simulation or optimization models, allowing the management model to be separate from the groundwater simulation model. Other terms used in the literature to refer to response functions are "influence" functions (Schwarz 1971; Schwarz 1976), "algebraic technological" functions (Maddock 1972; Maddock and Lacher 1991a) and "discrete kernel" functions (Morel-Seytoux and Daly 1975).

The fundamental assumption of the response function approach is that the groundwater system behaves linearly over the relevant range (Schwarz 1971; Schwarz 1976). The confined groundwater flow equation is a linear equation. Unconfined aquifers can use the confined flow equation if changes in head are small compared to the thickness of the aquifer (Reilly 2001). The assumption of linearity in modeling groundwater flow is often realistic (Bear 1979, p. 152; Reilly et al. 1987). Linearity allows application of the principles of a) proportionality and b) superposition. Thus a) pumping *i* ("i" for intensity) times the unit stress causes *i* times the unit drawdown and b) an aquifer's reaction to a combination of stresses is the sum of the reactions to each separate stress. Application of the linearity assumption is also contingent on the stability of the boundary conditions over time.

To potentially evaluate an aquifer's response to a combination of stresses, one could pre-calculate the aquifers response at every point to stresses applied at all locations. This defeats the goal of finding a computationally economical way to synthesize an aquifer's response. Using a numerical groundwater model that discretizes the aquifer into a series of two or three-dimensional blocks is a first simplification of the system. For management models, a further simplification involves applying stresses only at specific locations (i.e., where the installation of pumps or recharge basins is considered) and analyzing their effects on piezometric head at selected strategic places (e.g. near water bodies). For example when evaluating a pumping scheme, potential stress locations can be referred to as "managed pumping nodes" and important water level nodes as "control head nodes" (Greenwald 1998).

The steady-state spatial superposition principle is described mathematically by:

$$h_i = umh_i + \sum_j a_{ij} * (\frac{Q_j}{unitQ_j})$$

where h_i is the head at control location *i* resulting from the managed stresses,

umh_i is the unmanaged head (those levels that result from no managed stresses) at point *i*,

a_{ij} is the steady-state response or "influence" coefficient; it is the change in head at point *i* due to the stress Q_{unit} applied at point *j*,

Q_j is the amount pumped at node *j*, and

$unitQ_j$ is the unit pumping rate at node *j* (the unit stress applied in the external simulation model).

The change of head at i due to the management strategy is given by $h_i^T - umh_i^T$; for pumping, Q is negative and so is the change in head due to pumping. *Q* is a general source/sink term with *Q<0* for pumping and *Q>0* for recharge. Because we may be pumping at a controlled head location *i*, the set of *j* pumping nodes in that case includes node *i*. The steady-state equation is appropriate to analyze long-term equilibriums. For studies that focus on the short-term of strongly dynamic system, the transient formulation must be used.

The transient formulation is:

$$h_i^T = umh_i^T + \sum_{t=1}^{T}\sum_j a_{ij}^{T-t+1} * \left(\frac{Q_j^t}{unitQ_j}\right) \tag{3}$$

where h_i^T is the head at control location *i* at the end of period *T*,

umh_i^T is the unmanaged head at *i* at the end of period *T*,

a_{ij}^{T-t+1} is the transient response or "influence" coefficient measuring the change in head at point *i* during time period *T* due to the stress $unitQ_j$ applied at point *j* during period *t*,

Q_j^t is the stress applied at node *j* during period *t*, and

$unitQ_j$ is the unit pumping rate at node *j* (the unit stress applied in the external simulation model).

A supplementary summation over the number of time periods (*T*) and a time superscript in the pumping rate and in the dynamic response coefficient are introduced in the transient formulation. The expression to the right of the summation is the change in head at i induced by pumping at j during period *t*, *T-t+1* periods after that pumping started. The superscript, (*T-t+1*), represents the number of periods between the end of final period *T* and the beginning of period *t*. For example if *T=5*, pumping during the first period (*t=1*) will be multiplied by the response coefficient a_{ij}^5 to quantify its drawdown effect at the end of period 5.

Finally, the response function expressions above have the form of a discrete-time convolution integral (Maddock 1972; Heidari 1982; Chow et al. 1988, p. 204). This equation can be applied to linear systems which have a "cause and effect" structure. It is the fundamental equation for solution of a linear system on a continuous time scale. A convolution is an integral that expresses the amount of overlap of one function *a* as it is shifted over another function *Q*. The convolution integral is written:

$$O(t) = \int_0^t Q(\tau)a(t-\tau)d\tau$$

where *O(t)* is the output function which quantifies the system's response (e.g. drawdown) to a stress (e.g. pumping/recharge),

$Q(\tau)$ is the system input, also called the stress or excitation,

$a(t-\tau)$ is the response function or "integral kernel" or "Green's function", where τ is a dummy variable of integration.

The general expression for a discrete time convolution is (Chow et al. 1988, p. 211):

$$O^T = \sum_{t=1}^{T} Q^t \; a^{T-t+1}$$

Generic computer programs exist to build response function databases (matrices) using groundwater simulation models. For example AQMAN (Lefkoff and Gorelick 1987) and MODRSP (Maddock and Lacher 1991b) build response matrices without using them in either simulation or optimization models. Although the response function method is used most often in mathematical programming (optimization) models (section 1.3.2), it is also used in conjunctive use simulation systems as an efficient simulator (e.g. Illangasekare and Morelseytoux 1982; Fredericks et al. 1998; Miller et al. 2003). The source of the efficiency is that only a subset of the groundwater model results, i.e. selected control point head or flux responses caused by unit stresses, is integrated into the conjunctive use simulator.

Some techniques are available to further facilitate use of response functions in conjunctive use simulation models. These include use of the artificial steady-state variant and an associated scanning grid (Illangasekare et al. 1984). Use of reinitialization and artificial steady state stresses for transient simulation (Illangasekare et al. 1984) can reduce the length of response functions, potentially down to single coefficients (one time period). To understand the reinitialization process suppose a groundwater model cell sustains managed stresses $Q(t=1)$, $Q(t=2)$, … . The artificial steady-state stress is that stress which if applied at a node would maintain a constant head (i.e. $h(t=1) = h(t=2)$). To solve for it one must solve the steady-state groundwater flow equation in the domain affected by the initial ($t=1$) stress. At $t=2$, $Q(t=1)$ is no longer considered, rather the artificial steady-state stress which would cause $h(t=2) = h(t=1)$ is calculated, $Qss(t=1)$. In $t=2$, rather than multiplying $Q(t=2)$ by the response coefficient, the difference between $Qss(t=1)$ and $Q(t=2)$ is multiplied by the response coefficient. Since $Qss(t=1)$ contains the information about $Q(t=1)$, $Q(t=1)$ must not be considered at $t=2$ and thus single time-period response coefficients can be used instead of multi-period response functions. In many cases the managed stresses will only affect neighboring cells and a small "scanning grid" steady-state groundwater model (Illangasekare et al. 1984) can be solved rather than the full grid. In summary, by calculating the artificial steady state stress at the end of each time period, single time-step response functions can be used in lieu of full transient multi-period responses. These techniques are useful when groundwater responses to stresses are slow, implying a large summation over index t (i.e. a large T in equation 8). This simplifies the use of response functions as the memory of past flows need not be retained in the model.

2.2.4. Comparison of Simulation Methods

Andreu and Sahuquillo (1987) compare the relative efficiencies of different distributed groundwater simulation techniques for inclusion in systems models. They compare sequential time-marching, response function and the eigenvalue methods by deriving equations for the number of floating point operations (flops) per time period required by each technique. Figure 6 shows which technique is more efficient for different relative quantities of control locations (c) and total number of groundwater cells (N). Control locations are those locations where a groundwater head result is needed in the management model. The figure shows that response functions are most efficient when control locations are sparse (<4%) and the classical time marching simulation is best when groundwater response at most cells is relevant (man control locations). The eigenvalue method is best for cases in between those two extremes. The paper claims that use of reinitialization (i.e. use of response functions 1 time-step long with calculation of the "artificial steady state") is no more efficient then sequential time marching because "heads need to be computed in all cells for every time period." This is not the case if the scanning grid is used, in which case only heads in the vicinity of control locations must be modeled at each reinitialization. However, if scanning grids are not used, it is probable given the flop equations of Andreu and Sahuquillo that sequential time-marching will be as or more efficient that response functions with reinitialization.

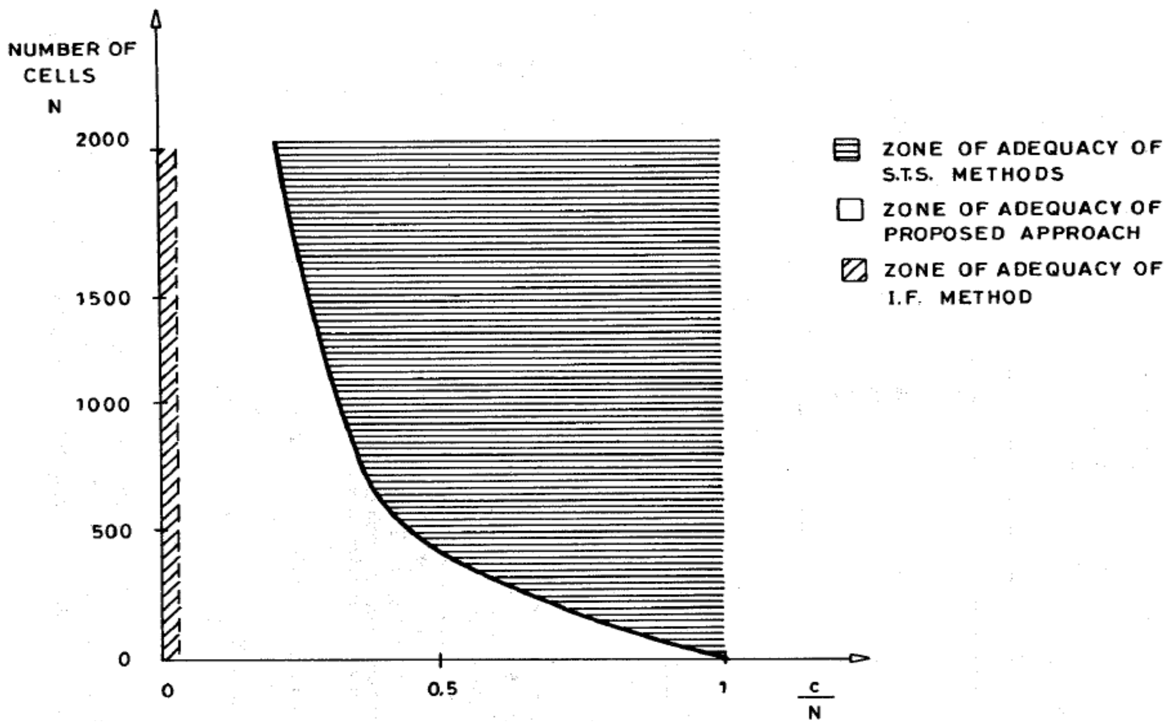

Figure 6. Zones of numerical efficiency of three distributed groundwater simulation methods for long time horizons. S.T.S. refers to sequential time-marching, the proposed method is the eigenvalue method, and the I.F. stands for influence function (i.e. response function) method. The x-axis refers to the proportion of control locations (c) relative to the total number of grid cells, N.

Source: Andreu and Sahuquillo (1987)

2.3. Groundwater Optimization

Groundwater models can be incorporated into optimization model constraint sets using either embedding or response function approaches.

2.3.1. Embedding Methods

With embedding methods a numerical model of groundwater flow is placed in the constraint set of the optimization model. The three methods described below were reviewed in the simulation section; the same equations are used here but passed to an optimization solver rather than to simulation solver.

Storage Coefficient Equation Method

This method models basic head responses to groundwater stresses. Equation 4 is incorporated in the model constraint set, providing *h(t)* as a function of net stresses, *Q(t)*, on the groundwater unit.

Sequential Time-Marching Method

In the sequential time-marching method the spatially and temporally discretized approximations of the groundwater flow equation are embedded into the constraint set as equality constraints (Aguado et al. 1974; Alley 1976; Willis and Newman 1977).

Equations 3 and 4 for steady-state and transient time marching simulation are given below in indexed notation for inclusion as constraints:

$$\sum_j G_{i,j} * h_j = rhs_i + q_i \quad \forall i$$

$$\sum_j \left(\Delta t * G_{i,j} + D_{i,j}\right) * h_j^{t+1} = \sum_j D_{i,j} * h_j^t + \left(rhs_i + q_i^t\right) * \Delta t \qquad \forall i,t$$

where q_i^t = pumping at model nodes *i* (or alias index *j*) during time period *t* (decision variable)

h_j^t = hydraulic head at location *i* (or *j*) during time period *t* (decision variable)

Gi,j = conductance matrix element *i,j*

Di,j = capacitance matrix element *i,j*

rhsi = conductance matrix elements that depend on boundary conditions

Δt = length of time period *t*.

Inequality constraints on minimum and/or maximum head levels and stresses can be added to constraint sets. Because the embedded approach models the flow equation at each discretized node, it may lead to unnecessarily large constraint sets if the modeled aquifer is large, or finely discretized, and the number of managed variables relatively small (Gorelick 1983; Peralta et al. 1991). A model with *N* variable head nodes will contain *N* hydraulic head decision variables and between one and *N* net stress decision variables (recharge/pumping actions are considered only at managed stress nodes). If the problem is transient with *T* periods, the maximum number of decision variables and linear constraints will be at least *2NT*.

The embedding method is advantageous if the model is relatively small and contains many possible pumping locations and drawdown limitations (Gorelick 1983; Peralta et al. 1991). Practical applications of the embedding of confined aquifers in linear programs appear in

Aguado et al. (1974), Yazdanian and Peralta (1986) and Peralta and Killian (1985). Applications to non-linear systems such as unconfined aquifers are found in Aguado et al. (1974), Gorelick et al. (1984), Willis and Finney (1985), Wanakule et al. (1986), Rastogi (1989), and Gharbi and Peralta (1994).

Eigenvalue Method

Another way of embedding distributed groundwater models into optimization models is to use a spatially discretized but time-continuous groundwater model (e.g. Willis and Newman 1977; Willis 1979; Willis 1984; Willis and Liu 1984; Pulido-Velazquez et al. 2006). The following indexed notation of equations 6 and 7 can be used to embed the efficient eigenvalue method (Andreu and Sahuquillo 1987) into a constraint set (e.g. Pulido-Velazquez et al. 2006).

$$h_{cv}^{t} = \sum_{i} ared_{cv,i} * l_{i}^{t} \qquad \forall cv, t$$

$$l_{i}^{t} = e_{i,i} * l_{i}^{t-1} + \sum_{bs} f_{i,bs} * QI_{bs}^{t} \qquad \forall i, t$$

where h^{t}_{cv} = mean hydraulic head in groundwater sub-basin *cv* at time *t* (index *cv* refers to control variables, which can be piezometric head or any linear combination of head values)

$ared_{cv,i}$ = element of cv^{th} row and the i^{th} column of the reduced eigenvector matrix (index *i* enumerates all groundwater model cells)

l_{i}^{t} = element *i* of the aquifer state vector at time period *t*

ei,i = element *i* of the diagonal matrix of eigenvalues

$f_{i,bs}$ = i^{th} row and bs^{th} column of the pre-calculated F matrix

QI_{bs}^{t} = element bs of the basic stress intensity vector at time *t* (index bs refers to basic stresses)

The efficient eigenvalue method has the accuracy and theoretical rigor of the embedding method but without the necessity of incorporating constraints for all groundwater model nodes.

2.3.2. Response Function Methods

Using response functions to represent steady-state or transient groundwater responses in optimization models is common (e.g. Heidari (1982), Elwell and Lall (1988), Nishikawa (1998), and Larson et al. (2001)). The equations used are the same that were listed in section 1.2.3.3 on response function simulation. The method is applied in many groundwater management contexts such as flow direction and piezometric surface design, groundwater-surface water interaction, subsidence management, pollution source control and groundwater quality management. The response function approach has also been used to optimize conjunctively managed surface and groundwater resources. Notable applications include Reichard (1987), Basagouglou and Mariño (1999), Belaineh et al. (1999), and Barlow et al. (2003). Fredericks et al. (1998) and Miller et al. (2003) use response functions with an iterative optimization formulation solved for each time period of a simulation model that considers stream-aquifer interaction.

Several computer programs construct response function databases (matrices) and allow the user to use them in optimization models. Such software systems include MODMAN (Greenwald 1998), SOMOS (Peralta 2001), MODOFC (Ahlfeld and Mulligan 2000) and GWM (Ahlfeld et al. 2005). MODMAN and SOMOS implement links to commercial optimization packages while MODOFC and GWM perform the optimization internally.

Reinitialization and artificial steady-state variations of the response function method described at the end of the simulation section could be applied for optimization. Whether reinitialization makes sense for optimization models depends on the length of the reinitialization period. Each reinitialization is computationally equivalent to running a steady-state groundwater model over the modeled domain. If reinitialization occurs at each time period the response function method is computationally inefficient since it is equivalent to embedding the full groundwater model using sequential time-marching. A potentially useful application would be deterministic optimization in which the full period of analysis is divided into shorter optimization model time-horizons of length N (e.g. Diaz et al. 2000). Reinitialization would occur at the end of each optimized time horizon (e.g. after N time periods). This would require response functions of length N; responses beyond that length would not be needed since the artificial steady-state variables would be calculated at the end of the N-period reinitialization.

2.3.3. Stream-Aquifer Interaction

When stream-aquifer interactions are important in a water resource system, it is beneficial to include them in integrated models. With the embedding method, stream-aquifer interaction will depend on what boundary conditions are used in the groundwater model to represent the stream. Most commonly a head-dependant flow (Cauchy type) boundary is selected, in which case the following nonlinear (discontinuous) constraint equation (McDonald and Harbaugh 1988; LaBolle et al. 2003) can be added to model both hydraulically connected and disconnected stream-aquifer flow:

$$Q_{riv} = G_{riv}\left[h_{riv} - max\left(h_{aq}, RBOT\right)\right]$$

where Q_{riv} = flow from the stream into the aquifer (> 0 for a loosing stream, recharging aquifer),

G_{riv} is the streambed conductance given by $Griv = KA/b$,

h_{riv} is the elevation of the free water surface in the river,

h_{aq} is the piezometric head in the aquifer cell which contains a stream,

$RBOT$ is the elevation of the streambed layer bottom for which the conductance was provided.

This equation has been embedded into management models in the steady-state case (e.g. Peralta and Datta 1990) and in the transient case (e.g. Gharbi and Peralta 1994).

In the context of stream-aquifer management studies the response function method has been used the most. Initial papers by Maddock (1974) and Morel-Seytoux and Daly (1975) provide expressions for stream-aquifer flow that closely resemble the linear equation of piezometric head in response to stresses. Applications of this method include Illangasekare and Morel-Seytoux (1982), Fredericks et al. (1998), Basagouglou and Mariño (1999), Belaineh et al. (1999), Miller et al. (2003), and Barlow et al. (2003).

With the eigenvalue method stream-aquifer interactions can be represented in 2 ways: declaring a basic stress which is comprised of unit recharge rates for all cells overlain with a stream or selecting flow exchange through cells of specified head boundary condition as a control variable. The first case was used in Pulido-Velasquez et al. (Pulido-Velazquez et al. 2006).

Finally, a variety of analytical equations exist for modeling stream-aquifer exchange that can be integrated into a management model (Glover and Balmer 1954; Jenkins 1968; Sahuquillo 1983b; Hantush and Marino 1989; Hantush et al. 2002; Pulido-Velazquez et al. 2005).

2.3.4. Comparison of Optimization Methods

Andreu and Sahuquillo (1987) discuss computational requirements of using sequential time-marching, response function and eigenvalue simulation methods. The article derives equations for the number of floating point operations involved in previous calculations, per time step calculations and storage calculations. In the case of optimization models such operation counts cannot be derived as much depends on the particular algorithms of each optimization solver. Peralta et al. (1991) compared embedding sequential time-marching and response matrix techniques in the steady-state case. Several review articles discuss the merits of each method, especially Gorelick (1983). In general it is found that for problems where pumping is considered in a large proportion of groundwater model cells, e.g. 25% in the case of Peralta et al. (1991), the embedding method adds fewer equations to the constraint set of an optimization model. While embedding methods are more efficient for small systems (Peralta et al. 1991; Peralta et al. 1992), response function techniques are more widespread. Advantages, disadvantages and recommended uses for each method are found in Table 2. Chapter 3 of this report is a computational comparison of embedding optimization methods using a 2-D model of the Sacramento Valley, California.

Table 2. Advantages, Disadvantages and recommended uses of groundwater flow optimization methods.

	Advantages	Disadvantages	When to Use
Embedding sequential time-marching	• Rigorous, can model unconfined aquifers & other non-linear equations • Doesn't require memory of past stresses	• Large sets of equations • Some reported instabilities in optimizing large transient problems	• Dense models (many control and stress locations) • Nonlinear equations needed • Sustainable yield steady-state problems
Eigenvalue	• Reduces problem to control and	• Assumes linearity • Mathematically	• Medium problems (in between dense

	management variables • Preprocessed from a calibrated model • Doesn't require memory of past stresses	complex, non intuitive	and sparse) • When maintaining time memory of past stresses is impractical
Response function	• Reduces problem to control and management variables • Preprocessed from a calibrated model • Conceptually simple • Response functions useful by themselves	• Assumes linearity • Transient formulation requires memory of past stresses	• Sparse problems (few control and stress locations)

2.4. Conclusions

Water resource systems models can choose from a wide variety of groundwater formulations in both simulation and optimization modes. Differences between simulation and optimization modeling were reviewed. Both modeling types must choose between lumped basin (single- or multi-cell models) or spatially distributed formulations. Distributed models can use either numerical approximations of the groundwater flow equation (in 1,2 or 3 dimensions) or response function databases of groundwater response to local stresses. Incorporating distributed formulations into systems model comes at a much larger computation cost but enables representation of local hydrogeologic conditions. Because water resource management models increasingly strive to represent local conditions, distributed formulations will increasingly be used in both engineering and economic models.

3.0 Computational Comparison of Groundwater Optimization Embedding Techniques—Application to the Sacramento Valley

3.1. Introduction

This chapter describes an upscaled regional groundwater model of California's Sacramento Valley and implements three different numerical techniques to simulate and optimize the system. Different options to represent groundwater within water resource systems models were described in chapter 2. Two basic strategies are distinguished: lumped single- or multi-cell models and spatially distributed models.

In lumped groundwater representations a single groundwater level is valid over a specified region or sub-basin. Flow within sub-basins is not considered although flow between adjacent basins can be modeled using the Darcy equation. Regional groundwater levels in such models are either fixed or modeled dynamically using regional storage coefficients. Various single- and multi-cell aquifer models are described in chapter 1. Recent examples of these methods used in California water management models include CALSIM2 (Draper et al. 2004), CALVIN (Draper et al. 2003) and WEAP (Joyce et al. in press). In this chapter a multi-cell model is implemented using the storage coefficient equation without considering flow between basins and is compared to distributed models.

Spatially distributed groundwater models are numerical approximations of groundwater flow analytical equations. As described in chapter 2, embedding or response function approaches are available to incorporate a distributed numerical groundwater model into management models. The response function approach is the one most commonly applied in the literature and in practice; in California it was recently applied in CALSIM III (Stockholm Environment Institute 2007). The response function method differs fundamentally from the lesser-known techniques evaluated in this study, all of which embed groundwater model equations directly into the mathematical program constraint set. This chapter implements and investigates the speed and reliability of a sequential time-marching model, using an implicit finite-difference scheme, and a continuous-time model, solved by eigenvalue methods. The different methods are built and tested in simulation mode then embedded into an optimization model constraint set. The techniques are implemented using the Sacramento Valley Groundwater Model (SVGM), an upscaled version of a published calibrated model of California's Central Valley (CVRASA1). Both of these models are described below. SVGM is good example of a regional groundwater model that could be integrated into a regional water resource system "management model". In fact such a management model incorporating SVGM can be found in Appendix B.

3.2. Groundwater in the Central Valley

The Central Valley is an elongated topographic basin (640 km long, 80 km average width) surrounded by several mountain ranges. It is drained by two major rivers, the Sacramento River in the North (Figure 7) and the San Joaquin River in the South. The valley is underlain by a heterogeneous alluvial-fill aquifer system consisting of discontinuous beds of clay, silt, sand, and gravel. The thickness of these deposits averages about 730 m and increases from north to south, with a maximum thickness of more than 2700 m near Bakersfield. Deposits are continuous except for the Sutter Butte volcanic plug in the Sacramento Valley. Groundwater pumping has drastically changed natural recharge and discharge rates to the aquifer system. In the 1960's the recharge rate increased 5-fold from percolated irrigation water (Bertoldi et al. 1991). Rather than being discharged by evapotranspiration and baseflow as occurred before extensive well pumping, most discharge now occurs through pumping. Total flow through the system has increased to more than six times pre-stressed conditions to more than 15 km^3 (Bertoldi et al. 1991). Vertical movement of groundwater has also been greatly enhanced by more than 100,000 irrigation wells perforated at various levels.

Figure 7. The Sacramento Valley lies in the northern portion of California's Central Valley.

Source: CDWR (2003a)

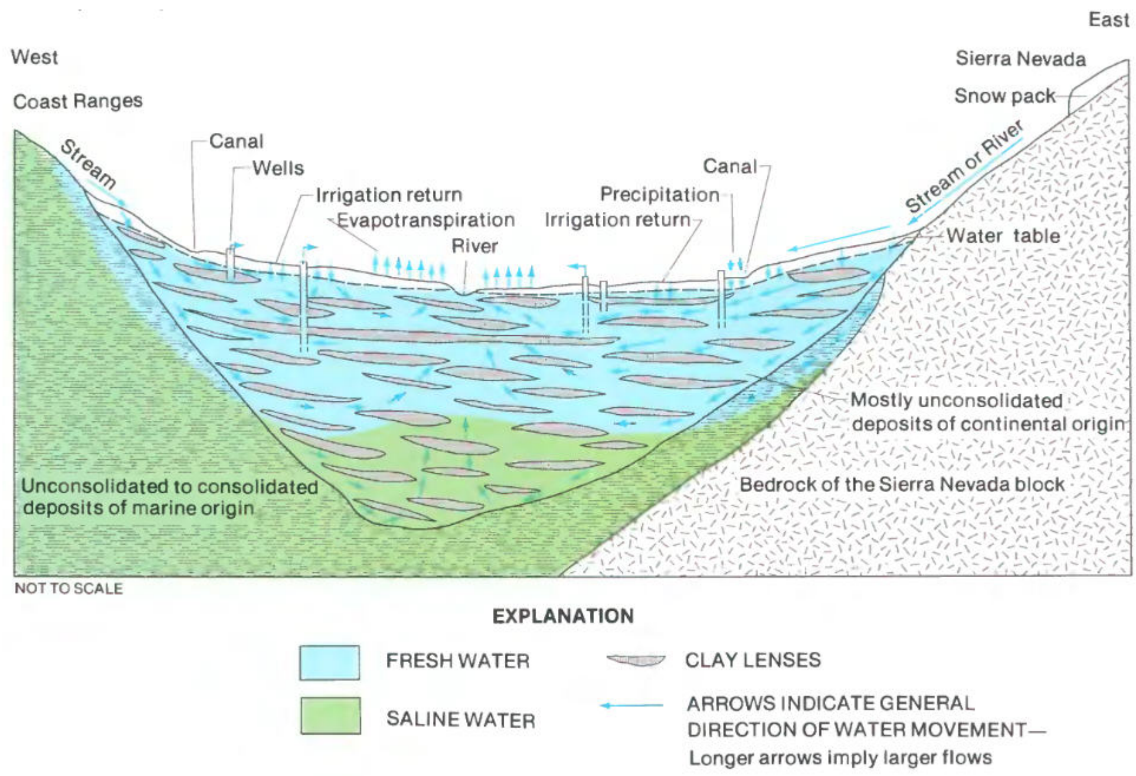

Figure 8. Cross-sectional view of Central Valley hydrogeology and recharge/discharge fluxes

Source: Williamson et al. (1989)

3.3. CVRASA1 Groundwater Model

The Central Valley Regional Aquifer-System Analysis (CVRASA1) groundwater model was built by the U.S. Geological Survey during the 1980's (Williamson et al. 1985; Williamson et al. 1989). The model uses a finite-difference scheme for spatial and temporal discretization of the groundwater flow equation. Originally developed with the program of Trescott (1975), a later version of CVRASA1 using MODFLOW (McDonald and Harbaugh 1988) is available from the Sacramento USGS office.

The quasi-three-dimensional regional model has four layers and at most 529 square grid cells per layer; each cell is 6 by 6 miles (9.66 km). The vertical dimensions of the blocks vary; their depths are incorporated into depth-averaged aquifer parameters. The model simulates heterogeneity in the aquifer system by varying aquifer properties from block to block and averaging to represent the heterogeneity within each block. Given the large size of each block (almost 100 km2) only broad regional hydrogeologic characteristics are described.

Transmissivity (T) values are time-invariant in all model cells (as in confined aquifers). This entails an estimated error of maximum 12% (Williamson et al. 1989) (P 16) for unconfined aquifer cells where T should change with head.

Layers 1 (bottom) through 4 (top layer) encompass different hydrologic conditions (Figure 9). All pumping occurs in topmost layers 3 and 4; the boundary between them is where most wells are no longer perforated. When present, the Corcoran Clay Member of the Tulare Formation also serves as a boundary between the two top layers. Layer 4 represents the mostly unconfined superficial aquifer which is in contact with surface water bodies and rivers. Sixty one percent of pumping occurs in this top layer in CVRASA1. Layer 3 represents deeper pumping in mostly confined zones. The bottom layer (# 1) consists of the continental deposits below the depth penetrated by any production wells.

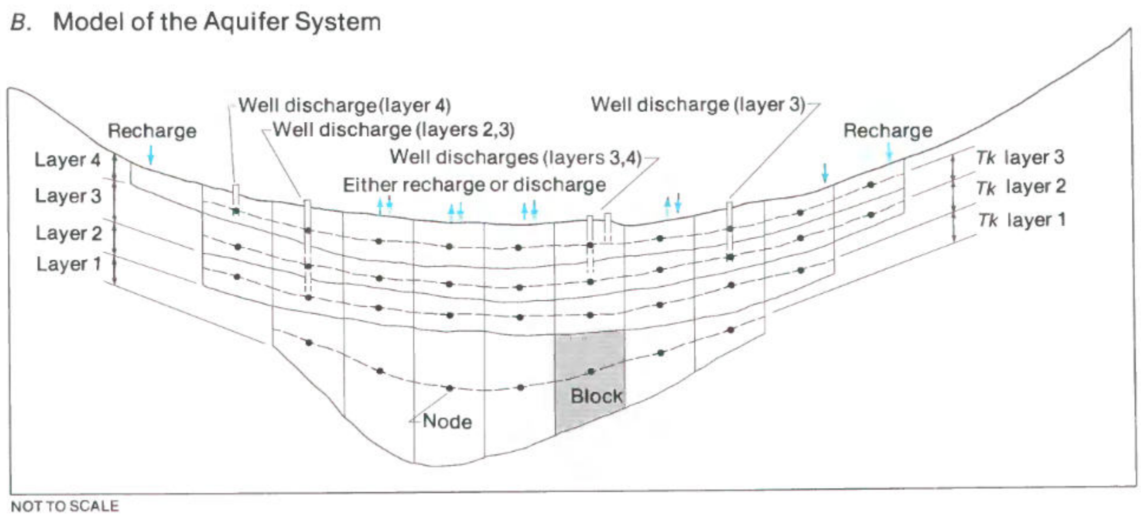

Figure 9. Cross-sectional view of Central Valley in CVRASA1 showing four layers and general patterns of recharge, discharge, and groundwater flow

Source: Williamson et al. (1989)

The Central Valley model is surrounded by no flow boundary conditions except at the Sacramento-San Joaquin Delta where there are 3 specified head cells in layer (4 Figure 10). Williamson et al. (1989) describe the boundaries as follows:

> "Generally, the boundaries along the west side of the valley and beneath the aquifer system represent less permeable marine deposits; along the east side, the boundary is represented by less permeable igneous or metamorphic rocks. At the south end of the Central Valley, the boundary of the modeled aquifer system is the White Wolf fault, which acts as a barrier to flow. At the north end, the boundary is the Red Bluff arch, which is a series of low-lying hills consisting of northeast-trending anticlines and synclines. The series of hills acts as a barrier to ground- water flow. In addition, both the Sutter Buttes and the Kettleman Hills within the valley restrict ground-water flow and were assumed virtually impermeable"

Aquifer-system properties such as thickness, hydraulic conductivity, and storage coefficient were determined by the USGS using methods that could be applied throughout the valley so as not to bias results. Transmissivity values (Table 3) were calculated for each layer using data on horizontal hydraulic conductivity (from pump tests and driller's logs) and model block thickness. Storage coefficients are much higher in the unconfined layer 4 where they are equivalent to specific yield. This is reflected somewhat in model results; piezometric levels show less seasonal variation in the top unconfined layer than in lower confined layers. A leakance (Tk) coefficient representing vertical hydraulic conductivity is used in CVRASA1 to model vertical flow between model cells in different layers. Leakance is the ratio of vertical conductivity to the thickness of the confining beds values. Calibration showed modeled heads are more sensitive to leakance parameters than to any other aquifer parameter. Leakance terms were estimated physically but subsequently varied to calibrate the model to observed values.

Table 3. Aquifer characteristics of the Central Valley by CVRASA1 model layer

	Transmissivity (ft^2/day)			Storage Coefficient		
	Avg	**Max**	**Min**	**Avg**	**Max**	**Min**
Layer 4	0.020	0.168	0.00003	0.092	0.200	0.02000
Layer 3	0.029	0.165	0.00026	0.002	0.020	0.00015
Layer 2	0.034	0.195	0.00007	0.003	0.030	0.00015
Layer 1	0.090	0.521	0.00003	0.009	0.046	0.00003

The CVRASA1 model simulates 16.5 years of historical flow between 1961 and 1977 with a 6 month time step. CVRASA1 uses 2 stress terms: the first is a constant calibration term and the second is a time-varying net stress term based on historic water budget calculations.

The dynamic net recharge/discharge (net stress) term is built upon an in-depth regional water budget summarized in the Appendix A of Williamson et al. (1989). Terms estimated during the modeled period in Appendix A include; (1) excess precipitation, (2) ungaged runoff from small streams, (3) river losses (+, or positive) and gains (-, or negative), (4) evapotranspiration of applied irrigation water, (5) surface water diverted to irrigation districts, (6) agricultural pumpage, and (7) municipal pumpage. Based on a budget per grid cell, a net recharge/discharge term per 6-month period is used to stress the model.

To better fit observed water-table elevations, local adjustments in the net recharge/discharge term were necessary. A static calibration stress term for each 6-month period was added to improve similarity between observed and simulated piezometric levels.

River leakage and evapotranspiration are not calculated in the model using a head-dependent boundary condition. The authors explain this choice saying:

> "By regression analysis, the authors found that the dominant factors affecting recharge and discharge rates in the aquifer system are the amount of surface-water flow, land use, and canal systems; these factors affect net recharge/discharge more than the head change in the aquifer."

River losses and gains were calculated based on historical streamflow and other hydrologic data using the water budget method (Mullen and Nady 1985). During calibration, the authors realized estimates of river losses/gains and small stream recharge were too large and were divided by 5. The authors justify this action saying: "This adjustment was necessary because of systematic errors in estimating stream losses/gains, local recharge and discharge within a model block, and inability of the model to simulate the real aquifer system."

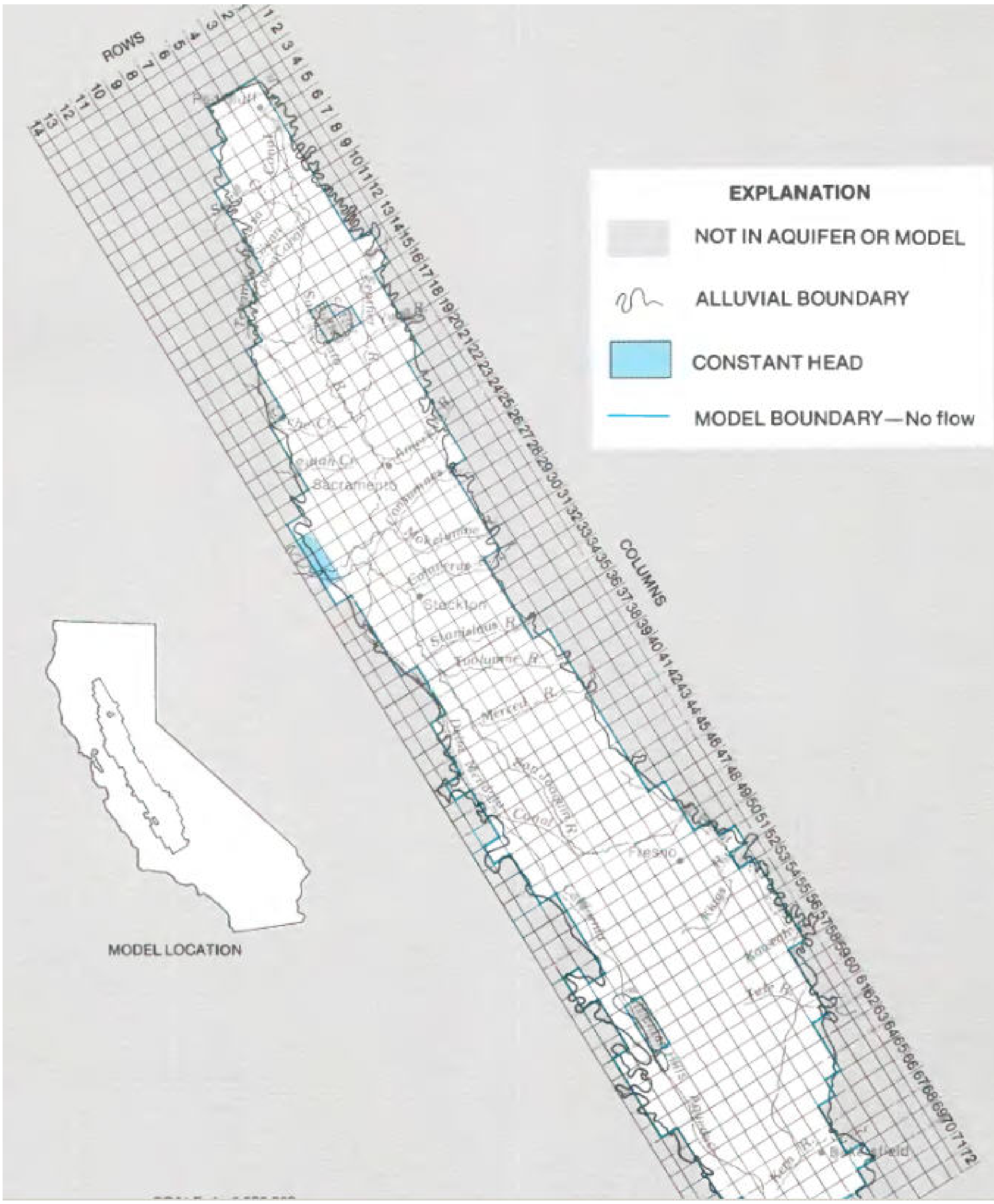

Figure 10. CVRASA1 finite-difference groundwater model grid

Source: Williamson et al. (1989)

3.4. Sacramento Valley Groundwater Model (SVGM)

The Sacramento Valley Groundwater Model (SVGM) is built from part of CVRASA1 and upscaled for use as a management model or as a component of an integrated systems model (e.g. Appendix B). It was built using the northern-most 170 cells of CVRASA1, converting it from three-dimensions (3D) to 2D and by changing the time-step from 6-months to 1 month, but retaining the same horizontal 2D grid. The SVGM grid with its three specified head cells (at the Sacramento-San Joaquin River Delta) is displayed in Figure 11.

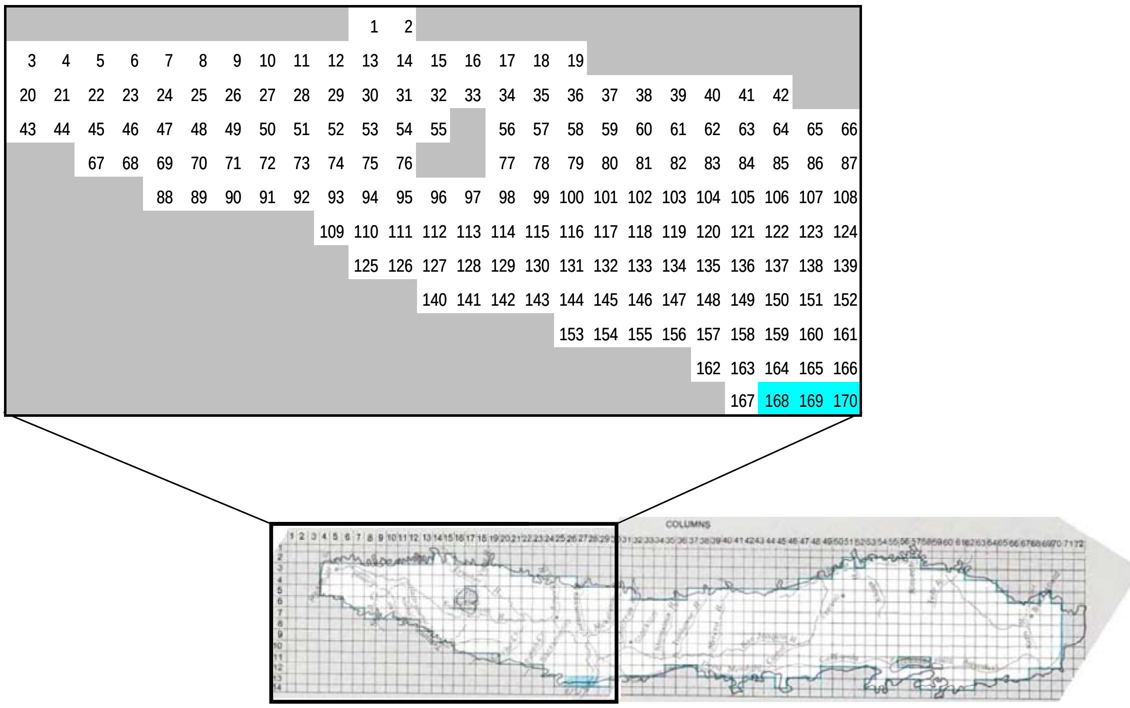

Figure 11. Sacramento Valley Groundwater Model (SVGM) grid with its 167 variable head 10 x 10 km cells and 3 specified head cells. SVGM is a 2D version of the northern portion of the CVRASA1 USGS groundwater model.

3.4.1. Upscaling From 3D to 2D

Converting the model from 3D to 2D involved combining the 4 superimposed layers of CVRASA1. This change simplifies the distributed model for use in management models where computational speed is required. Because both transmissivity and storage coefficients are depth-averaged parameters, their value for the combined 1-4 layer is the sum of the parameter in each layer. Leakance coefficients between layers (Tk) are not used in the 2D version. Average, minimum and maximum values for T and S for the combined-layer SVGM are compared to individual CVRASA1 layers in Table 4. Most importantly, the storage coefficient of the combined layer is similar to the specific yield of the unconfined layer (layer 4). This means the combined layer will essentially be modeling the top unconfined layer; this is confirmed in simulation results presented later in this section.

Table 4. Aquifer characteristics for the Sacramento Valley (cells 1-170) from CVRASA1 and Sacramento Valley Groundwater Model (SVGM)

Sac. Valley	Transmissivity (ft^2/day)			Storage Coefficient		
	Avg	Max	Min	Avg	Max	Min
Layer 1-4	0.060	0.27	0.00137	0.090	0.172	0.0441
Layer 4	0.013	0.05	0.00018	0.082	0.170	0.0200
Layer 3	0.015	0.06	0.00026	0.002	0.004	0.0003
Layer 2	0.011	0.09	0.00007	0.002	0.006	0.0003
Layer 1	0.026	0.11	0.00003	0.005	0.030	3E-05

Potential concerns with upscaling include loosing accuracy in estimating groundwater heads and generally misrepresenting the system. For example, well pumping from the confined layer 3 should have lower and more variable heads due to the layer's smaller storage coefficient. This is relevant since 41% of CVRASA1 pumpage occurs in layer 3. Another concern is that stream-aquifer interaction cannot be considered if the upper unconfined layer is not modeled separately. Both concerns are valid and up to a point sacrificed in a trade-off for gains in management model simplicity and speed. Several observations however help argue that the loss of information from the 2D simplification is only moderate. The first is that results from several CVRASA1 cells indicate that piezometric head levels and their seasonal variability differs relatively little between different layers (see cells 21, 31 or even 61 of Figure 14). Secondly, the coarse CVRASA1 100 km2 grid cell resolution means precise stream-aquifer modeling is not realistic; therefore highly accurate modeling of the upper unconfined aquifer is less important.

3.4.2. Historical Stresses

CVRASA1 simulates from 1961 to 1977 using historical stresses (i.e. recharge and discharge rates based on historical time-series data) which are combined into a single net stress term (1 for each cell during each 6-month time period). Data on the different water balance components that make up the net stresses are described in Appendix A of Williamson et al. (1989) although the full data set is no longer available. Because the management model in Appendix B is monthly and needs to separate managed fluxes (pumpage and agricultural percolation) from hydrologic recharge (rainfall-fed and stream-fed recharge), it was useful to build two distinct monthly stress time-series: a 'hydrologic' stress term composed of historical rain, stream and canal recharge and a 'management' stress term comprising historical pumpage, agricultural percolation, and the CVRASA1 calibration term. The hydrologic stress term was taken from the Central Valley Groundwater Surface Water Model (CVGSM); it consists of monthly historic rain-fed recharge and stream and canal recharge rates (USBR 1997). Although SVGM does not represent stream-aquifer exchange dynamically, including a fixed recharge term assures this part of the water balance is represented, albeit in a simplified fashion. Because CVGSM is a finite element code that uses triangular elements, water balance terms were averaged by groundwater sub-basin (Figure 12) and then redistributed evenly amongst the SVGM finite-difference cells of each sub-basin.

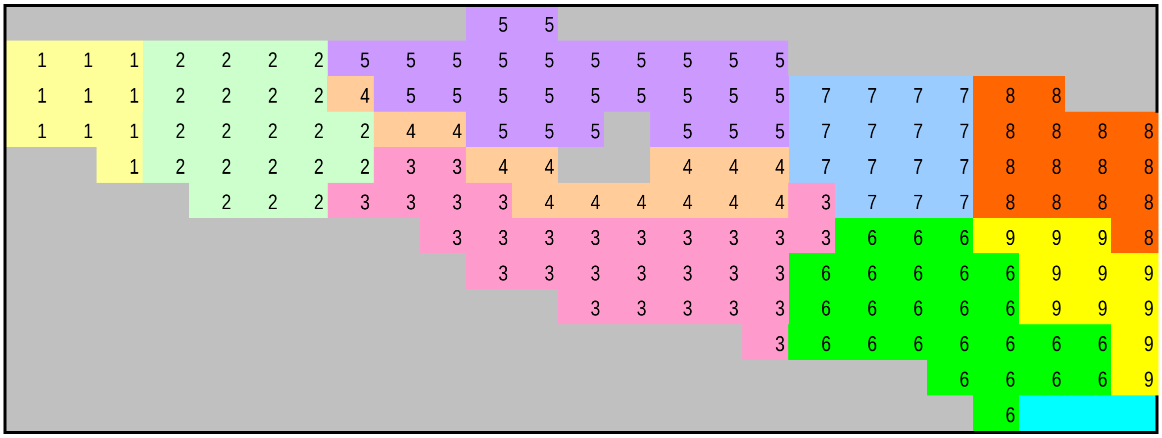

Figure 12. Groundwater sub-basins whose boundaries coincide with the Central Valley Production Model (CVMP) regions

The second 'management' stress term combines all other historic stresses into 1 term: pumpage, agricultural percolation, and the CVRASA1 calibration term. This stress term was calculated such that the combination of both the hydrologic and managed stress terms would be the same as the unique CVRASA1 historical stresses. The 3 stress terms are displayed in Figure 13. The hydrologic stress term will stay the same in the management model whereas the managed stress term will become a decision variable of the management model.

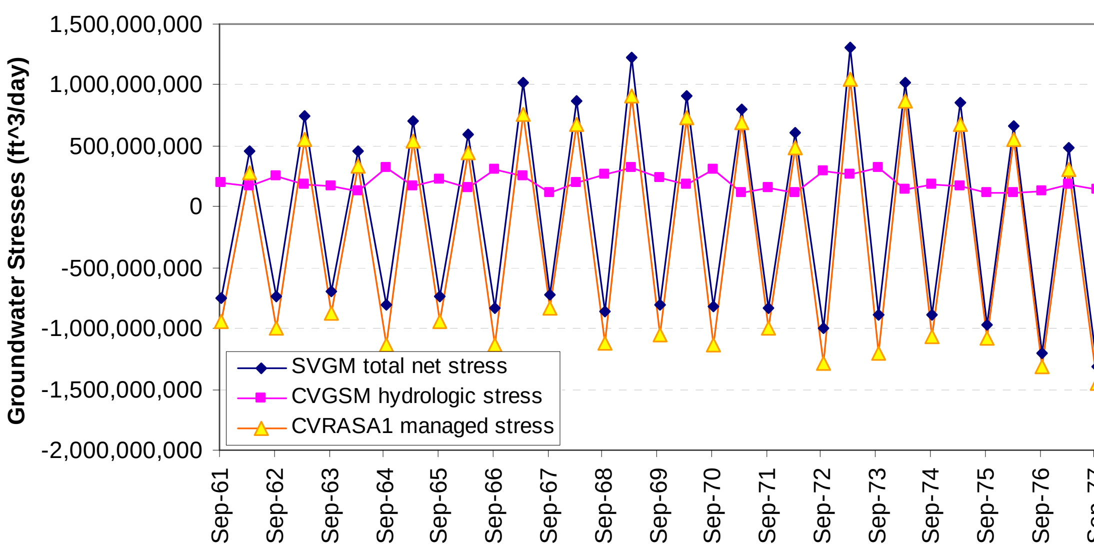

Figure 13. The SVGM total net stress term is composed of hydrologic stresses (rain, stream, canal recharge) taken from CVGSM and managed stresses (pumping, irrigation water percolation, calibration term) taken from CVRASA1.

The single CVRASA1 historical stress time-series has a 6-month time-step while SVGM runs on a monthly time-step. Therefore the CVRASA1 managed stress term needed to be broken into monthly intervals. Because groundwater pumping data is scarce in California another data source was needed for the breakdown to monthly managed stresses, be they positive (irrigation water recharge) or negative (groundwater extraction). To estimate the breakdown of pumping during the growing (summer) season it was assumed that pumping follows economic incentives: during months when water has the highest agricultural economic value, the most groundwater pumping occurs. Breakdown of pumping in the summer was according to water delivery targets of CALVIN model (Draper et al. 2003) sub-basins. For each month and water demand region, the CALVIN model has a water delivery target at which no scarcity costs will be incurred. In this way months with a higher water allocation target produce a similarly higher pumping rate. The calculated rates appear in Table 5.

Table 5. Breakdown of pumping rates in summer as derived from CALVIN water allocation targets.

REGION:	1	2	3	4	5	6	7	8	9	Average
APR	13%	11%	18%	16%	17%	17%	17%	10%	11%	14%
MAY	17%	18%	17%	16%	18%	18%	18%	16%	15%	17%
JUN	17%	19%	22%	23%	22%	20%	21%	21%	25%	21%
JUL	22%	23%	23%	24%	22%	22%	21%	25%	27%	23%
AUG	19%	18%	16%	18%	17%	15%	17%	19%	17%	17%
SEP	12%	11%	3%	4%	5%	8%	6%	9%	6%	7%

For the September through March winter season, Figure 13 shows CVRASA1 data identifies a high recharge rate. These high rates account for percolation of applied water but they also make up for differences between the hydrologic input time-series of the CVGSM and CVRASA1 models. Breakup of winter recharge rates was unknown and approximated as 0.1, 0.15, 0.25, 0.25, 0.15, and 0.1 for the months of October through March.

3.4.3. SVGM Numerical Model and Results

SVGM runs using a custom MATLAB code implementing an implicit scheme finite-difference groundwater model, analogous to MODFLOW (McDonald and Harbaugh 1988). Basic descriptions of the model's equations are in chapter 2. Unconfined aquifers are modeled as linear confined aquifers (with constant transmissivity) so that associated management models can use linear programming. This is valid at the regional scale for deep unconfined aquifers where changes in head due to stresses are small compared to the saturated thickness of the aquifer (Reilly et al. 1987).

Before running SVGM (with monthly stresses), the custom finite-difference code was run with the CVRASA1 6-month stresses but combining layers 1-4. The run shows that the custom finite difference code works correctly and shows how combined layers 1-4 simulated head levels compare to CVRASA1 results for individual layers. Results are shown in Figure 14 for individual cells (see Figure 11 for cell numbering) and in Figure 15 aggregated by groundwater sub-basins (see Figure 12 for sub-basin numbering). Individual model cells show a wide variety of behaviors. Cells 61 and 97 follow the expected pattern of a relatively steady unconfined layer

4 with deeper and more variable water levels in confined layers 1-3. Many cells however show layer 4 varying more widely than lower confined layers (e.g. 1, 48, 56, 107); in other cells all layers show similar levels and seasonal variation is comparable (e.g. 31). For results aggregated by sub-basin (Figure 15) similar observations are valid; sub-basins 1, 2 and 7 show expected behavior while most other sub-basin show similar magnitudes of seasonal variability, with heads staying between 5 and 15 feet apart.

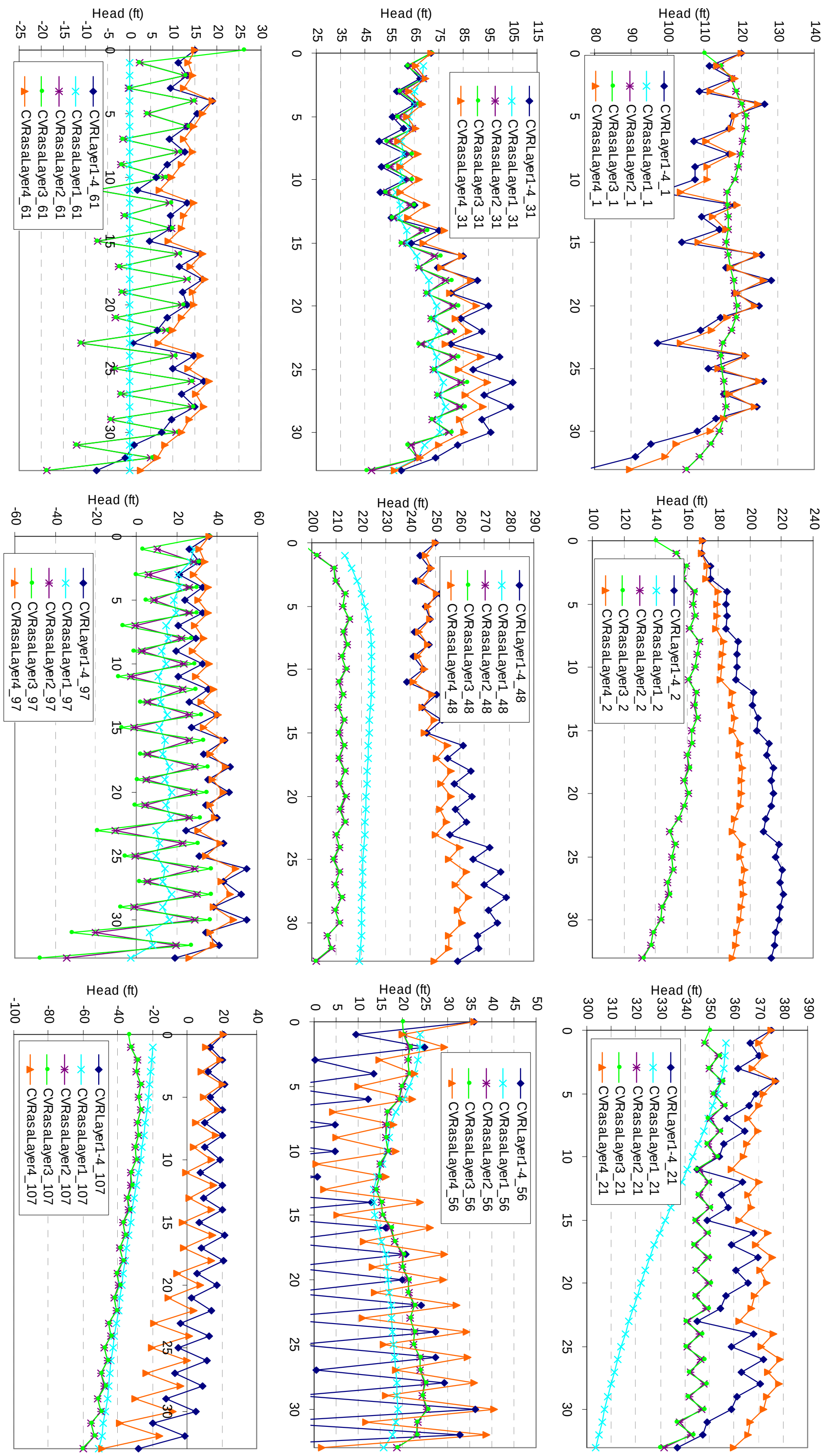

Figure 14. Semi-annual CVRASA1 individual cell comparisons: combined layers 1-4 and individual layers (cell number is last number in legend).

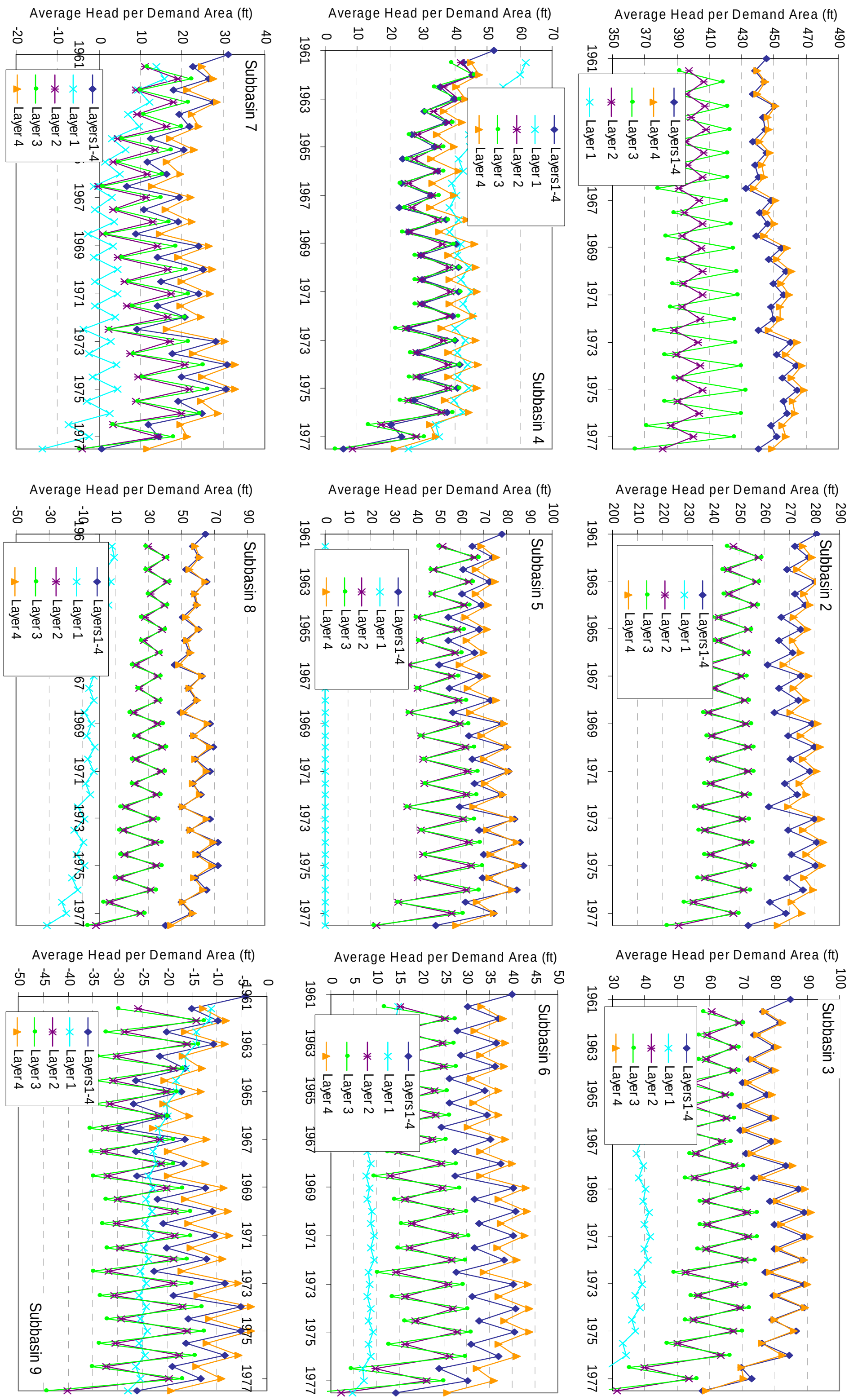

Figure 15. Sub-basin results for CVRASA1 with combined layers 1-4 and individual layers

Results for both individual groundwater model grid cells and aggregated sub-basins show that the combined layer 1-4 behaves very similarly to unconfined layer 4. The 2D upscaled model basically represents the top unconfined layer. The fact that SVGM models the aquifer as confined is acceptable considering seasonal head variations rarely exceed 5 m while the saturated aquifer thickness averages 730m. Because 60% of pumping in the Sacramento Valley occurs in this superficial layer (according to CVRASA1 data) and because heads of all layers behave reasonably similarly at the sub-basin scale, we tentatively assume that the 2D model has effectively simplified the system with relatively little loss of information. Finally, monthly downscaling of stresses in SVGM was successful and results closely resemble those of the 6-month CVRASA1 combined layer model (Figure 16).

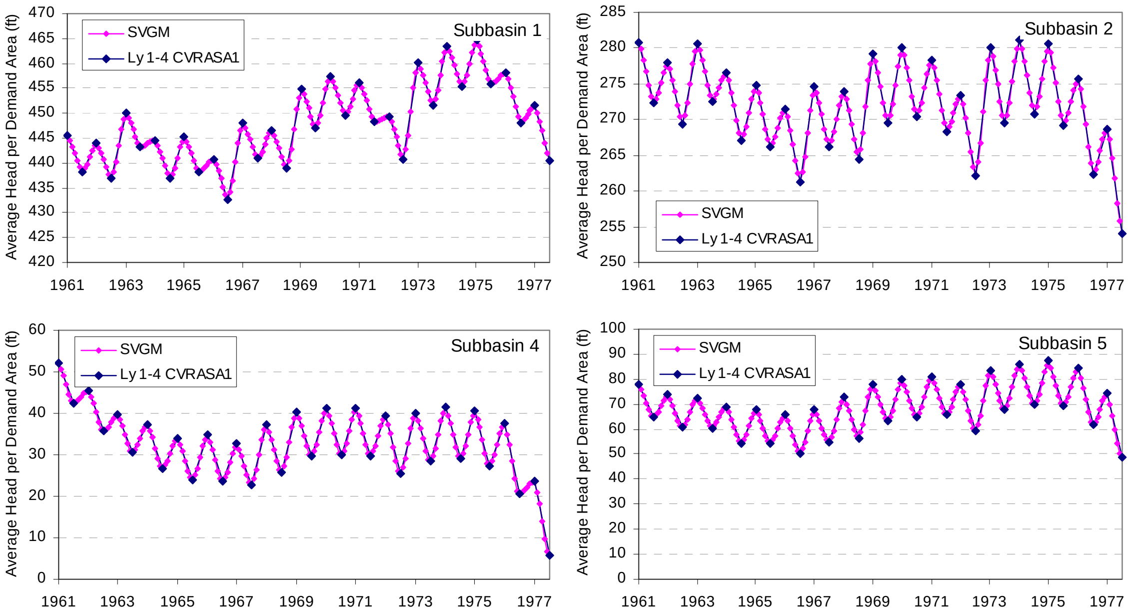

Figure 16. SVGM monthly simulation results for selected groundwater sub-basins 1,2,4 and 5. SVGM results verify with combined layer 1-4 CVRASA1 results (6-month time-step)

3.5. Simulating SVGM With Alternative Methods

Because SVGM is designed to serve as a component of a management model, several options were examined to further simplify the model for greater computational speed. Spatial representation has been simplified by upscaling from 3- to 2D, but the numerical modeling scheme (implicit scheme finite-difference) has not changed from that of CVRASA1 (same as MODFLOW). Other possibilities exist however to further simplify the model by using more efficient formulations available both in simulation and optimization. These methods were introduced in Chapter 2. Two methods are applied to simulate SVGM, the eigenvalue method and the storage coefficient method.

3.5.1. Eigenvalue Method

The eigenvalue method (Sahuquillo 1983a; Andreu and Sahuquillo 1987) uses an efficient numerical scheme that solves the spatially discretized but time-continuous version of the groundwater flow partial differential equations. Chapter 2 describes the method and provides references. The full continuous-time model was first implemented without using control variables or basic stresses as a test; results were the same as for the implicit time-marching scheme. The full continuous-time solution took 0.5 seconds to simulate 198 time periods while the implicit finite-difference scheme took 0.7 seconds. The full continuous-time method is faster than the finite-difference scheme because it uses only matrix multiplication to progress through the simulation time horizon rather than needing to solve a system of equations at each time step (as does the finite-difference scheme).

The eigenvalue method is more efficient than the full continuous-time solution through its use of control variables and basic stresses (Andreu and Sahuquillo 1987). Management models are rarely interested in head values or groundwater fluxes at each model grid cell. More commonly heads or fluxes in specific individual cells or in subsets of cells are of interest. Analogously, managed stresses being considered by the model usually occur only in specific cells or over areas composed of a subset of cells. The eigenvalue method takes advantage of these properties of most simulated systems to make the continuous-time solution more efficient.

Control variables can be heads or fluxes at individual cells or linear combinations thereof. In our implementation of the eigenvalue method to SVGM (abbreviated SVGM-EV) the average monthly head over each of the 9 groundwater sub-basins are the control variables. Head levels in individual cells overlain by streams could be declared as control variables if stream-aquifer interaction were considered.

Basic stresses of 1 ft3/day applied homogenously were implemented for each sub-basin. This allows simulating any stress (recharge or pumping) applied homogenously over an entire groundwater sub-basin. Heterogeneous basic stresses over groups of cells can be used with this methodology although this fixes the relative weights for each cell. To model stream-aquifer interaction basic stresses for individual cells can be created although this is not considered here.

In summary SVGM-EV uses 9 control variables and 9 basic stresses. The full continuous-time solution used the equivalent of 167 distinct control variables (head at every cell) and 167 individual basic stresses (stress at every cell). Efficiency gains are large for the eigenvalue method vs. the full continuous-time formulation (considering all individual model cells). For each time step the full continuous-time model multiplies vectors of length 167 with the A and X matrices (167x167); in the eigenvalue method the equivalent reduced A and F matrices are smaller (9x167 and 167x10 respectively) and multiplied by smaller vectors (length 9 and 10) for each time step. Efficiency gains of the eigenvalue method are also discussed in chapter 2.

SVGM-EV results per control variable (sub-basin) are almost identical to average heads per sub-basin as simulated with the combined layer CVRASA1 model (implicit finite-difference scheme) as seen in Figure 17. The SVGM-EV method worked as expected; results per control variable should be the same as averaged results from a MODFLOW-style model.

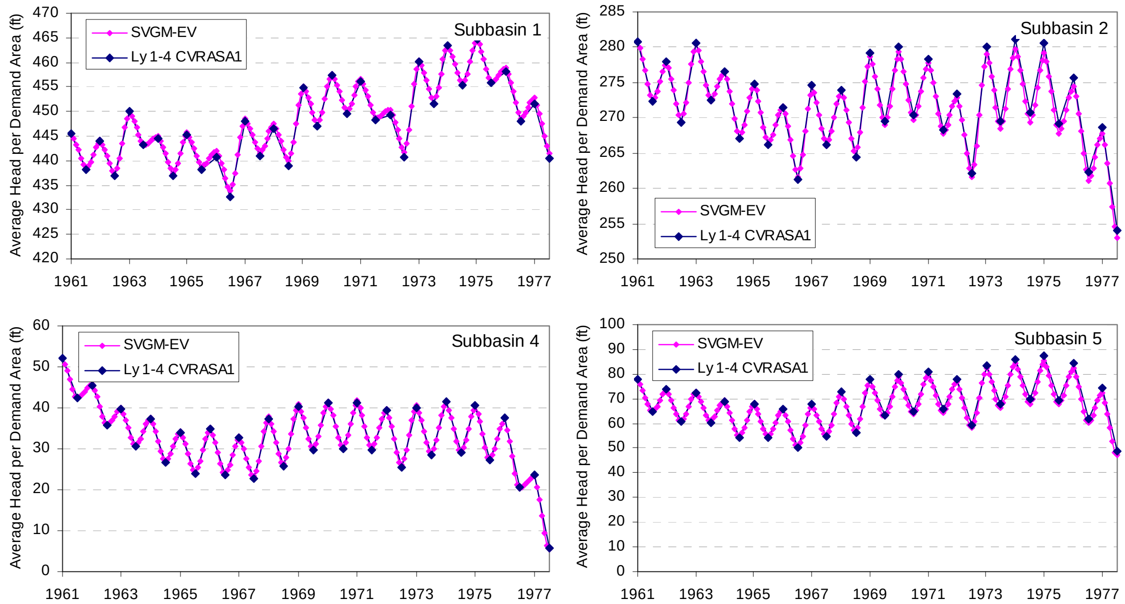

Figure 17. SVGM-EV historical simulation results for selected groundwater sub-basins 1,2,4 and 5. Groundwater sub-basin results using the eigenvalue method are nearly identical to the combined layer 1-4 CVRASA1 model.

3.5.2. Storage Coefficient Method

The storage coefficient equation can be used to model piezometric head levels in single- or multi-cell aquifer models as reviewed in chapter 2. The storage coefficient relates the volume of water released (or absorbed) from (into) storage per unit surface area of aquifer per unit change in hydraulic head in a confined aquifer. For gravity drainage in unconfined aquifers its equivalent is the specific yield. This approach is not considered a spatially distributed model if flow between adjacent cells is not considered.

The storage coefficient model (abbreviated SVGM-SC) represents groundwater sub-basins 1 through 9 as single-cell aquifers without inter-connections. Storage coefficients were taken from the combined layer CVRASA1 model. Results in Figure 18 show SVGM-SC obtains results similar to the CVRASA1 combined layer model.

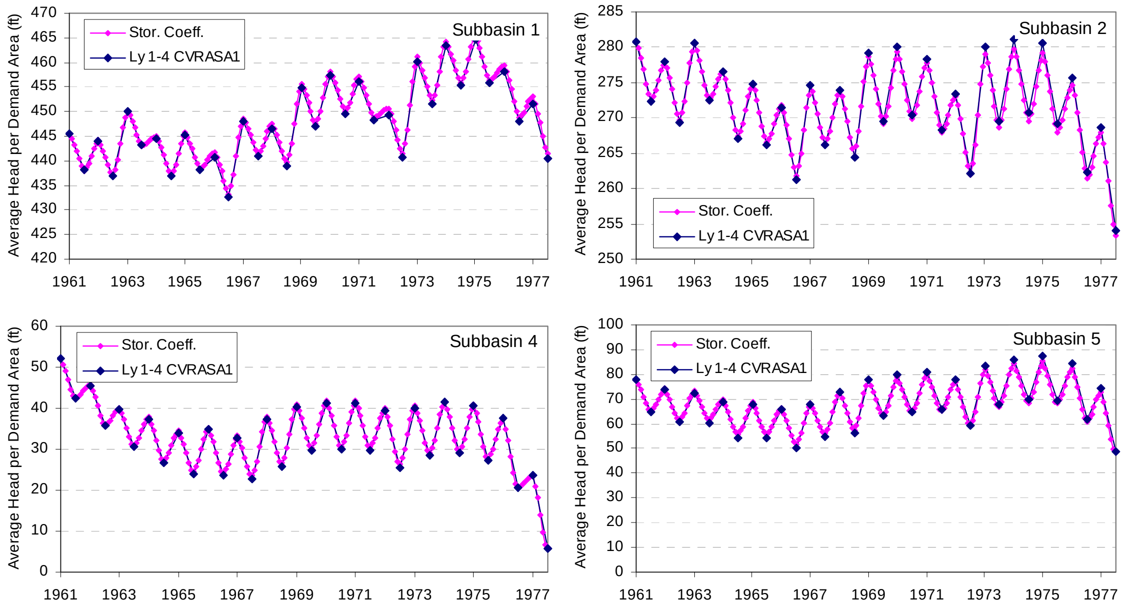

Figure 18: SVGM-SC historical simulation results using the storage coefficient method for selected groundwater sub-basins 1,2,4 and 5. Groundwater sub-basin results using the storage coefficient method are nearly the same as the combined layer 1-4 CVRASA1 model.

The storage coefficient method is the most efficient with only 9 compact groundwater equations per time step. If a management model only requires this level of detail, the storage coefficient method may suffice and SVGM-SC can be chosen. The disadvantage of the method is lack of flexibility: model scale and detail are fixed and limited. Boundary conditions such as stream-aquifer interaction or specified heads can never be represented. Although flow between neighboring basins is not considered here, it can be modeled using the Darcy equation. In our case flow between sub-basins doesn't strongly affect head levels as there is barely a difference between SVGM-SC and the CVRASA1 combined layer model results.

So far only simulation models have been discussed. For the rest of this chapter generic (maximize pumping) optimization formulations using all 3 groundwater representations are implemented and evaluated. The next section optimizes with the sequential time-marching method, the section after that uses the alternative eigenvalue and storage coefficient approaches.

3.6. Optimizing SVGM Using the Sequential Time-Marching Method

Chapter 2 describes the theory and implementation of the sequential time-marching optimization (STMO) method. This approach passes a full set of equations from a numerical model to the constraint set of a mathematical program which is solved by an optimization solver. The purpose of this section is to present a simple application of the sequential time-marching method to the SVGM simulation model. Model runs of this section are labeled SVGM-STM. Two simple optimization formulations are solved: 1. maximize pumping subject to historical stress constraints and 2. maximize pumping subject to historical head level constraints. In principle both formulations should arrive at the same solution as the pure simulation SVGM model. In practice the first formulation does but the second had numerical problems obtaining an optimal solution identical to historical pumping.

3.6.1. Maximize Pumping Subject to Fixed Stresses

This is essentially a simulation model posed and solved as an optimization model; the system of equations is fully determined and there is only one solution. Pumping is to be maximized subject to a fixed pumping time-series. Since there are no other constraints the optimal pumping stresses will be those specified in the constraint set. The model solves 33,067 equations (167 cells * 198 time periods and 1 objective function) and the same number of variables. Solving this model aims to verify that the optimization model formulation and solver are working correctly and to reveal the numerical efficiency of the method. The transient implicit finite-difference groundwater model can be encoded into the Generalized Algebraic Modeling System (GAMS) (Brooke et al. 2006) as:

$$\underset{\{Q_i^t, H_i^t\}}{Max} \sum_t \sum_i q_i^t$$

$$S.T. \sum_j \left(\Delta t * g_{i,j} + d_{i,j}\right) * H_j^{t+1} = \sum_j d_{i,j} * H_j^t + \left(rhs_i + q_i^t\right) * \Delta t \qquad \forall i,t$$

$$H_i^t = historicalh_i^t \quad \forall i,t$$

with decision variables and state variables (capitalized):

H_i^t = hydraulic head at cell *i* at time *t*

and data inputs:

q_i^t = fixed time-series of historical pumping stresses at cell *i* at time *t*

$g_{i,j}$ = conductance matrix

$d_{i,j}$ = capacitance matrix

rhs_i = right hand side vector elements that depend on boundary conditions

Δt = length of time period *t*

$historicalh_i^t$ = fixed pumping time series at cell *i* at time *t*

The SVGM model was simulated in GAMS using MINOS 5.51 (Murtagh and Saunders 1998) in 7.2 seconds (with a 2GHz, 2 GB RAM computer). Head results are the same as those calculated by the SVGM simulation model in MATLAB. The purpose of solving this model is to verify that its formulation in GAMS is correctly posed.

3.6.2. Maximize Pumping Subject to Historical Simulated Head Constraints

Here we maximize extraction subject to historical simulated piezometric head levels at all cells for each 198 monthly time periods. There are now two decision variables for each cell (*Q* and *H*) for a total of 66,132 decision variables and 99,199 equations (another 33,067 head constraints and one objective function). Optimal results should produce the same heads and extraction levels as SVGM. To embed SVGM into a linear program with the sequential time-marching method (SVGM-STM model), Q and H are decision variables as below:

$$\underset{\{Q_i^t, H_i^t\}}{Max} \sum_t \sum_i Q_i^t$$

$$S.T. \ \sum_j \left(\Delta t * g_{i,j} + d_{i,j}\right) * H_j^{t+1} = \sum_j d_{i,j} * H_j^t + \left(rhs_i + Q_i^t\right) * \Delta t \qquad \forall i,t$$

$$H_i^t \geq \min h_i^t \quad \forall i,t$$

where Q^t_i = pumping at cell i at time t

H^t_i = hydraulic head at cell i at time t

gi,j = conductance matrix

di,j = capacitance matrix

rhsi = right hand side vector elements that depend on boundary conditions

Δt = length of time period t

$minh^t_i$ = minimum allowed hydraulic heads at cell i at time t (this time-series is the SVGM simulated historical record).

The SVGM-STM problem was solved using two linear program solvers (CONOPT, MINOS) in GAMS and with and without using an initial solution identical to the optimal one. The 4 runs detailed in Table 6 are conducted to examine the robustness of optimizing a moderately sized problem using the sequential time-marching optimization method.

Table 6. Discussion of SVGM-STM(Opt) results with respect to SVGM simulation, figure references and run times.

	With Initial Solution	Without Initial Solution
MINOS 5.51	Optimal heads are the same as simulated cells except cell 167 (Figure 19) (14 seconds)	Head results are close to historic H but never the same (Figure 20) (3.6 minutes)
CONOPT 3	Some moderate to strong instability (Figure 21) (14 minutes)*	Time limit reached—infeasible (31.4 minutes)

* An option file beyond the default settings was needed to reach a feasible solution

Results confirmed reports in the literature (Tung and Koltermann 1985) that instabilities sometimes occur with the sequential time-marching optimization method. None of the 4 optimization model run produced optimal head values that were equal to the SVGM simulated historical levels. Table 4 provides the details of each optimization run performed in GAMS. The figures mentioned in the table compare SVGM and SVGM-STM values at selected cells. These attempts to optimize a medium size problem using the sequential time-marching optimization method show the method is susceptible to optimization solver errors (Figure 19). The sequential time-marching optimization method should be used with prudence and results checked against a known bench mark scenario, as done here (Figures 20 and 21), before being trusted to calculate optimal pumping patterns.

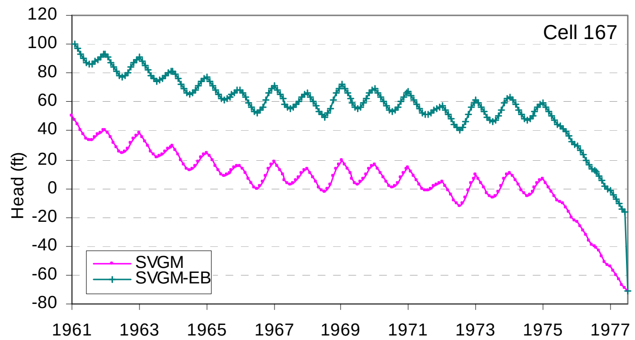

Figure 19. Cell 167 is the only cell where SVGM and SVGM-STM (labeled as –EB) gave different results when MINOS was used for optimization with an initial solution of optimal pumping levels.

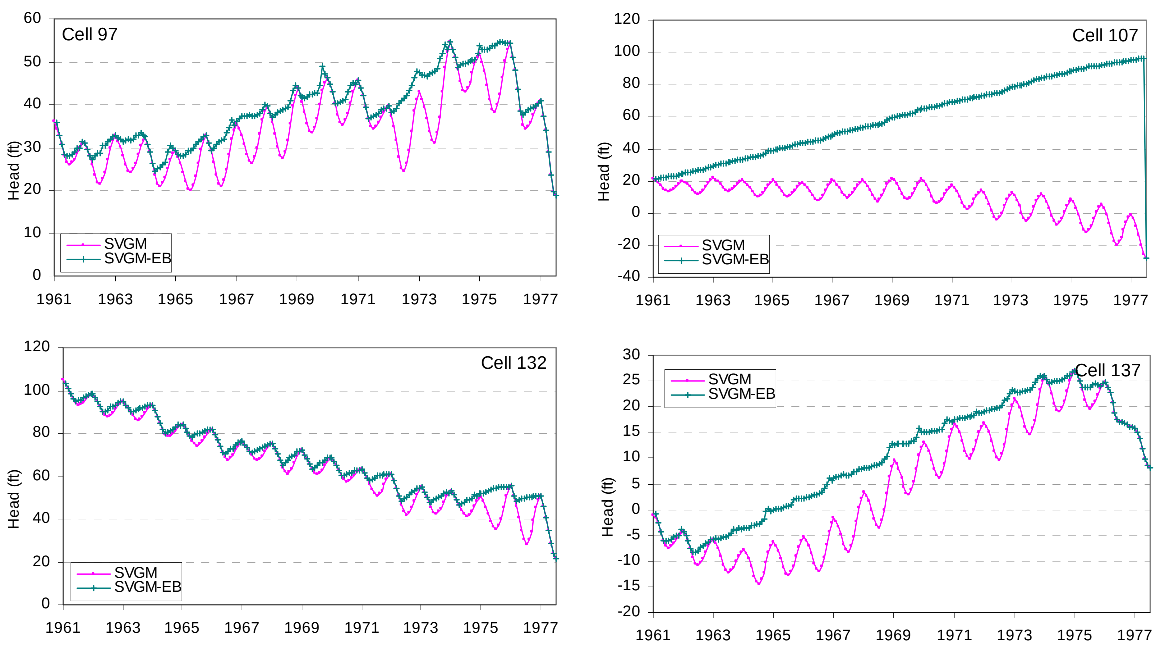

Figure 20. SVGM sequential time-marching optimization method (SVGM-STM, labeled here as -EB) GAMS model (MINOS, with no initial solution) compared to SVGM simulation. In most cells the solver shows minor or major instabilities.

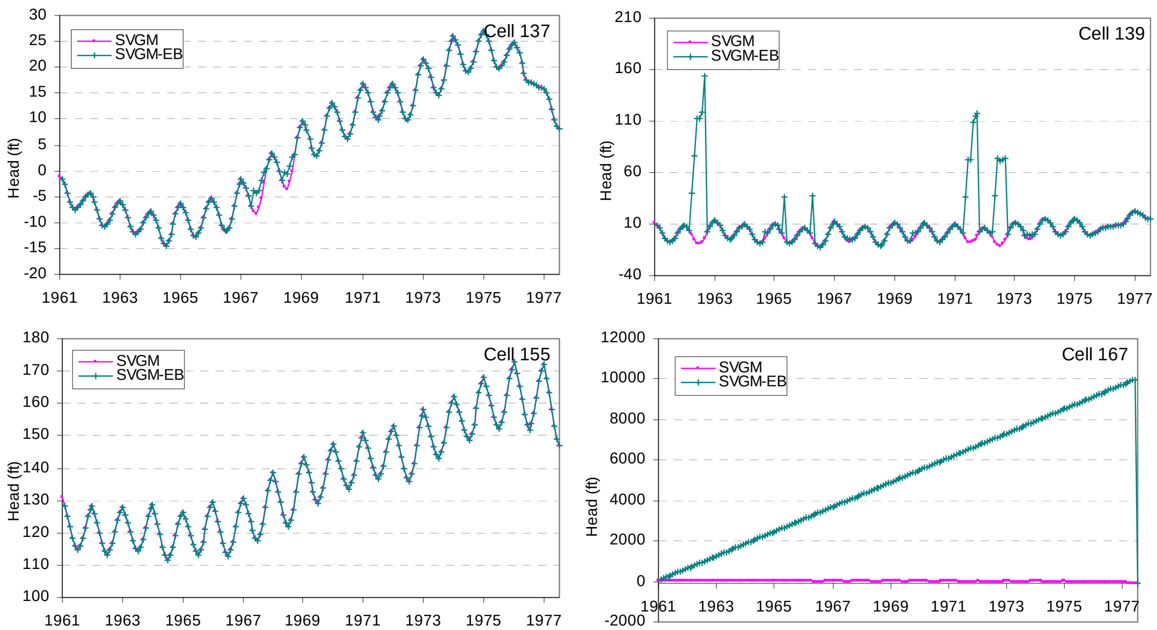

Figure 21. SVGM sequential time-marching optimization method (SVGM-STM, labeled here as -EB) GAMS model (CONOPT, with optimal initial solution) compared to SVGM simulation. In most cells the solver finds historical heads (e.g. cell 155), in others minor (cell 137, 139) or major (cell 167) instabilities occur.

3.7. Optimizing SVGM Using Alternative Efficient Methods

Embedding the full implicit finite-difference groundwater model into a linear program created a large problem (circa 100,000 constraint equations) vulnerable to instabilities and errors with major commercial solvers. Some management models require average regional groundwater levels and do not use information on individual cells. Therefore it is worth investigating more efficient and possibly more effective methods to optimize the groundwater model. The two methods applied here are the eigenvalue (EV) method and the storage coefficient (SC) method.

3.7.1. Eigenvalue Method

The eigenvalue method is employed in optimization here using the same control variables – basic stresses used in the eigenvalue simulation section. A description of the method and its equations appear in chapter 2. There are 9 control variables (the 9 groundwater sub-basins) and 9 basic stresses (unit stress distributed homogenously over each sub-basin). As with the sequential time-marching optimization method, two optimization formulations are used: 1. maximize extraction subject to the fixed historical withdrawal rates and 2. maximize pumping subject to historical head constraints. The first model is a simulation model solved to make sure the eigenvalue method is correctly translated into GAMS. MINOS correctly solves the full model without an initial solution in 3.4 seconds, as opposed to 7.2 seconds for simulating the full sequential time-marching method using GAMS.

The second formulation maximizes pumping subject to historical head levels. As with the sequential time-marching optimization method, this formulation should assure that optimal head levels equal historical ones. The formulation is written:

$$\underset{\{QI^t_{bs},H^t_{cv}\}}{Max} \quad \sum_t \sum_{bs} QI^{\,t}_{bs}$$

$$S.T. \quad H^t_{cv} = \sum_i ared_{cv,i} * l^t_i \qquad \forall cv, t$$

$$l^t_i = e_{i,i} * l^{t-1}_i + \sum_{bs} f_{i,bs} * QI^{\,t}_{bs} \qquad \forall i, t$$

$$H^t_{cv} \geq \min h^t_{cv} \quad \forall cv, t$$

where QI^t_{bs} = element *bs* of the basic stress intensity vector at time *t* (index *bs* refers to basic stresses because if a management model only considers stresses applied over entire groundwater sub-basins, the total number of control variables and of basic stresses is the same)

H^t_{cv} = mean hydraulic head in groundwater sub-basin *cv* at time *t* (index *cv* refers to control variables, in this case average head in sub-basins)

$ared_{cv,i}$ = element of cv^{th} row and the i^{th} column of the reduced eigenvector matrix (index *i* enumerates all groundwater model cells)

l^t_i = element *i* of the aquifer state vector at time period *t*

$e_{i,i}$ = element *i* of the diagonal matrix of eigenvalues

$f_{i,bs}$ = i^{th} row and bs^{th} column of the pre-calculated F matrix (see chapter 2)

$minh^t_i$ = minimum allowed hydraulic heads at cell *i* at time *t* (this time-series is the SVGM simulated historical record averaged per sub-basin).

Unfortunately, results (Figure 22) demonstrate that this formulation leads to severe numerical problems with the LP solver (in this case MINOS). Furthermore the only way to find the feasible solution graphed below was to provide an initial solution identical to the optimal one and to provide bounds for appropriate basic stress intensities. Although the number of equations sent to the solver is smaller with the eigenvalue method than with the sequential time-marching method, 36,631 equations instead of 99,199, the SVGM-EV takes longer to solve when it manages to do so. When using MINOS with an optimal initial solution, SVGM-STM solved in 14 seconds while SVGM-EV took almost 3 minutes. The main reason for this difference is that embedding the full finite-difference model produces sparse and relatively well structured (banded) matrices, whereas the eigenvalue method matrices can be densely populated, sometimes with very small numbers (particularly the reduced A matrix (ared above) and the F matrix (*f* above)). The resulting model is diagnosed by GAMS as being poorly scaled. GAMS computes the number of non-zero elements in the problem matrix it sends to solvers. For the

SVGM-EM, there are 314,258 non-zero elements while the SVGM-EV model, although it has much fewer equations, has more than twice the number of non-zero elements (650,660).

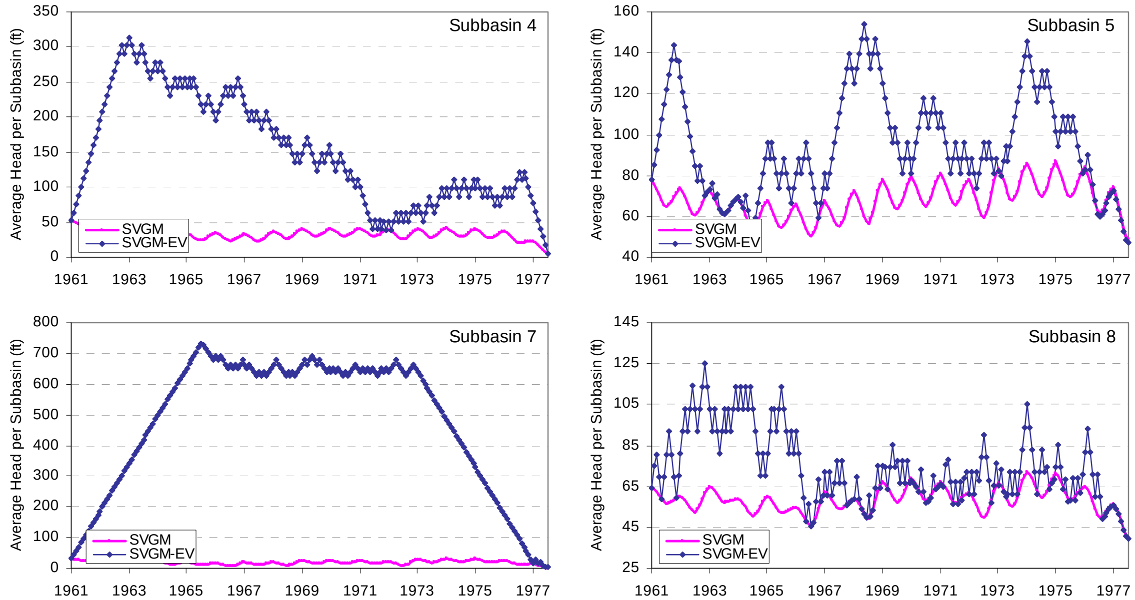

Figure 22. SVGM-EV results compared to historical simulated levels (SVGM) show the severe numerical problems encountered by the LP solver (MINOS) when using the eigenvalue method.

These observations indicate that in the case of the SVGM, the eigenvalue method leads to more optimization solver problems than the sequential time-marching method – already considered at risk for numerical optimization problems. Further study may show that there are ways to condition the eigenvalue method problem to make it more easily solved by optimization solvers. The eigenvalue method has been used successfully in an optimization model by Pulido-Velazquez et al. (2006), albeit on a smaller model (19 groundwater cells, and 10 years of monthly time-steps).

Inclusion of partial differential equations in optimization model constraint sets has long been recognized as challenging and the field is under continual development (Biegler et al. 2003). In particular, regularization of the ill-posed problem, as is standard practice in the groundwater inverse problem (McLaughlin and Townley 1996) could also be investigated. The solver instability that can be encountered while embedding distributed groundwater models into mathematical programs may encourage the use of others methods such as the response function method. However even using response functions unsolvable management problems can arise due to response nonlinearity and discontinuous derivatives (Ahlfeld et al. 2007).

3.7.2. Storage Coefficient Method

As described in Chapter 2, the storage coefficient equation can be used to dynamically model regional changes in piezometric head when the error due to neglecting flow between sub-basins is small. Earlier in this chapter, a simulation model using this method obtained results very similar to traditional groundwater simulation methods for the groundwater sub-basins of the

Sacramento Valley. Here we investigate whether this method would serve equally well in the management optimization model.

The storage coefficient method was implemented in GAMS first with a pure simulation formulation to assure it obtained similar results as the MATLAB code. As with the other techniques the simulation formulation involved maximizing extraction subject to historical stresses. Results were identical to the SVGM-SC simulation results displayed in Figure 18.

The second optimization model maximizes extraction subject to historical average heads per sub-basin:

$$\underset{\{Q_g^t, H_g^t\}}{Max} \sum_t \sum_g Q_g^t$$

$$S.T. \quad H_g^t = H_g^{t-1} + \frac{Q_g^t}{sc_g * area_g} \qquad \forall g, t$$

$$H_g^t \geq \min h_g^t \quad \forall g, t$$

where Q_g^t = mean stress in groundwater sub-basin *g* at time *t*

H^t_g = mean hydraulic head in groundwater sub-basin *g* at time *t*

sc_g = mean storage coefficient of sub-basin *g*

$area_g$ = surface area of sub-basin *g*

$minh^t_g$ = minimum allowed hydraulic heads at sub-basin *g* at time *t* (this time-series is the SVGM simulated historical record averaged per sub-basin).

Unlike the analogous optimization model results of the sequential time-marching and eigenvalue methods, here head results exactly match historical head constraints (Figure 23).

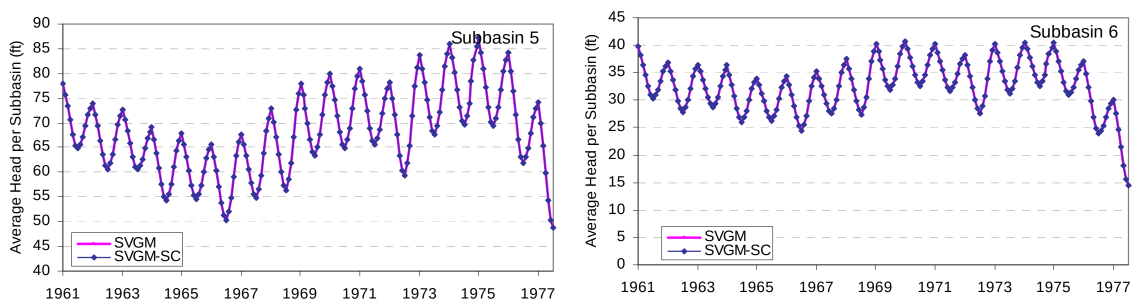

Figure 23. Average head per sub-basin in the SVGM-SC optimization model; SVGM historical head levels were used to contain the model.

The question then becomes, how much error is induced by using the simplified storage coefficient equation approach. This question is addressed by comparing historical stresses with optimized stresses solved for by the simplified model constrained to match historical head levels. Differences between the two are relatively small as showed in Figure 24. Differences between optimized and historical monthly stresses ranged from 0% to 36%, with an average 5.3% difference. In addition to this relative good accuracy, the model is also small (3,565 equations and variables) and fast (0.003 seconds to build and solve the LP model in GAMS using MINOS with no initial solution).

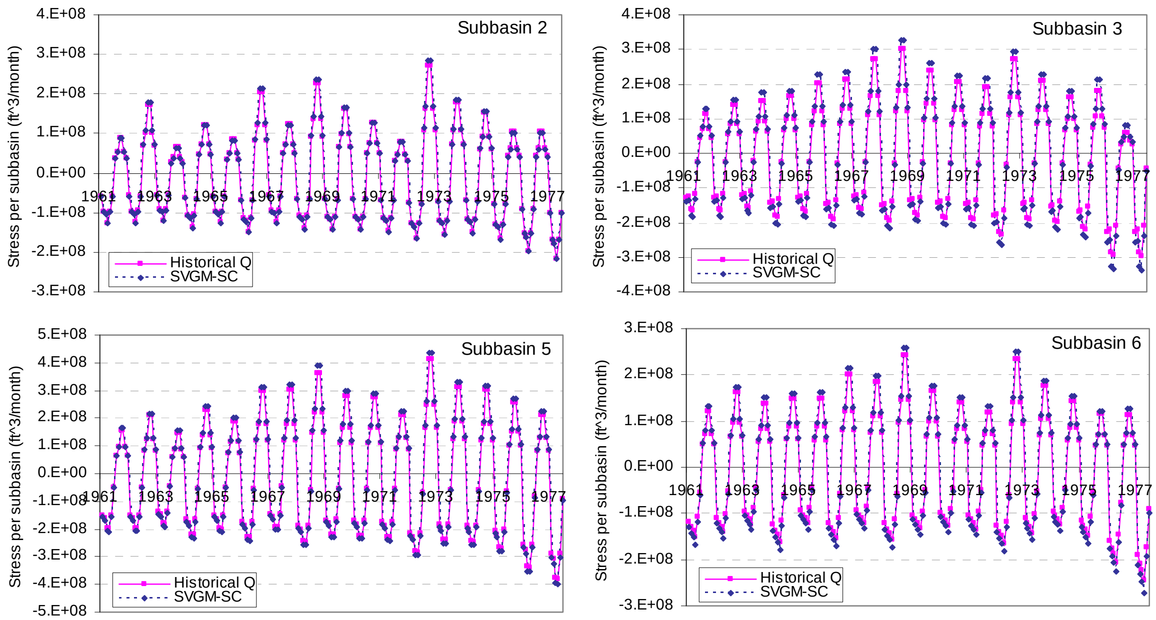

Figure 24. Stresses solved for in the SVGM-SC optimization model are similar to SVGM historical stresses.

3.8. Discussion

Table 7 summarizes runtimes and stability of the major model runs. The sequential time-marching method obtains results at all 167 model nodes. Run times should be compared mindful that eigenvalue and storage coefficient applications obtain results only for the 9 groundwater sub-basins.

Results show that optimization of distributed groundwater models by incorporating the discretized flow equations into the mathematical program constraint set remain a challenge. Neither the embedding method nor the eigenvalue method proved to be fully reliable optimization formulations for the SVGM model. The conceptually simpler, faster, and relatively accurate storage coefficient method performed well.

Table 7. Model run times and stability

	MATLAB	GAMS	
	Simulation	Simulation	Optimization
Sequential time-marching (STM) method	0.72 sec. stable	7.2 sec. stable	3.6 min. moderate stability
Eigenvalue (EV) method	~ 0.02 sec. stable	3.4 sec. stable	2.9 min. (init. sol.) poor stability
Storage coefficient (SC) method	~ 0.01 sec. stable	0.003 sec. stable	0.003 sec. stable

NB. All runs performed on a 2 GHz, 2 GB RAM computer. All GAMS models are run in MINOS 5.51 without initial solution (unless indicated with 'init. sol.') and with default solver settings. MATLAB runtimes do not include data manipulation and matrix construction while GAMS runtimes do. Symbol '~' denotes approximate runtimes.

Each method presents advantages and disadvantages and should be selected depending on the modeling task at hand. The sequential time-marching method for both simulation and optimization is most valuable when spatial detail in all parts of the model is desirable. Embedding the time-marching finite-difference scheme into the optimization model showed moderate reliability but improved when a good initial solution was provided. For simulation the eigenvalue method is much faster than conventional time-marching simulation while providing the flexibility to incorporate detail beyond sub-basin resolution where it is necessary. However, optimization using SVGM-EV the method was unstable and slow. Perhaps future efforts can resolve these problems for large-scale optimization uses. For a management model that requires a fast reliable way to optimize groundwater levels by sub-basin (e.g. to represent regional pumping costs), the simplified storage coefficient equation method is the most favorable choice among those tested in this study. The response function method, although not implemented here, is also a candidate technique for such management models.

3.9. Conclusions

Groundwater and its management are central to the continued productivity of the agricultural and urban sectors in California's Central Valley. In response several groundwater models of the Central Valley have been created by different agencies. An early USGS model named CVRASA1 modeled the Central Valley from 1961 to 1977 using 4 layers with 529 6x6 mile grid cells per layer. For the present study CVRASA1 was reduced to the northern Sacramento Valley and upscaled to 2D in order to be efficiently included into management (optimization) models. The combined layer model is simulated and compared to CVRASA1 layer by layer results. Results show that combining the four layers is almost equivalent to modeling only the topmost unconfined aquifer. This was deemed acceptable because in CVRASA1 60% of pumping occurs in the top layer and because head levels of lower confined CVRASA1 layers and their seasonal variation are relatively similar to the top layer.

Once built the model was simulated and optimized using three different approaches: implicit finite-difference scheme, eigenvalue method, and storage coefficient method. The first two ‘distributed’ methods solve the groundwater flow equation over a modeled domain, the third “multi-cell” method solves for groundwater levels of independent sub-basins, neglecting their interconnection. In the multi-cell case optimization results are monthly heads and stresses for 9 groundwater sub-basins of the Sacramento Valley. These sub-basins are a plausible discretization level for a water resource system management model. Results of each method were compared in terms of speed and stability of results. The embedding methods showed some instability when incorporated into a linear program maximizing extraction subject to historical head levels. The storage coefficient method was both stable and fast. It was selected for inclusion in the management model since its loss of accuracy at the sub-basin level is small compared to the gains it offers in stability and speed.

4.0 Conclusions

Effective water resources management in the 21st century will benefit from modeling tools that suggest integrated, sustainable and economically efficient solutions. This report focuses on options for representing groundwater within integrated models of regional water resource systems.

Two applications (Appendices A and B) describe simple multi-cell groundwater models embedded into regional hydroeconomic optimization models. Hydroeconomic models represent hydrologic engineered systems while explicitly considering the economic nature of water demands and costs rather than considering them as fixed quantities. Combining civil engineering, economics and hydrologic science, the hydroeconomic approach is well positioned as a tool for integrated water resource management. Hydroeconomic models should serve as a guide in the policy making process, revealing where innovative and dynamic policies can replace older institutional arrangements for increased performance. As water scarcity and lack of new supplies increase, resource managers will increasingly turn to such tools which reveal in a relatively transparent way where greater efficiency in water use can be attained.

4.1. Summary

Water resource systems models can include a variety of groundwater formulations in both simulation and optimization modes. Simulation models answer “what if” questions most appropriate for detailed predictive models (such as operational models). Optimization formulations search for the solution that best satisfies an objective function subject to a set of constraint equations representing physical laws or operational and institutional limitations. Optimization may be a valuable complement to simulation when investigating new operational or management policies.

Model formulation and scale is chosen depending on the type of inferences that will be made with the model. Scale refers to the choice of model domain and the spatial discretization used to subdivide the modeled region. When representing groundwater in regional water resource system models, the choice is between lumped (single- or multi-cell models) or spatially distributed formulations. Simple representations of groundwater can be quite sufficient for integrated system modeling, and provide great computational advantages relative to incorporating fully spatial models. Models that are expected to produce locally relevant and hydrologically verifiable results will benefit from upgrading their groundwater representations to spatially distributed sort. Levels of discretization impose the groundwater formulation. Lumped formulations implement simple mass balance water volume accounting or rely on the storage coefficient equation to track basin-wide average groundwater level. Spatially discretized approaches using grids or meshes to subdivide the aquifer solve numerically discretized forms of the groundwater flow equations. The groundwater flow equations can be discretized in space and time (time-marching) or only in space (time-continuous). In both simulation and optimization, the groundwater model can be solved endogenously or exogenously (using a database of unit stress groundwater response functions, also called discrete kernels). The response function method, available for simulation and optimization, assumes system linearity in order to apply additive superposition of groundwater stress effects. This assumption is valid for confined aquifers or for unconfined aquifers in which the saturated thickness is an order of magnitude larger than changes in the saturated thickness. The various lumped and distributed formulations and their applications in the literature were described and compared and their potential for integration into water resource system models was discussed in Chapter 2.

In Chapter 3 a computational comparison of three different approaches to simulating and optimizing California's Sacramento Valley was presented. Data was derived from a regional USGS groundwater model, CVRASA1. CVRASA1 modeled the Central Valley from 1961 to 1977 using 4 layers with 529 6x6 mile grid cells per layer. The groundwater model was reduced to the northern Sacramento Valley and upscaled to 2D as it might be for efficient inclusion in a management model. The combined layer model is simulated and compared to CVRASA1 layer by layer results. Results showed that combining the four layers is practically equivalent to modeling only the topmost unconfined aquifer.

Once built the model was simulated and optimized using three different approaches: one lumped (multi-cell) and two distributed formulations. The implemented groundwater formulations are: multi-cell storage coefficient method, implicit finite-difference time-marching scheme, and continuous-time eigenvalue method. All groundwater formulations were incorporated into a linear program maximizing extraction subject to keeping groundwater head above or at historical (simulated) head levels. This formulation serves only to test the optimization formulation, as maximizing withdrawals subject to minimal heads should reproduce historically simulated groundwater levels. Results of each method were compared in terms of speed and stability of results. The time-marching groundwater model produced correct results at all cells but a select few, where optimized groundwater levels deviated from simulated results. The time-continuous eigenvalue method showed major instability when incorporated into the linear program using two separate commercial solvers. The storage coefficient method was both stable and fast and its loss of accuracy at the sub-basin level is small compared to the gains it offers in stability and speed.

These results underlined the difficulty of embedding distributed groundwater models directly into mathematical programming model constraint sets. This result confirms the difficulties reported in the mathematical and groundwater modeling literature of including discretized systems of partial differential equations into mathematical programs. Further research will have to test techniques, such as regularization, that could make embedding groundwater models more stable.

4.2. Applications

Two hydroeconomic applications are included in the appendices. Appendix A investigates the economic effects and water management actions that accompany ceasing groundwater overdraft in California's Tulare basin. A portion of the CALVIN model (Draper et al. 2003) is used, updated to reflect recent conjunctive use infrastructure investments undertaken by the region's water banks. Previous economic studies of the Tulare Basin showed optimal pumping depths had been reached in much of the basin, causing sustainable use to become a relevant policy. Results show that when overdraft is prohibited and water transfers are allowed, groundwater banking using conjunctive use infrastructure built between 1990 and 2005 largely annuls the cost of not overdrafting.

Appendix B presents a conjunctive use management model which integrates the contributions from each previous chapter. A method for optimizing conjunctive use systems with an economic objective and limited hydrologic insight is described. The work extends existing models (Draper 2001; Draper et al. 2003) in that groundwater piezometric head and pumping costs are modeled, resulting in a non-linear formulation. Limited foresight in achieved by dividing the modeled time horizon into annual segments and optimizing them individually using a carry-over storage value function to prevent end of period drainage. Each annual model is linked to the previous one by end of year storage. Optimal carry-over storage value function parameters are derived by repeated evaluation of the model within search algorithms. A proof of concept application of the methodology to the Redding Basin in California's Northern Central Valley is implemented. Results show the potential of the method as a tool to improve conjunctive use water management in California.

4.3. Implications for Modeling in California

In California many government agencies maintain water resource system models. Several integrated models of regional water resource systems in California and their groundwater formulations were reviewed. Until recently, lumped multi-cell groundwater formulations have been the norm in system models. CALSIM-III, the latest version of the California Department of Water Resources monthly reservoir system simulation model applied to California, has been updated to simulate spatial groundwater heads by using response functions (NHI 2005; SEI 2007). The unit responses are derived from C2VSim, a finite-element groundwater model of the Central Valley. Two variants of the response function method were used in each report to link surface and groundwater in CALSIM. The first report used transient multi-period response functions (the standard transient response function formulation described in Chapter 2) while the second report uses re-initialization and the concept of artificial steady-state stresses to make the CALSIM link more efficient.

The need for CALSIM to maintain in memory all groundwater stresses and surface flows of previous years so as to propagate their time-lagged effects on groundwater decreased the effectiveness and efficiency of the standard transient response function method (NHI 2005). In this respect the use of reinitialization and artificial steady-state techniques (described in Chapter 2) improved the efficiency of CALSIM with spatially discretized groundwater. In addition, a linked simulation–optimization approach (both CALSIM and C2VSim are run at each monthly time step) has been implemented by CDWR. In theory, if stresses are considered in each cell of the groundwater model (no scanning grid), the conventional sequential time marching simulation involves less floating point operations (FLOPS) than the response function method (Andreu and Sahuquillo 1987). This suggests that in principle the linked simulation-optimization approach, where CALSIM exchanges information on a time-step by time-step basis with C2VSIM, is more efficient. In practice however, implementing spatial groundwater results in CALSIM is a complex software project, the practical implementation of which will ultimately decide which methodology produces a faster running CALSIM. In addition, the assumptions and numerical techniques each effort will implement, for example with respect to non-linear processes such as stream-aquifer interaction, will also determine the speed at which CALSIM can integrate the spatial groundwater model.

The present work has demonstrated that embedding even simple linear spatially discretized groundwater formulations into optimization model constraint sets can cause major numerical instability. Unless practitioners can deploy specialized techniques to make the system of discretized partial differential equations more amendable to classical mathematical programming solvers, we recommend implementation of response function methods given that they are well understood, conceptually simple and have been programmed into several freely available software. This being said, systems that comport non linearities will never be easy to optimize without recourse to supplementary methods, such as iteration. Many groundwater optimization problems containing transport or variable density flow are still considered open problems (Bear 2003).

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Appendix A

Ending Groundwater Overdraft in the Tulare Basin

Introduction

Groundwater overdraft occurs when extraction exceeds both natural and induced aquifer recharge over long periods. While ultimately unsustainable and invariably having detrimental effects, overdrafting aquifers is common and may be temporarily beneficial within a long-term water management strategy. Once a region chooses to end overdrafting, water management must change if increased water scarcity is to be avoided. Integrated water management models allow aquifers and overdraft to be analyzed as part of a regional water supply system. Incorporating economics into the model establishes a framework for evaluating the costs and effects of groundwater management actions on the entire system. This economic-engineering approach is applied in a case-study of the Tulare Basin in California, USA (44,000 km2, population 2 million), where previous economic studies showed optimal pumping depths have been reached. A hydro-economic optimization model is used to study the economic effects and water management actions that accompany ending overdraft. Results show that when overdraft is prohibited, groundwater banking using conjunctive use infrastructure built between 1990 and 2005 largely annuls the cost of not overdrafting. The integrated economic-engineering approach quantifies effects of groundwater policies on complex regional water resource systems and suggests promising strategies for reducing the economic costs of ending aquifer overexploitation.

This study assumes a decision was made to stop overdrafting groundwater. This choice could be imposed by an agency or court order, it could be based on localized negative hydrogeological and environmental effects of overdraft, and it could be supported by dynamic economic models that suggest optimal regional groundwater depths have been achieved or surpassed. The latter two reasons are relevant for the Tulare Basin. Government or court intervention is not currently envisaged. Adopting a sustainable regional groundwater policy would most likely occur progressively and involve changing water use incentives within the context of water and water storage markets currently active in the region. A model is used to investigate the economic and water management effects of ending overdraft. In particular, the value of new conjunctive use infrastructure built by groundwater banks between 1990 and 2005 is investigated. The model identifies the combination of water demand reduction, surface water substitution, and conjunctive use solutions (Table 1) that allow the system to adapt at the lowest cost to a no overdraft policy.

Tulare Basin, California

The Tulare Lake Hydrologic Region is a closed hydrologic basin comprising the southern portion of California's Central Valley. The valley is underlain by a series of interconnected mostly unconfined aquifers of area 21,500 km^2 and average depth of 1.3 km. These aquifers have been historically overexploited, resulting in low piezometric levels and high land-subsidence in some locations (Planert and Williams 1995).

The Tulare Basin is one of California's most productive agricultural regions. Water consumption is high (circa 16,000 million m^3/year) but natural inflows are limited. The Tulare Basin is the state's greatest water-importing region, importing annually between 4,000 and 7,000 million m³ (Mm³). Major water imports arrive from the San Joaquin River (through Friant-Kern Canal) and Northern California (from the California Aqueduct) (see Figure A1). Since the 1990s, some imported water is stored during wet periods by artificial recharge in groundwater banks managed by local irrigation districts (e.g. Semi-tropic, Kern, Arvin-Edison). Despite the water imports, overdraft continues in the basin. Annual change in groundwater storage varies widely depending on rainfall; in 1998 it was estimated at + 1,000 Mm³ but at -5,000 Mm3 in 2001 (CDWR 2005). Average annual groundwater overdraft was estimated at 380 Mm3 (USBR 1997) for the period 1921-1993.

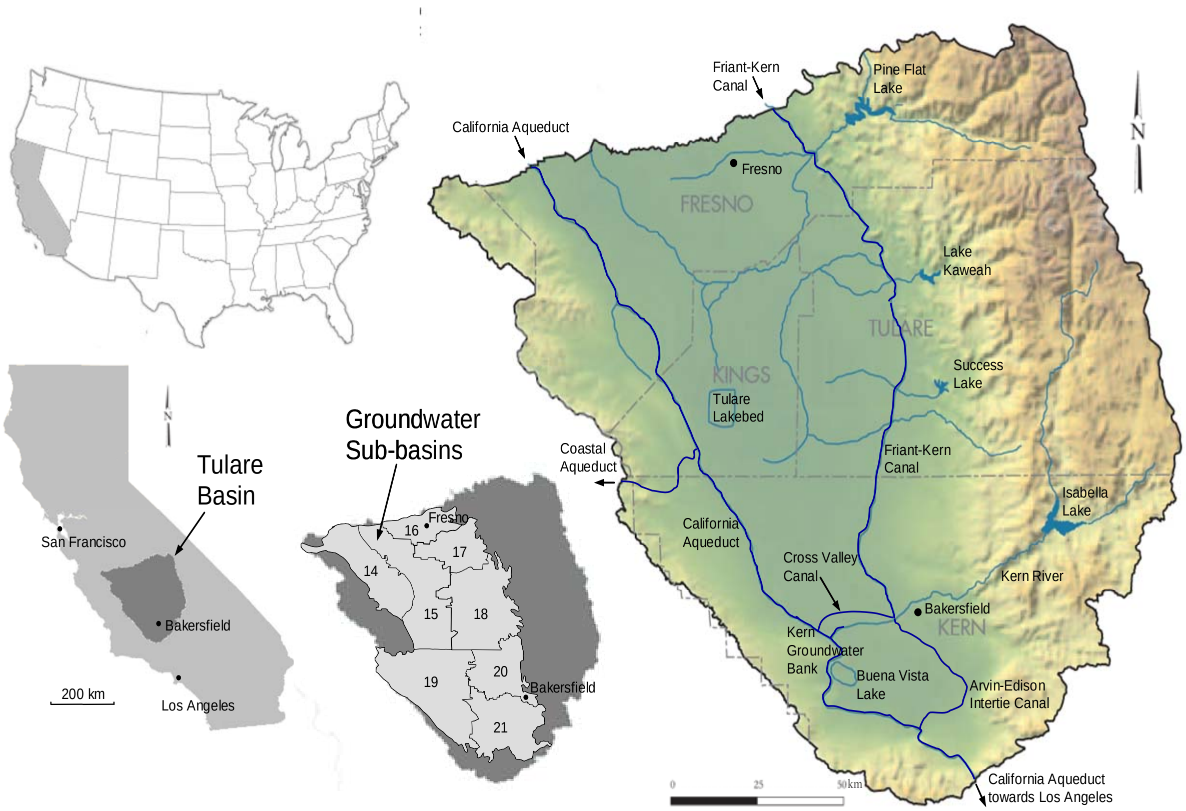

Figure A1. The Tulare Basin is a closed hydrological basin in the southern part of California's Central Valley. The small Tulare outline map shows the extent of Tulare groundwater sub-basins. The large map shows major canals and natural watercourses on a background of topography. Counties are written in uppercase.

Source: Modified from CDWR 2005

Regional Groundwater Levels

Pumping lifts have increased since 1945 when intensive exploitation began from in some cases artesian conditions to present lifts that can exceed 100m (KFMC 2005; CDWR 2005). Table A1 provides average regional pumping heads and other regional groundwater basin data from the early 1990s (initial storage estimated for 1920). Pumping heads listed include effects of local drawdown and well losses. Interseasonal variability of piezometric head ranges from approximately 15m for unconfined aquifers to 50m for confined layers (KFMC 2005;

Haslebacher, T., Kern County Water Agency, personal communication, 2006). Piezometric head under the regions' groundwater banks can be much higher than regional levels.

Table A1: Pumping and storage data for each groundwater sub-basin

Groundwater Sub-basins	Pumping Head	Pumping Cost	Pumping Capacity	Initial Storage	Ending Storage	Overdraft
	(m)	($/m^3)	(Mm^3)	(Mm^3)	(Mm^3)	(Mm^3)
Westside (basin 14)	116	0.062	411	63001	56449	6552
Tulare Lake (basin 15)	71	0.038	503	86954	86857	97
Fresno (basin 16)	45	0.024	75	7844	0	7844
Kings River (basin 17)	48	0.026	188	9018	8641	377
Tule-Kaweah (basin 18)	69	0.037	430	50296	41874	8422
W Kern (basin 19)	105	0.055	211	53145	53148	-2
NE Kern (basin 20)	102	0.054	133	27914	28868	-953
S Kern (basin 21)	106	0.056	282	63642	58700	4943
Total Overdraft (Mm^3) over 72 year period:						27,280

Source: Pumping heads from CDWR (1994), initial and ending storage from USBR (1997), data and costs discussed further in Jenkins et al. (2001). Pumping capacity in Mm3 is per month.

Several studies have considered groundwater management in the southern Central Valley using dynamic economic models (Howitt 1979; Knapp and Vaux 1982; Feinerman and Knapp 1983; Knapp and Olson 1995; Schuck and Green 2002; Knapp et al. 2003). Howitt (1979) published a preliminary estimate of optimal depth to groundwater for three sub-basins within the northern Tulare Basin (Kings, Kaweah, Tule,) averaging 57 m (62, 51, 60m respectively). Feinerman and Knapp (1983) and Knapp and Olson (1995) estimate benefits of optimal control for Kern County and recommended pumping depths. Their first study estimates regional steady-state pumping lifts at 169m and 127m under competition and optimal control regimes respectively.

Both studies for which steady state pumping depths are given above used a pumping cost of 0.045 $/kWh and a 5% discount rate. Results were sensitive to both discount rate and energy cost, with recommended pumping depths increasing with higher discount rates and decreasing with higher energy costs. These results suggest levels during the early 1990s were at or approaching their economic optimums. Groundwater levels in most areas not covered by groundwater banks have since further decreased.

A decision-maker with a strong interest in regional sustainability (e.g., 3% discount rate) could argue that groundwater depths have passed their optimal levels. Besides the economic optimality of regional groundwater depth, environmental and political factors would most likely influence this decision. This position is the starting point of this study and opens a new question: how should system operation best adapt to a new sustainable groundwater use policy? Because economics motivates most water use in the region and was a key factor in deciding to halt overdraft, economic considerations should also influence how the system is newly operated. This is the working hypothesis of the model runs presented in the next section where economic water demands drive operations and allocations.

Method

A sustainable groundwater use policy is analyzed using a portion of an economic-engineering optimization model of California's water supply system. The model named CALVIN (California Value Integrated Network) (Jenkins et al. 2001; Draper et al. 2003) provides time series of optimal surface and groundwater monthly operations, water use and allocations to maximize statewide net economic benefits. This includes changing surface and groundwater operations and reallocating or marketing water to maximize regional economic net benefits within environmental flow constraints. The model employs all available water management options to economically adapt to the presence or absence of groundwater overdraft. Because CALVIN's objective function is economic, time series of shadow values provide the monetary value of relaxing model constraints throughout the network.

The model optimizes over a 72-year historical hydrologic record (1921-1993) for a particular level of infrastructure, population and land use development (projected to the year 2020 in this case). Conjunctive use infrastructure is modeled at 1990 and 2005 levels. Water demands are represented as economic penalty functions, based on agricultural and urban demand curves for water. Agricultural demand curves were derived using the State-wide Agricultural Production (SWAP) model (Howitt et al. 2001) and urban demand curves were taken from published sources (Jenkins et al. 2001, Appendix B). Operating costs for pumping, artificial recharge, desalination and water treatment also are represented. CALVIN employs the HEC-PRM (USACE 1999) "Prescriptive Reservoir Model" software as its computational and organizational core. HEC-PRM uses a computationally efficient generalized network flow linear optimization formulation that represents the system as a network of nodes and links. "Generalized" refers to the possibility of including gain/loss multipliers on network flows. Use of a linear model guarantees a unique globally optimal solution while use of the network flow algorithm offers more than a 10-fold increase in solution speed relative to a standard linear program solver (Labadie 2004). The model minimizes costs subject to flow continuity at nodes and capacity constraints on links; it can be written Min $\sum_i \sum_j c_{ij} X_{ij}$ subject to $\sum_i X_{ji} = \sum_i a_{ij} X_{ij} + b_j \ \forall j$, $X_{ij} \le u_{ij} \ \forall ij, \quad X_{ij} \ge l_{ij} \ \forall ij$, where X_{ij} is flow leaving node i towards node j (link ij), c_{ij} = costs of flow through link ij (scarcity costs or operational costs), b_j = external inflows to node j, a_{ij} = gain/loss coefficient on flows in link ij, u_{ij} = upper bound (capacity) on link ij, and l_{ij} = lower bound on link ij. This restricted network flow formulation precludes including other constraints such as groundwater response equations. The number of links (1,617) multiplied by the number of time periods (864) gives the number of X_{ij} decision variables (1,397,088). The full CALVIN model solves in approximately 12 hours using an initial solution on a 2 GHz PC.

For this study the California-wide model has been reduced to the Tulare Basin region (292,896 decisions). Inter-regional boundary conditions include fixed inflow time series (San Joaquin River diversion and the California Aqueduct in the north) and fixed outflows (California Aqueduct deliveries to southern California). These regional boundary flows are marked in Figure 1. The Tulare Basin is divided into eight water demand areas, each with agricultural and urban water demands. Each water demand area overlies one of eight semi-interconnected groundwater sub-basins. The eight sub-basins of the Tulare Basin as well as the water storage and conveyance network appear in Figures A1 and A2.

As a large-scale regional economic-engineering model, CALVIN tracks groundwater volume in each sub-basin and does not represent the piezometric surface (groundwater level) as would a spatially distributed groundwater model. These limitations are discussed in Jenkins et al. (2001, Appendix J) and Pulido-Velázquez et al. (2004). Pumping lifts are constant for each sub-basin and based on averaged water levels from the 1990s. The model does not dynamically represent groundwater flow within and between the groundwater sub-basins or stream-aquifer interactions. Instead, it uses fixed series of flows between sub-basins derived from simulation of the historical period by a spatially distributed groundwater model: the Central Valley Groundwater–Surface Water Model (CVGSM) (USBR 1997). Groundwater recharge time series also are taken from the CVGSM model, although deep percolation of irrigation water is dynamically modeled.

Modeling Scenarios

Initial groundwater storage of the Tulare Basin was estimated at 362,000 Mm^3 (USBR 1997). A representative overdraft rate for each groundwater sub-basin was set based on extraction rates estimated during the 1990s (USBR 1997). This resulted in an estimated annual overdraft rate of 380 Mm3 for the entire basin. Applied over the 72-year modeled period, the modeled overdraft leads to 335,000 Mm3 of ending storage for the entire basin.

The model is run under four scenarios (Table A2), all with projected year 2020 water demands and using 72 years of historical monthly time series of inflows to represent hydrologic variability. "No overdraft" runs constrain ending groundwater basin storages to equal initial storage volumes. Model runs also contrast the conjunctive use infrastructure present in the early 1990s with current expanded infrastructure capacities (Figure A2). These include the Kern, Arvin-Edison and Semitropic irrigation district water banks and new conveyance links to the California Aqueduct (west-ward flow of the Cross Valley canal, Buena Vista Lake Pumping facility, and the Arvin-Edison intertie) (SAIC 2003). Figure A2 shows the framework of such infrastructure; more detail is given in the electronic supplementary material.

Table A2. Four modeled scenarios

Scenario	Description
ODCU-	Overdraft (OD) with less (1990 level) conj. use infrastructure (CU-)
ODCU+	Overdraft (OD) with more (2005 level) conj. use infrastructure (CU+)
NoODCU-	No Overdraft (NoOD) with less conj. use infrastructure (CU-)
NoODCU+	No Overdraft (NoOD) with more conj. use infrastructure (CU+)

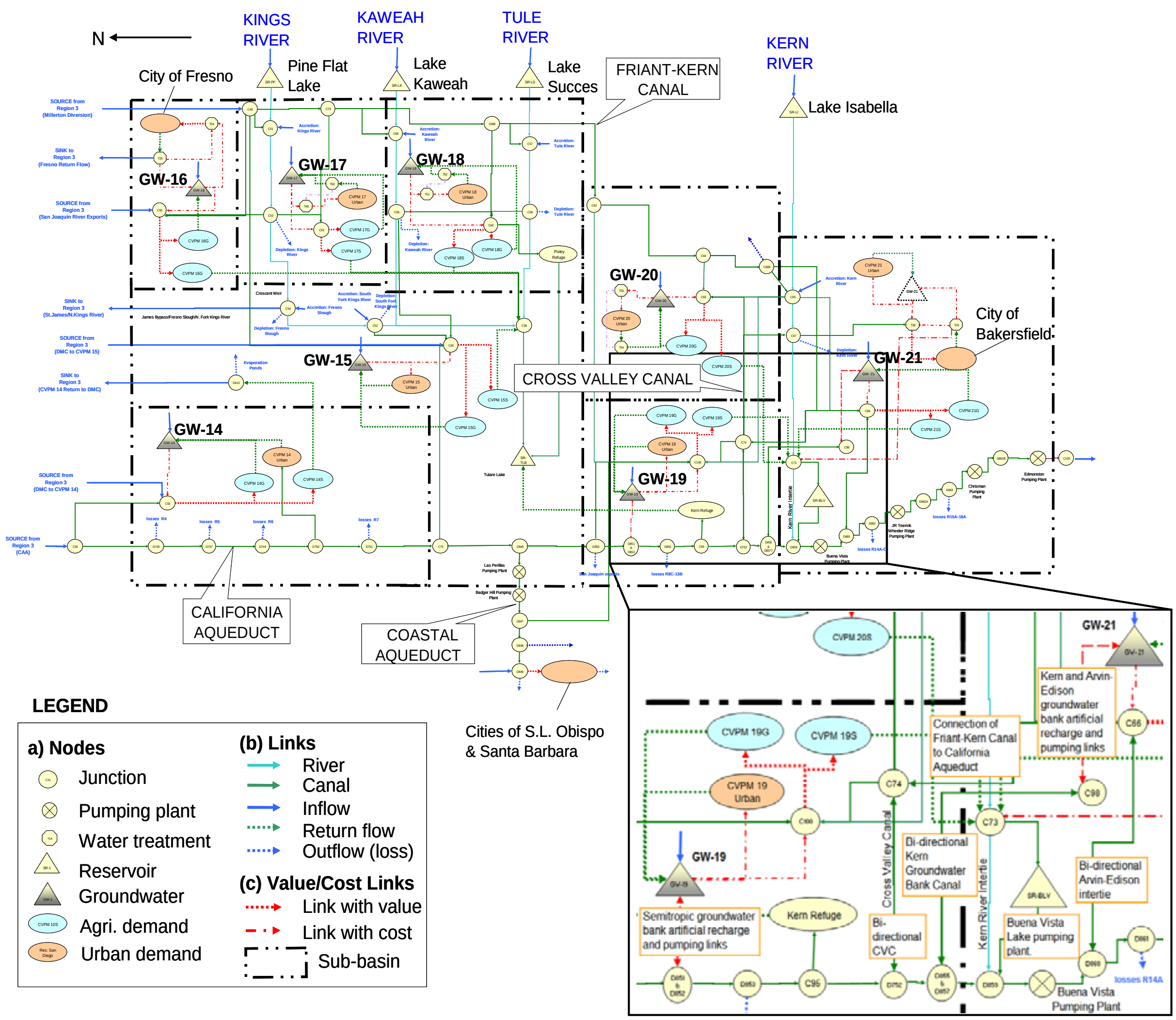

Figure A2. Schematic of Tulare Basin CALVIN model. Modeled conjunctive use infrastructure enhancements from 1990 to 2005 are labeled in the zoom box. A fully labeled network is found in the electronic supplementary material.

Results

The combination of model runs permits examination of the effects of overdraft policies on water scarcity (shortages), water scarcity costs, water users' willingness-to-pay for additional water, and the value of conjunctive use facilities (artificial recharge and pumping) within the region. Preliminary results of this study were described by Harou and Lund (2007).

Groundwater Storage

For each management alternative, groundwater storage in the Tulare Basin varies on seasonal and drought time scales. Figure A3 shows groundwater storage for southern Kern County. In addition to long-term trends of overdraft, there are also significant changes in recharge and withdrawal over seasonal and drought time-scales. Drier periods have steep decreases in storage. The time span of drought-related drawdown and refill cycles is commonly a decade or longer (Lettenmaier and Burges 1982; Pulido-Velázquez et al. 2004). For all sub-basins, cyclical patterns of groundwater storage follow similar trends with and without conjunctive use infrastructure. However, conjunctive use operations have a greater role for off-setting agricultural water scarcity as demonstrated in the next section.

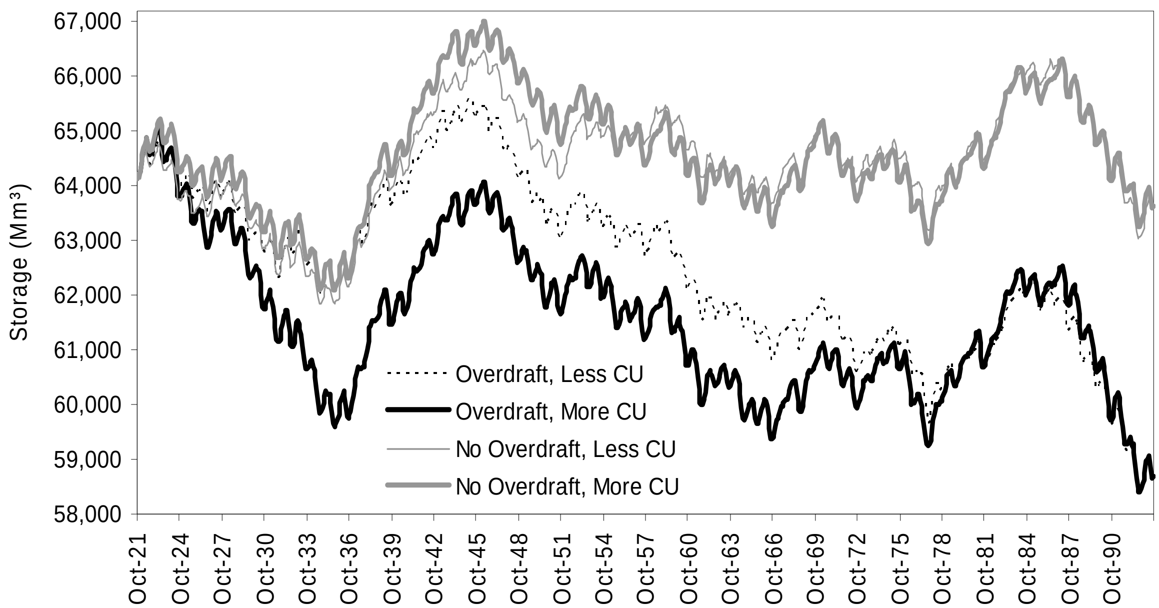

Figure A3. Groundwater storage (million m^3) in southern Kern County (sub-basin 21) for the four management scenarios. "Less CU" refers to the less conjunctive use infrastructure (1990 level) and "More CU" refers to 2005 levels.

Scarcity and Its Costs

The model provides time series of optimal flows at each location in the network. The economic penalties used in the optimization imply a water supply target for each demand and economic costs of not providing a full target delivery. Because available water is limited and less than the region's full demands, any physically feasible water allocation will cause water scarcity to some water users. The cost of this scarcity is evaluated using the economic penalties.

Overall scarcity results from the four optimized alternatives are summarized in Table A3. The scarcity cost from ending an overdraft of 380 Mm^3/year in the Tulare Basin would be roughly US $31 million/year (48 minus 17 in Table A4) with current conjunctive use infrastructure and $59 million/year with 1990 infrastructure levels. Both estimates assume the best-case economically optimal operations from CALVIN. Dividing those costs by 380 Mm^3 gives the average supplemental cost incurred per cubic meter not used due to the no overdraft policy; it is 0.08 $/m3 with current conjunctive use infrastructure and 0.16 $/m^3 with less infrastructure. Assuming optimal operations which take most advantage of the new infrastructure, the average cost of reducing each cubic meter of overdraft is half of what it would be without the expanded conjunctive use infrastructure.

Table A3. Summary values for scarcity and scarcity costs in the Tulare Basin under the four scenarios.

	Average Annual Water Scarcity (Mm^3)		Annual Scarcity Cost (million US $)	
	CU+*	**CU-**	**CU+**	**CU-**
Overdraft	318	779	17	59
No Overdraft	728	1241	48	118

* "CU+" more (2005 level) conjunctive use infrastructure, "CU-" less (1990 level) conjunctive use infrastructure.

Deliveries, water scarcity and scarcity costs of individual water demand areas are summarized in Table A4. The urban areas of Fresno, Bakersfield and Santa Barbara suffer no water scarcity; their high willingness-to-pay for water and relatively low water use allow them to purchase enough water to avoid shortages. The ability to purchase water from agricultural areas eliminates the need for expensive desalination (at a conservative cost of $1.1/m³). Agricultural water-right holders sell or lease part of their water to farmers or cities with higher willingness-to-pay. Agricultural sector water scarcities roughly double without overdraft.

Water scarcity and scarcity cost estimates are available at each location on the network for each monthly time step. Water scarcity and scarcity cost vary seasonally, following seasonal patterns of local water demands and availability (Figure A4). As seen in Table A4, the no overdraft with current conjunctive use scenario (NoODCU+) faces scarcity costs similar to the scenario allowing overdraft but with less conjunctive use (ODCU-).

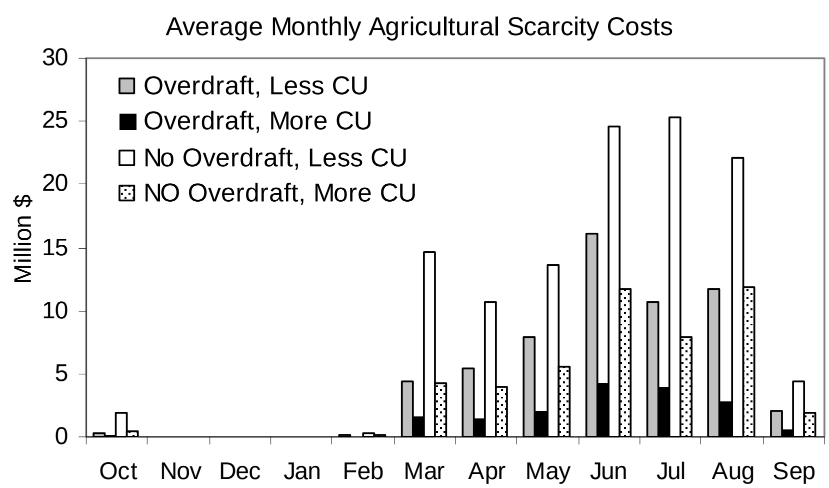

Figure A4. Average monthly agricultural water scarcity costs (US dollars)

Source?

le A4. Average deliveries, scarcities, and scarcity costs for economic water demand areas for each modeled scenario

	Target (Mm³)	Delivery (Mm³)				Scarcity (Mm³)				Scarcity Cost (1,000 $/year)			
		OD CU-**	OD CU+	NoOD CU-	NoOD CU+	OD CU-	OD CU+	NoOD CU-	NoOD CU+	OD CU-	OD CU+	NoODC U-	NoOD CU+
Agricultural Demands *													
stside (14)	1,845	1,681	1,836	1,681	1,704	164	9	164	141	19,569	2,064	19,572	13,709
are Lake (15)	2,457	2,375	2,457	2,320	2,366	82	0	137	91	3,235	0	8,791	3,609
sno (16)	611	605	605	583	605	6	6	29	6	149	149	2,383	173
gs River (17)	1,030	1,012	1,012	979	983	17	17	50	46	445	445	3,839	3,435
e-Kaweah (18)	2,664	2,291	2,462	2,216	2,360	374	203	448	305	39,684	12,470	51,498	28,390
Kern (19)	1,180	1,120	1,161	1,023	1,119	60	19	156	61	3,939	1,253	20,443	3,980
Kern (20)	835	787	797	722	787	47	38	113	48	3,465	2,771	16,631	3,522
ern (21)	1,433	1,405	1,407	1,289	1,404	28	27	144	29	1,755	1,640	21,848	1,784
l Agricultural demands	12,055	11,276	11,737	10,814	11,327	779	318	1,241	728	72,242	20,792	145,004	58,608
Urban Demands													
sno	469	469	469	469	469	0	0	0	0	0	0	0	0
ersfield	321	321	321	321	321	0	0	0	0	0	0	0	0
ta Barbara, San L.O.	171	171	171	171	171	0	0	0	0	0	0	0	0
l Urban Demands	961	961	961	961	961	0	0	0	0	0	0	0	0
l all water demands	13,017	12,238	12,698	11,775	12,289	779	318	1,241	728	72,242	20,792	145,004	58,608

icultural demand sub-basin numbers are in parentheses.

enarios: “OD” Overdraft, “NoOD” No Overdraft, “CU+” more (2005 level) conjunctive use infrastructure, “CU-” less (1990 level) conjunctive use infrastructure.

Willingness-to-Pay for More Water

Water demand areas that experience water scarcity would economically benefit from additional supplies. Marginal willingness-to-pay (WTP) reflects what demand areas would be willing to pay for an extra m^3 given their current allocation in each model run. Marginal WTP is estimated as the slope of the economic benefit function at the delivered water quantity. If the delivery falls on a discontinuity in the piece-wise linear benefit function, the lower slope is taken so values represent a lower bound on the WTP for an additional unit of water. Demand areas receiving full deliveries have no scarcity (in this case the urban demands) and no marginal WTP (Table A5).

Table A5. Monthly average and 72-year maximum marginal willingness to pay for additional water at water demand sites

	Average Marginal WTP ($/m^3)				Maximum Marginal WTP ($/m^3)			
	OD CU-**	OD CU+	NoODCU-	NoOD CU+	OD CU-	OD CU+	NoODCU-	NoOD CU+
Agricultural Demands *								
Westside (14)	0.05	0	0.05	0.04	0.45	0.45	0.45	0.45
Tulare Lake (15)	0.02	0	0.04	0.02	0.03	0	0.09	0.03
Fresno (16)	0.01	0.01	0.05	0.01	0.02	0.02	0.08	0.08
Kings River (17)	0.02	0.01	0.06	0.05	0.03	0.03	0.08	0.08
Tule-Kaweah (18)	0.06	0.03	0.08	0.05	0.13	0.05	0.13	0.13
W Kern (19)	0.03	0.01	0.09	0.04	0.05	0.05	0.14	0.05
NE Kern (20)	0.04	0.03	0.10	0.04	0.14	0.06	0.18	0.08
S Kern (21)	0.03	0.03	0.09	0.03	0.05	0.05	0.14	0.05
Urban Demands								
Fresno	0	0	0	0	0	0	0	0
Bakersfield	0	0	0	0	0	0	0	0
Santa Barbara, San Luis Obispo	0	0	0	0	0	0	0	0

* Agricultural demand sub-basin numbers are in parentheses.

** Scenarios: "OD" Overdraft, "NoOD" No Overdraft, "CU+" more (2005 level) conjunctive use infrastructure, "CU-" less (1990 level) conjunctive use infrastructure.

Table A6 provides average monthly values for additional water imports at the northern boundary of the Tulare Basin. These values are the shadow values of mass balance constraints at border nodes which receive the fixed inflows. Water is worth slightly more in the eastern Friant-Kern canal system due to its proximity to water demand areas and lower pumping costs. Average marginal WTP (Table A6) is slightly less than the marginal value of additional surface water imports (Table A6) because of the way it is estimated. In both cases additional water is worth more when less conjunctive use infrastructure is available because there is less opportunity to store and convey water within the basin. Conjunctive use infrastructure decreases the value of additional imported supplies because it allows the basin to store more water in wet periods for use within the basin in dry periods.

Table A6. Average value ($\$/m^3$) of additional imported water at the northern boundaries of the Tulare Basin

	ODCU-*	ODCU+	NoODCU-	NoODCU+
Friant Kern Canal	0.10	0.04	0.14	0.10
California Aqueduct (Westlands)	0.10	0.02	0.13	0.07

* Scenarios: "OD" Overdraft, "NoOD" No Overdraft, "CU+" more (2005 level) conjunctive use infrastructure, "CU-" less (1990 level) conjunctive use infrastructure.

Figure A5 focuses on the value of additional California Aqueduct imports during the 1976-77 drought. At the height of the drought, historic aqueduct flow was at its lowest and all alternatives show similar economic values for further imports. During more normal years, 1976 and 1978, differences in the value of additional supplies returns to those in Table A6.

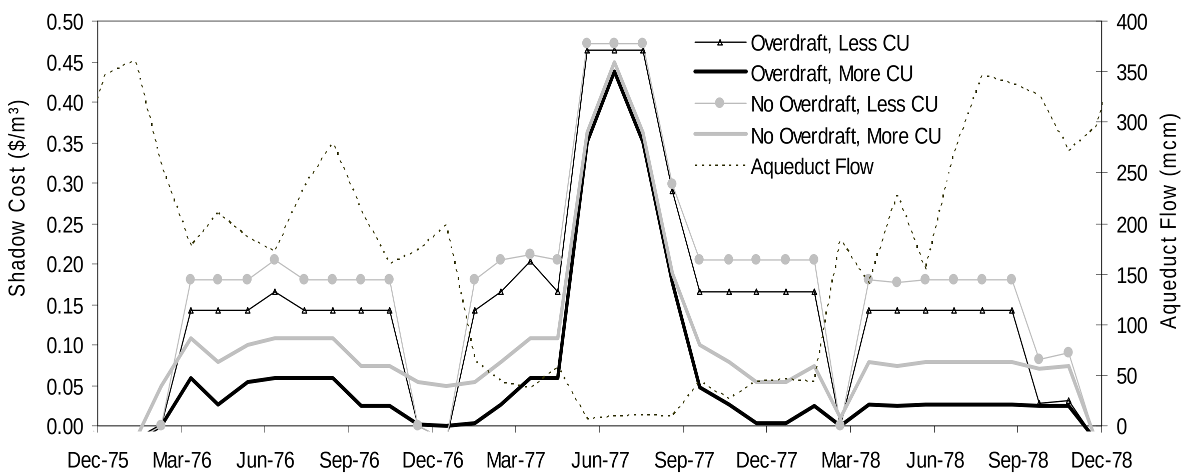

Figure A5. Marginal value of additional imports from the California Aqueduct (beyond historical deliveries) based on economic water demands in the Tulare Basin and historic deliveries to southern California during the period of the 1976-77 drought.

Value of Capacity Expansion

Shadow values on constraints pertaining to water storage and conveyance capacities provide estimates of the value of augmenting both storage and conveyance capacities. Table A7 shows average and maximum annual marginal value of added surface water storage capacity at the four major reservoirs in the Tulare Basin. These values could help identify promising locations for reservoir enlargement; the smaller reservoirs Kaweah and Success show the highest expansion benefits. Conjunctive use infrastructure added in the basin over the last 15 years decreases the value of increasing reservoir capacity, replacing it with better use of groundwater storage.

Table A7. Average of maximum monthly shadow values of each hydrologic year ($/tcm)

	Current summer max. capacity	Average annual maximum monthly shadow values ($/tcm**)			
	(Mm^3)	ODCU-*	ODCU+	NoODCU-	NoODCU+
Pine Flat Lake	1,230	0.6	0.5	1.7	1.5
Lake Kaweah	179	173	55	203	119
Lake Success	102	152	51	192	117
Lake Isabella	693	25	10	32	11

* Scenarios: "OD" Overdraft, "NoOD" No Overdraft, "CU+" more (2005 level) conjunctive use infrastructure, "CU-" less (1990 level) conjunctive use infrastructure.

** tcm: thousand cubic meter

Recharge capacity is usually more valuable than pumping capacity in the region's two modeled groundwater banks (Figure A6). This is because groundwater banking infrastructure is connected to the California aqueduct, which in the model contains on its southern border a fixed historic outflow time series. Southern California, if represented economically as in the statewide CALVIN model, would increase the shadow value on groundwater bank pumping capacities, better reflecting the value of the Tulare Basin as California's conjunctive use hub. Recharged groundwater in CALVIN can be used for regional export or to satisfy local economic or fixed water demands. This double function explains the high value of expanding percolation facilities, especially without overdraft.

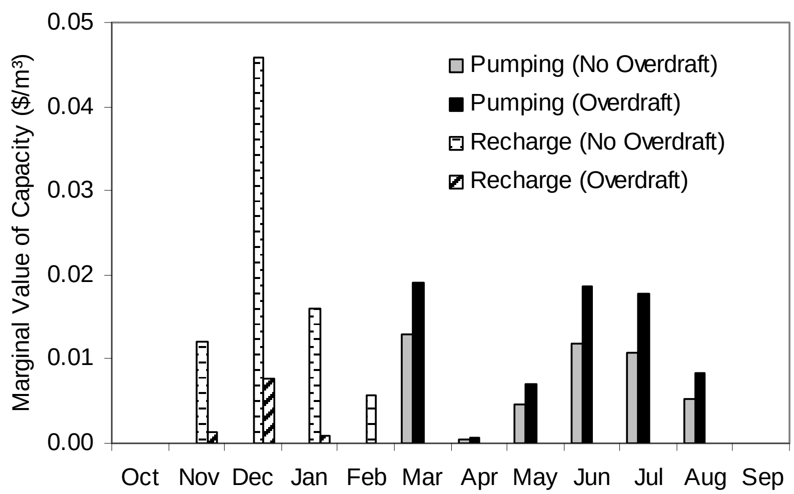

Figure A6. Shadow values showing the monthly economic value of more artificial groundwater recharge and groundwater pumping capacity in the Kern and Arvin-Edison groundwater banks (groundwater sub-basin 21).

The value of artificial recharge varies seasonally (Figure A6) and annually (Figure A7). In dry years percolation basins have little value. Both figures show that prohibiting overdraft often more than triples the economic value of additional recharge capacity. Although the value of further recharge is strongly affected by existence of overdraft, actual operations are not. Figure

A7 shows that the system is constrained enough by water demands and fixed exports that actual recharge rates recommended by the model are similar. Slightly less recharge occurs with overdraft since these alternatives have more water available for deliveries.

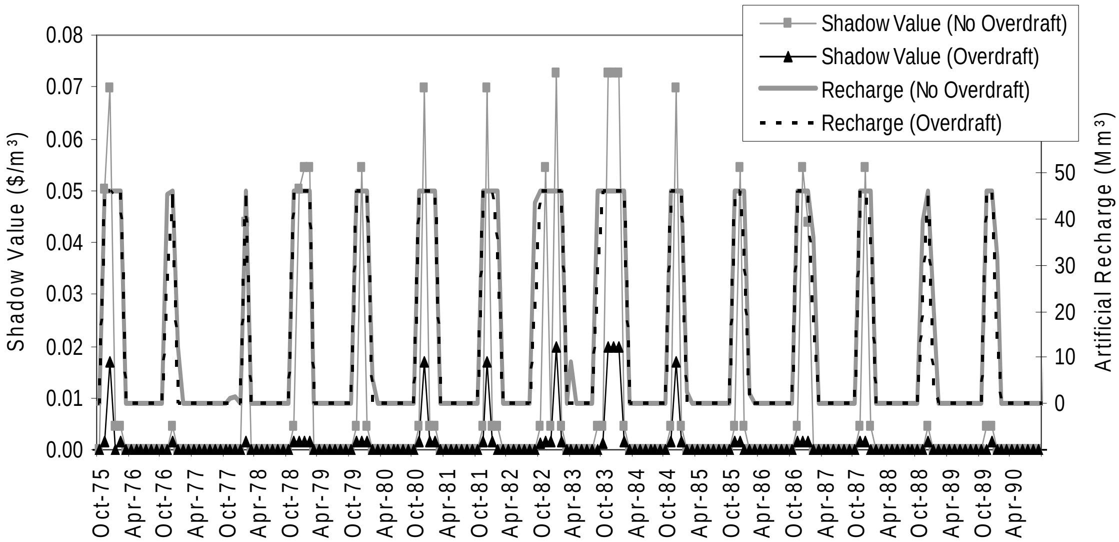

Figure A7. Artificial groundwater recharge flows and shadow values from 1975-1990 for the aggregated Kern and Arvin Edison groundwater bank (sub-basin 21) with 2005 level conjunctive use infrastructure.

Modeling Limitations and Discussion of Results

Several key assumptions and limitations of the model and its application are useful in interpreting its results. CALVIN, as a multi-period optimization model, suggests water allocations, operations and costs that reflect perfect hydrologic foresight. In systems with significant over-year storage such as the Tulare Basin, the sequence of storages will differ from those obtained with a simulation model and performance for some years will be optimistic (Draper 2001).

In CALVIN both groundwater levels and flows between sub-basins are fixed based on historic simulation results from an outside groundwater model. The more basin-scale groundwater volumes differ from historical storages, the more inaccurate the historical inter-basin flows will be. Groundwater pumping costs are static and do not vary with storage. Thus the model does not provide insights into one major benefit of ending overdraft – stabilizing pumping costs. The effect of pumping costs on optimal system management could be investigated by examining sensitivity to different fixed regional groundwater levels but results would remain tempered by the static nature of the analysis. A spatially lumped but dynamic treatment of groundwater effects for Kern County considering water transfers is given by Knapp et al. (2003).

Finally, all model runs portray perfectly optimized operations without institutional constraints. They represent an economic-engineering ideal, which might be difficult to achieve institutionally. Results, including operations and economic information should therefore be considered a best-case. The model assumes perfect operation for minimizing economic penalties. CALVIN's limitations are discussed by Draper et al. (2003).

For the specific application of CALVIN to the Tulare Basin, an important limitation is the fixed historic boundary flows. The Tulare Basin is a major hub for inter-regional water transfers and storage in California's statewide water system. Tulare Basin operations are strongly influenced by the boundary inflows and outflows. If California were to manage water resources as suggested by the statewide CALVIN model, inflows and outflows of the Tulare Basin region would probably differ from historical flows. More importantly, a fixed historical Tulare Basin export to southern California does not represent water demands in southern California economically. Because the Tulare Basin will increasingly be called to serve as a groundwater bank for southern California, this simplified representation of southern California demands understates the value of Tulare Basin water supplies and infrastructure. An easy way to correct this would be to simply extend this study to all of central and southern California. Apart from increasing run times and the complexity and quantity of interpretable results, this further study would be feasible using the full state CALVIN model in its present form. A benefit of the current case-study is that fixed boundary flows reduce the effects of the model's perfect hydrologic foresight.

Another modeling issue is the scale of groundwater analysis: the existence and extent of overdraft varies throughout the Tulare Basin; the large modeled sub-basins cannot capture this spatial variability. Figure A8 shows the observed evolution of depth to groundwater in 330 Kern County wells from 1990 to 2006. This plot reveals long-term changes in groundwater level vary widely throughout the region. Also of interest is that groundwater levels rose in 59% of wells probably due to increased recharge from groundwater banks. This observation supports the claim made that the most realistic institutional solution for ending overdraft in the Tulare Basin would be the increasing incentives brought about by the ability to store and sell water.

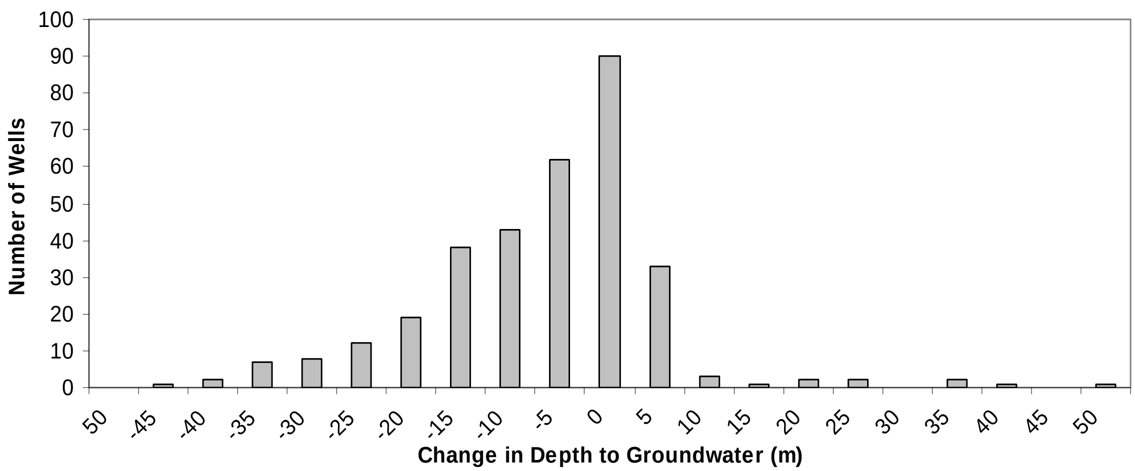

Figure A8. Change in depth to groundwater in 330 wells of Kern County from 1990 to 2006. Negative values indicate rising groundwater levels (e.g., 62 well levels rose between 0 and 5m during the 16-year period).

Source: CDWR (2006)

In some parts of the Tulare Basin, recommended steady state levels have not been reached, which questions why these areas should be lumped in with those with a graver overdraft problem. Knapp and Olson (1995) consider conjunctive use with stochastic surface flows and produce a probability distribution of optimal steady state pumping lifts for Kern County centered on 145m or 178m (optimal control or common property regime respectively). These steady-state depths are 9 to 17m greater than those suggested by Feinerman and Knapp (1983) when not considering conjunctive use. Since these depths are 40 to 70m more than the current regional depth to groundwater, perhaps some areas (e.g. in Kern County) should not reduce pumping? These remarks point to the relevance of spatial variation when managing aquifer overexploitation. However, to analyze the Tulare Basin as a whole and evaluate different strategies, a coarse level of spatial aggregation is used here (groundwater sub-basins average 3,000 km^2).

Having noted key limitations and simplifications, the study does provide a best-case cost estimate of enforcing a sustainable groundwater yield in a region with intensive groundwater use. The study underlines the importance of considering groundwater overdraft, and policies for reversing it, within the context of diverse system-wide water management activities and objectives. Overdraft is not easily reversed by considering groundwater management alone; for many semi-arid basins, this would be unrealistic. Results of the model help provide a perspective on how to end groundwater overdraft in a larger water management context. An example is given in the next paragraph.

Because the historical record for the Tulare Basin begins and ends in drought periods, the optimal trajectory of groundwater storage with no long-term depletion stays mostly well above current storage levels (Figure A3). This provides a specific case where initial drawdown of aquifers creates storage capacity that enables later conjunctive use. Initially overdrafting to make space for later flexible operations may be the optimal management strategy for some water resource systems. Whether this is the best option in practice will depend on existence of sources of water for recharge, institutional feasibility, ecological effects, hydrogeological characteristics and water quality.

Conclusions

While some groundwater overdraft may be beneficial depending on its hydrologic, environmental and economic consequences, its timing and duration, and on one's perspective, all overdraft must inevitably end. This can occur through inaction, with variable results, or by way of managed solutions. Sustainable managed solutions for renewable aquifers typically involve a combination of reduction of water demands, surface water substitution, and conjunctive use. In some situations temporary overexploitation may provide important economic benefits for the over-lying society and engineering functions, such as water quality protection and storage for conjunctive use schemes.

California's Tulare Basin has relied on overpumping groundwater for irrigation but will be faced with potential water quality problems, subsidence and inefficient pumping costs. A hydro-economic model examined how the region could adapt if it chooses to forego overdrafting. Results showed ceasing groundwater overdraft with current conjunctive use infrastructure would increase water scarcity by approximately 410 Mm3/year at an annual supplemental cost of $31 million. These are best-case figures since the model assumes economically optimal operations and perfect hydrologic foresight. This cost is two orders of magnitude less than the annual agricultural revenue generated by the counties of the Tulare Basin ($8,000 million). Seen in this light, costs incurred to end over-abstraction are significant but not catastrophic. However, if the inability to trade water during droughts led to urban or industrial water shortages, economic damage could be much larger. Water scarcity caused by ceasing overdraft was similar to the scarcity already present circa 1990 when basins were overexploited but had less conjunctive use infrastructure. Conjunctive use infrastructure built during the last 15 years greatly reduces the costs of ceasing to overdraft the region's aquifers. This suggests that system operation and in particular conjunctive use have a major role in making sustainable groundwater use policies a success. The methodology and findings of this study, although representative of areas in California's southern Central Valley, are less relevant to regions where water trades are impossible, surface water imports non-existent, or where groundwater basins are non renewable (limited possibility of artificial recharge). The study demonstrates groundwater overdraft problems can most effectively be addressed by considering them within the context of their hydrologic, economic and engineered water resource systems.

Acknowledgements

We thank Guilherme Marques, Manuel Pulido-Velázquez, Marion Jenkins, and Stacy Tanaka for useful discussions on conjunctive use and help with modeling conjunctive use for the Tulare Basin. The authors thank Charles Brush, Brian Wagner and an anonymous review for comments that improved the paper. Vicki Kretsinger is thanked for comments on an earlier version of this work.

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Appendix B

Conjunctive Use Hydro-Economic Optimization with Limited Foresight—Application to the Redding Basin

Introduction

Mathematical models have been integral to the planning and management of water resources in California (CDWR 2005). Because both surface water and groundwater resources are used in California to satisfy water demands, management models should represent both resources (Mariño 2001). Various simulation models of California's water resources are maintained by agencies. The modeling method presented here is a conjunctive use hydro-economic optimization model that builds on previous simulation and optimization modeling projects.

Groundwater Management in California's Central Valley

California's Central Valley is a large region (> 50,000 km^2) of intense irrigated agriculture ($13 billion annual revenue) with a growing population (> 5.5 million). Groundwater supplies between 30 and 40% of California's water use (CDWR 2003a). In the Central Valley groundwater plays has a greater role as a major source of irrigation water (up to 50%) (Bertoldi et al. 1991) and sometimes as the sole water supply (e.g., cities of Fresno and Davis). The modeling method described here is applied to the northern most basin of the Central Valley; however, using the same methodology and data sources it could be extended to the entire Sacramento Valley or even the entire Central Valley.

Context of Modeling Work

The model investigated here employs non-linear deterministic multi-period optimization and produces time-series of optimal allocations, flows and storages throughout the water resource network for the modeled time-horizon. If long historical or synthetic time series are used the technique is considered implicitly stochastic (Labadie 2004), as inflows cover a statistically representative range of hydrologic conditions. Optimal time-series are studied to learn how the system could be best operated to satisfy the objectives declared in the model's objective function; in some cases operating rules can be inferred. This method is classic in water resource system engineering (Bhaskar and Whitlatch 1980; Karamouz and Houck 1982; Karamouz et al. 1992; Lund and Ferreira 1996; Young 1967).

An issue with multi-period optimization is that it grants the model unrealistic perfect hydrologic foresight (i.e., the model can prepare for dry or wet periods long in advance). Our method addresses this following the approach of Draper (2001) where the network optimization model is run in sequential annual runs using carry-over storage value functions to prevent end-of-year system drainage. Sequential annual model runs decrease full time-horizon foresight to intra-annual (e.g. perfect within year forecasts) and also decrease the size of optimization models that use long synthetic or historical time series of inflows. This is especially valuable when non-linear groundwater processes are included in the model.

The starting point for the proposed model and methodology is the CALVIN model (Draper et al. 2003), the Draper dissertation (2001) and the updated Sacramento Valley GAMS hydro-economic model of Hansen (Hansen 2007). CALVIN and its representation of groundwater are described in Appendix A. Like Draper (2001) we derive and use carry-over storage value functions to limit hydrologic foresight in the optimization. Here the functions are derived with a nonlinear model (rather than a linear network flow model) and the model maximizes net benefits rather than minimizing penalty functions. In addition the model tracks groundwater level (head) per economic water demand area and considers variable pumping costs (a function of groundwater head).

Modeling Method and Formulation

The first and last subsections introduce and summarize the method. The 2nd and 3rd sections focus on objective function terms while the 4th and 5th sections focus on the constraint set and how it represents hydrologic flows (surface and groundwater).

Method

The proposed model optimizes an integrated network of hydrologic, water management and economic processes. Integrated refers to the inter-tied network of agricultural and urban economic water demands within an engineered system of surface reservoirs, natural rivers, canals, and aquifers. Hydrologic and water management processes included in the model are: reservoir operations, minimum environmental flows, distributed artificial groundwater recharge, distributed aquifer pumping, and agricultural return flows. Piezometric head response at the sub-basin level to pumping and recharge is estimated through the use of a lumped storage coefficient. A monthly time step common for water resource management planning models is used.

The proposed model is a hydroeconomic optimization model: flows are driven by urban and agricultural economic water demand curves and unit operating costs. Net economic benefits from water deliveries are maximized over the entire time horizon using annual carry-over storage value functions to reduce the model's hydrologic foresight (Draper 2001). Carry-over storage value functions estimate the economic value of leaving water in storage at the end of the growing season for future dry years. In this way, the modeling time-horizon is reduced to a series of sequential optimization problems (Diaz and Brown 1997; Draper 2001). The initial storage levels of one run being the ending storage levels of the previous run. The carry-over storage value function, assumed as a concave quadratic function after Draper (2001), is described by 2 parameters (see section 6.2.3). For each storage unit in the network, a particular set of value function parameters are optimal—i.e., using the optimal parameter set gives the highest net benefit summed over the full modeled time horizon. Various search algorithms are available for this search (section 6.2.5). Because value function parameter search involves repeated evaluation of the full model, a simplified groundwater approach is used in this phase (storage coefficient equation—see Chapter 2). Once optimal value function parameters are found they are used in a series of time-marching annual optimization models which implement a more detailed groundwater modeling approach.

Deterministic optimization models provide the set of optimal storage allocation, reservoir release and distributed groundwater pumping/recharge rates given a series of hydrologic inflows. The optimal set of operations given system inflows can be analyzed to learn how to better manage the system and in some cases to derive operational guidelines (Lund and Ferreira 1996). Optimal carryover storage value functions can be used to derive optimal hedging rules for surface storage (Draper and Lund 2004). These management rules reflect the economic objective function and its associated environmental and infrastructure constraints. Inflows can be historical, synthetic historical or based on climate change scenarios.

Economic Demands and Operating Costs

The hydroeconomic model optimizes allocation of water deliveries by maximizing total net benefits. Economic water demand curves are used to estimate economic benefits derived from water deliveries. A regional aggregate empirical demand curve for water presents the consumer's willingness to pay for water in that area. The integral of the demand curve (i.e., summing the area beneath it) forms an economic benefit function quantifying the gross local economic benefits from a water allocation. Water is allocated to maximize the gross region-wide economic benefits minus operating costs.

Operating costs are included with a unit cost term which multiplies flow in links that either generate revenue (e.g., in-stream hydropower) or incur costs (e.g., pumping stations, artificial recharge, treatment). Benefits and costs appear in the objective function (decision variables are capitalized):

$$Z = \sum_t agBenefits^t + \sum_t urbBenefits^t - \sum_t c_{ij} X_{ij}^t - \sum_t C_{gj} X_{gj}^t \quad \forall i, g, j \qquad (B1)$$

where Z = annual net benefits derived from water deliveries

$agBenefits^t$ = benefits derived from agricultural deliveries during month t

$urbBenefits^t$ = benefits derived from urban water deliveries during month t

c_{ij} = unit cost (or profit) incurred for flow from node i to node j

X_{ij}^t = flow during month t from node i to node j

C_{gj} = unit cost for pumping from groundwater sub-basin g to node j

X_{gj}^t = flow during month t from groundwater sub-basin node g to node j

Because the unit cost of pumping depends on groundwater levels, the cost parameter on links that join a groundwater sub-basin with another node (signifying pumping) is a decision variable. Unit pumping cost at groundwater sub-basin node g as a function of its average regional elevation and head level is given by:

$$C_{gj}^t = unitc * (elev_g - H_g^t) \qquad \forall g, d, t$$

where C_{gj}^{t} = cost per unit volume of pumping from groundwater node *g* to node *j* [$/$L^3$],

unitc = cost of pumping a unit of water volume up a unit of height [$/$L^3$ per L],

elevg = mean elevation of area represented by groundwater node *g* [L],

H_{g}^{t} = piezometric head at groundwater node g during period *t* [L],

This calculation of unit pumping cost causes non-linearity, as the cost is a function of piezometric head , itself a function of the net stress link that connects groundwater basins to the network, (equation B4).

Carry-Over Storage Value Functions

Reservoirs are rarely emptied at the end of an irrigation season. Typically a volume of water is left for minimum ecological river flows or other system requirements. Beyond this minimum amount, whatever water is left functions to buffer a potential drought in the following year. The volume of water stored beyond minimum requirements is called carry-over storage. It fills the reservoirs "conservation pool" and is limited by flood control which maintains storage capacity to absorb floods. Draper (2001) suggests a hypothetical function to model the value of carry-over storage (Figure B1). Additional storage always has positive value (slope is positive) but the value of additional storage decreases with storage (concave).

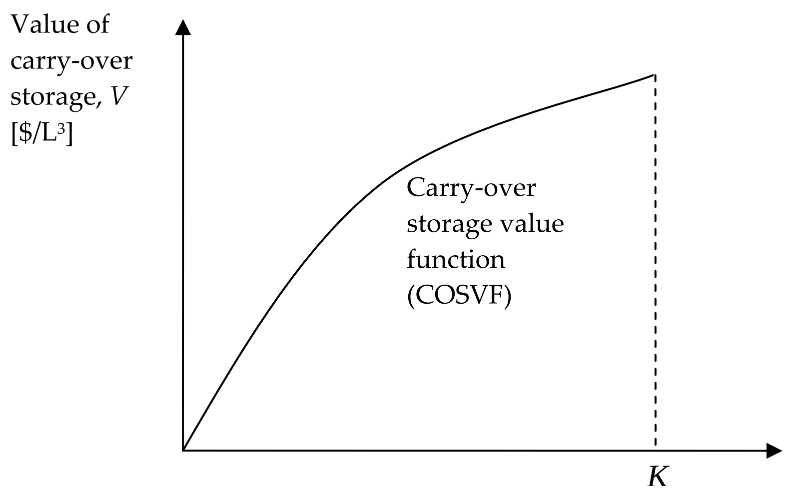

Figure B1: A carry-over storage value function characterizes the economic value of leaving water in storage at the end of the intensive use season.

Source: Draper (2001)

To limit the number of parameters necessary to represent the function, Draper (2001) assumes a quadratic function of form $V = aS^2 + bS + c$, where *V* is value of carry-over storage, *S* is carry-over storage and *a,b* and *c* are parameters of the quadratic function. Since when carry-over storage is zero it has no value, *c* = 0 and the term can be left out of the equation.

First and second derivatives of the value function are:

$$\frac{\partial V}{\partial S} = V' = 2aS + b \qquad (B2)$$

$$\frac{\partial^2 V}{\partial S^2} = V'' = 2a$$

The value function's positive slope and concavity imply a positive first derivative and negative second derivative, hence *b* is positive and *a* is negative. Because the value function represents aggregate value, its derivative (slope) indicates willingness-to-pay (WTP). Because plausible regional willingness-to-pay values are known (e.g., 0 to 500 $/AF in California), expressing quadratic function parameters in terms of willingness-to-pay establishes ranges of possible parameter values. To reveal the meaning of the quadratic constants Draper (2001) looks at the value of the derivative at $S = 0$ (no carry-over storage) and $S = K$ (maximum carry-over storage).

Evaluating equation B2 at S = 0, results in

$$b = V'\big|_{s=0}$$

Evaluating equation B2 $\frac{\partial V}{\partial S} = V' = 2aS + b$ at S=K, results in $V'\big|_{s=K} = 2aK + V'\big|_{s=o}$. Solving for a gives $a = \frac{V'\big|_{s=K} - V'\big|_{s=0}}{2K}$.

Following Draper (2001), we simplify notation by using the variable name WTP_{max} to represent $V'\big|_{s=0}$, slope of carry-over storage value at $S = 0$; and WTP_{min} to represent $V'\big|_{s=K}$, slope of carry-over storage value at $S = K$. Because willingness-to-pay at low storage will always be greater than at high storages, Figure B2 defines a feasible region for parameters WTP_{max} and WTP_{min}.

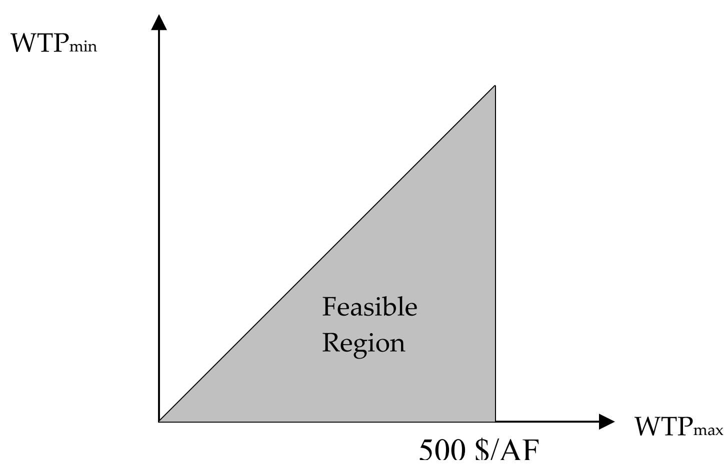

Figure B2: Feasible region for parameters WTP_{max} and WTP_{min}

Source: Draper (2001)

Identifying the feasible region reveals plausible parameter ranges. For example given a minimum and maximum willingness-to-pay of 0 to $500/AF ($500,000/TAF) and maximum carry-over storage, *K*, of 7,000 TAF, *b* ranges from 0 to 500,000 while *a* ranges from -500,000/(2*7,000) to 0/(2*7,000), i.e. from –35.7 to 0. The value function $V = -36S^2 + 500{,}000S$, assuming WTP_{max} = $500/AF and WTP_{min} = 0, is graphed below in Figure B3.

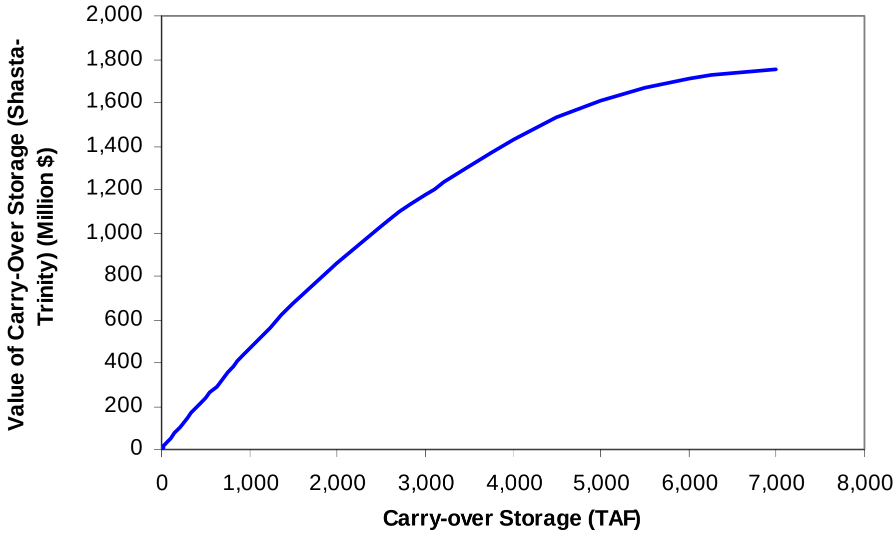

Figure B3: A carry-over storage value function (COSVF) for the Shasta-Trinity combined reservoir system assuming a range of WTP from 0 to 500 $/AF.

To value end of year storage in an annual model a term is added to the objective function, with the term COSBenefits:

$$COSBenefits = \sum_r a * (S_r^{sept})^2 + \sum_r b * S_r^{sept} \qquad (B3)$$

where S_r^{sept} is the September stored volume of reservoir *r* [L^3/t], and *a* and *b* are quadratic and linear term coefficients respectively of the carry-over storage value function.

Modeling Flow through the Network

The basic mathematical structure of the model is a flow network—in which nodes are connected by links (Jensen and Barnes 1980; Martin 1983). Nodes can be of storage or non-storage (junction) type; links can represent rivers, canals, flow into or out of aquifers. Storage is as a link over time in network flow programming. Hydrologic routing is not applied here, which implies flow travel times through the network must be less than the model's time-step (usually a month for water supply modeling).

The first constraint preserves mass balance at both storage and non-storage nodes:

$$S_i^{t+1} - S_i^t = \inf_i^t + \sum_j X_{ji}^t - \sum_j e_{ij} X_{ij}^t \qquad \forall\, i,t$$

where S_i^t = storage volume at storage node i during time period t [L^3/t],

$\inf_i^t$ = external inflows to node i during period t [L^3/t],

X_{ji}^t = flow from node j to node i during period t [L^3/t],

e_{ij} = linear gain/loss coefficient for the link from node i to node j [dimensionless],

For storage nodes, the left hand side is the change in storage from period t to period $t+1$. For non-storage nodes a maximum storage capacity constraint of zero at non-storage nodes ensures the left hand side of the equation equals zero. Variable $\inf_i^t$ is an external time series of inflows coming into node i; this could represent inflow from a river external to the model or runoff generated in the area between nodes j and i.

The net groundwater recharge/pumping rate (net stress) at each groundwater sub-basin is given in equation B4; it is used to estimate groundwater levels (equation B5).

$$NETX_g^t = \inf_g^t + \sum_j X_{jg}^t - \sum_j X_{gj}^t \qquad \forall g,t \qquad (B4)$$

where $NETX_g^t$ = net groundwater stress at groundwater sub-basin g [L^3/t],

inf_g^t = external inflows (recharge from precipitation) to g during period t [L^3/t],

X_{jg}^t = recharge flow into g (from artificial groundwater recharge and from percolation of applied agricultural water) during period t [L^3/t],

X_{gj}^t = groundwater pumping to node j (flow from GHN g to GDN d) during period t [L^3/t],

Capacity constraints on flow in links and on storage at nodes are listed below. When both min_{si} and max_{si} are zero, the node is a non-storage junction node.

$$minx_{ij} \le X_{ij}^t \le maxx_{ij} \quad \forall\, i,j,t$$

where $minx_{ij}$ = minimum flow from node i to node j [L^3/t],

$maxx_{ij}$ = maximum flow from node i to node j [L^3/t].

$$mins_i \le S_i^t \le maxs_i \quad \forall i,t$$

where $mins_i$ = minimum storage at node i [L^3],

$maxs_i$ = minimum storage at node i [L^3].

Numerical infeasibilities may appear making the network problem infeasible. To guarantee feasibility, artificial inflows are made available at each model node. To minimize their effect on results, these artificial flows have a high cost coefficient in the objective function. Infeasibility flows are added to the mass balance constraint:

$$S_i^{t+1} - S_i^t = \inf_i^t + \sum_j X_{ji}^t - \sum_j e_{ij} X_{ij}^t + INFEAS_i^t \qquad \forall\, i,t$$

where $INFEAS_i^t$ is the infeasibility inflow (> 0) at node i during time period *tI* These flows are minimized by adding the following term to the objective function:

$$\inf easCost^t = \sum_i (INFEAS_i^t) * mult \quad \forall t$$

where $infeasCost^t$ is the total cost of infeasibilities in the entire network during time period t and *mult* is a multiplier set empirically (10,000 is used in the application model presented later).

Representing Groundwater in the Network

As described in Chapter 2, groundwater can be included in system models at different levels of detail. Chapter 3 showed that in some cases, particularly when modeling scale is coarse and when site-specific phenomena such as stream-aquifer interaction are not considered, less detailed formulations can provide almost the same answers as more complicated formulations but at a lower computational cost. This was the case with the Sacramento Valley groundwater model presented in Chapter 3.

Embedding a storage coefficient formulation is the most economical method to model both lumped groundwater volume and head fluctuations. As summarized in the next section, an efficient formulation is necessary when repeated model runs are used to search for the carry-over storage value function parameters. The storage coefficient equation is repeated here for convenience.

$$H_g^t = H_g^{t-1} + \frac{NETX_g^t}{sc_g * area_g} \qquad \forall g,t \tag{B5}$$

where $NETX_g^t$ = net stress (net pumping, recharge term) in groundwater sub-basin g at time t

H_g^t = piezometric head in groundwater sub-basin g at time t

sc_g = mean storage coefficient of sub-basin g

$area_g$ = surface area of sub-basin g

Once optimal carry-over storage value function parameters have been selected, a more detailed series of annual optimization runs are completed. At that point, distributed models, such as embedding a sequential time-marching or eigenvalue formulation, are best. Stream-aquifer interchange also can be included if relevant.

Summary of Modeling Sequence

Our goal is to create a limited foresight deterministic-equivalent optimization model of an integrated water resource system. The modeled time horizon is separated into yearly optimization model runs, connected by initial and ending storage and groundwater head conditions. To prevent drainage of reservoirs at the end of each model run, carry-over storage value functions are used. Two major modeling phases can be distinguished: 1) search for optimal carry-over storage value function parameters and 2) sequential annual optimization using carry-over storage value functions to estimate economic value of end of year storage.

The search for value function parameters is carried out by repetitive execution within a search algorithm of an efficient form the conjunctive use model. A set of parameters are selected and linked annual models are run through the entire time horizon; net benefits in each year are recorded. At the end of a full time horizon, carry-over storage value and infeasibility costs are removed from the net benefits summed over each year. The optimal set of parameter values are those that produce the largest total net benefit over the entire modeled time horizon.

To decrease run times a simplified groundwater formulation is best used in this phase. In principle each source of storage (reservoir, aquifer) has unique carry-over storage value and should be modeled by its own quadratic function. In practice each parameter exponentially increases search run-times of the first modeling phase. Reservoirs can be aggregated by region to decrease the number of parameters. Another option is to represent storage value for some large storage elements (e.g., aquifers) with a linear value function (one parameter) (Draper 2001). Unlike surface reservoirs, aquifers are partially shielded from end of time-horizon drainage because of pumping capacity costs and limits, commonly large storage capacities, and pumping effects on future pumping costs.

Different search techniques can find optimal carry-over storage value function parameter values. The most basic is to lay a grid over the entire feasible solution space and run the model for each parameter value combination (Loucks et al. 1981). The advantage of this technique is that for parameters taken 2 at a time it produces a visual response surface. The method is inefficient for more than 2 or 3 parameters, although it can be useful to find initial solutions from which to start more sophisticated search techniques. Another option is to use a Nelder-Mead simplex search (Nelder and Mead 1965). Both search algorithms were implemented by Draper (2001).

Phase 2 begins when “optimal” carry-over storage value function parameter values have been found. In this phase those parameters are used in one last run to be scrutinized for its water management implications. Because only one model run is necessary in this phase, and since each optimization model is only a year in length, more detail can be included in this second phase. For example, if relevant to the specific modeling goals of a project, a spatially distributed groundwater formulation can be incorporated. The application presented next relies on a simple groundwater formulation in both the first and second modeling phases. The final run of phase 1 (in which we found the optimal carry-over storage value function parameters) is also the final run of phase 2; a separate groundwater formulation is not used.

Redding Basin Application

The method has the potential to be applied to a larger area; the implementation described here serves as a proof of concept.

Geography and Water Resource Network

The modeling approach is applied to the northern portion of the Sacramento River Valley, at the northern tip of California's Central Valley (region 1 of Figure B4). The Redding Basin contains the city of Redding and the Sacramento River headwaters, fed by Trinity and Whiskeytown Lakes (West) and Shasta Lake (East).

The Trinity-Whiskeytown and Shasta reservoirs are modeled in the network as two separate reservoirs. Their total storage is combined when considering carry-over storage to produce just one overall carryover storage value function.

Figure B5 details the water resource system, including one urban and one agricultural economic water demand nodes. This network is based on the CALVIN model (Draper et al. 2003), as simplified by Hansen (2007). The delimitation of the Redding basin (Depletion study area 58) comes from the California Department of Water Resource's CVGSM integrated model (USBR 1997). These regions are also used in the Central Valley Production Model (CVPM) that was developed as part of the CVPIA Programmatic Environmental Impact Statement (USBR 1997), as well as in CALVIN.

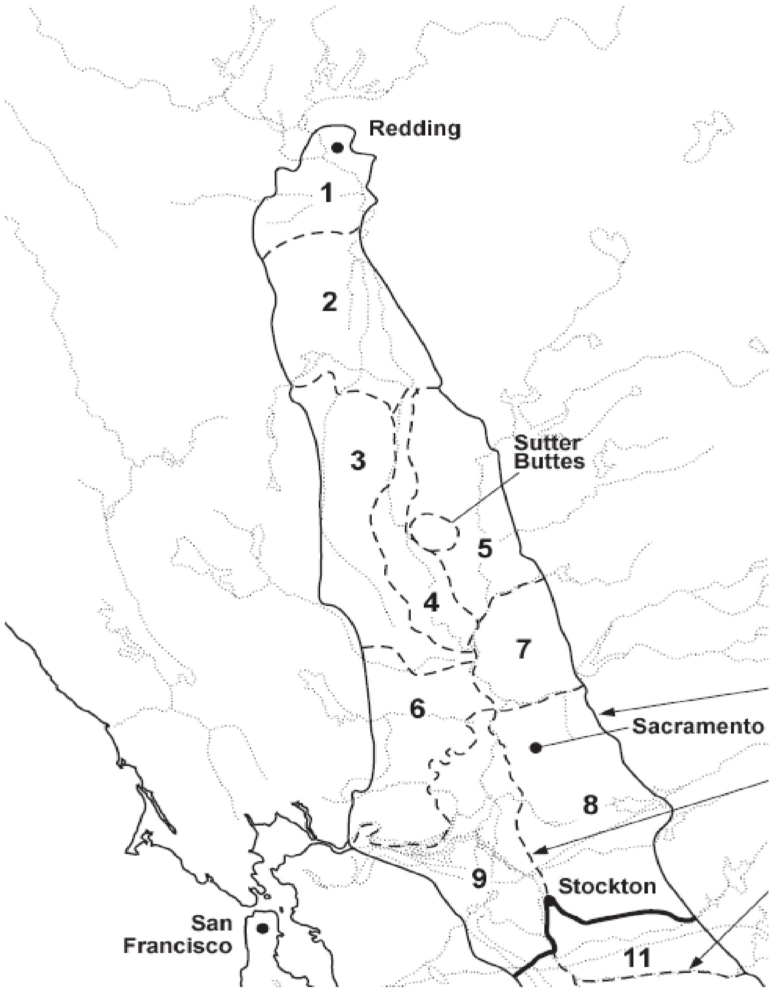

Figure B4. Subregions of Sacramento Valley corresponding to CVGSM subregion boundaries and SWAP model demand areas

Source: Modified from USBR (1997)

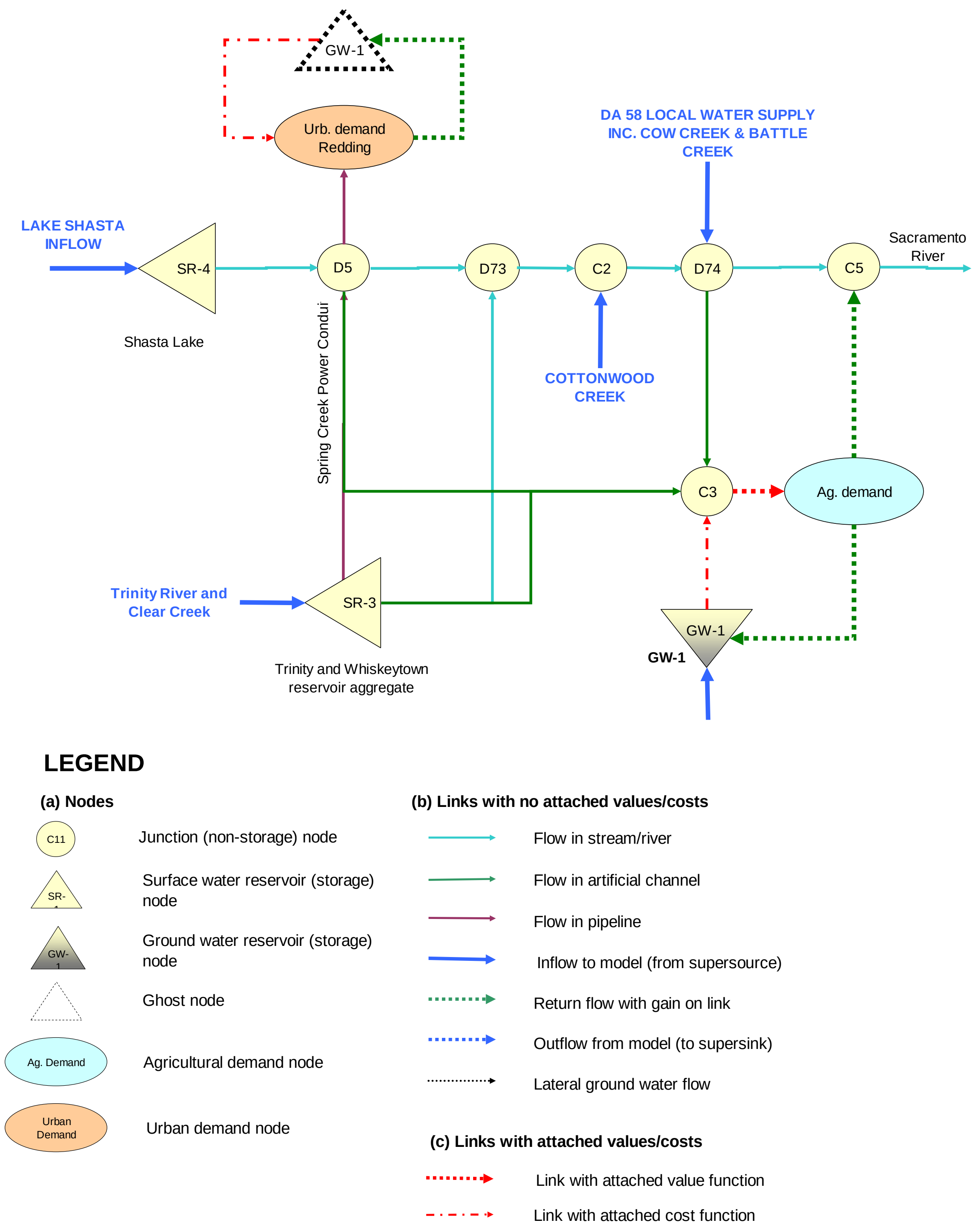

Figure B5. Network schematic of Redding Basin model. In other modeling studies this area corresponds to Depletion Study Area 58 (CVGSM, CALVIN) and to region 1 (SWAP).

Source: Adapted from CALVIN model schematic and Hansen (2007)

Data and Implementation

The model uses data from previous modeling studies. Conveyance capacities, operating costs, and basic network topology were obtained from the CALVIN model (Draper et al., 2003), adopting the simplifications of Hansen (2007). 72 years of historic hydrologic monthly time series for surface reservoirs inflows, groundwater recharge from precipitation are taken from the CALVIN model.

Agricultural economic benefit functions are derived from the Statewide Water and Agricultural Production model (SWAP) (Howitt et al. 2001) and were updated by Hansen (2007). SWAP is a mathematical program that reduces water deliveries incrementally and output shadow prices of water at each delivery level thus producing a demand curve (Table B1). Hansen (2007) fit a 2-parameter log function to this data of type: Price = $c + d*log(X)$, where c,d are monthly parameters (Table B2) and X is the monthly agricultural water delivery in TAF. The integral of the demand curve is the benefit function; in this case:

$$agBenefits = c*X + d*X(log(X)-1)$$

This definition for agricultural water delivery benefits is substituted in the model's objective function, equation B1.

Table B1. Water quantities (TAF), Shadow values and water demands for the Redding basin

Water	Shadow	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
183.8	$44,484	0	0	0.1	11.4	30.7	33.2	42.9	36.7	23.5	7.0	0	0
165.4	$49,657	0	0	0.1	10.3	27.6	29.9	38.6	33.0	21.1	6.3	0	0
147.0	$55,993	0	0	0.1	9.1	24.5	26.6	34.4	29.3	18.8	5.6	0	0
128.6	$63,942	0	0	0.1	8.0	21.5	23.3	30.1	25.7	16.4	4.9	0	0
110.3	$74,230	0	0	0.1	6.8	18.4	19.9	25.8	22.0	14.1	4.2	0	0

Source: SWAP results from Hansen (2007)

Table B2. Two parameters of the log function fitted to water demand data derived from the SWAP model.

	Log term	Constant
OCT	-57,984	156,343
APR	-57,938	184,847
MAY	-58,279	243,256
JUN	-58,248	247,795
JUL	-58,265	262,788
AUG	-58,225	253,479
SEP	-58,219	227,432
OCT	-61,076	218,764

Source: Hansen (2007)

Urban water delivery benefit functions are developed using observed average monthly deliveries and prices and elasticity of demand estimates from other studies. Table B3 provides the data used to create the benefit functions. *UXBAR* are representative fixed monthly urban water demands (CALVIN) and *UPBAR* is a representative urban water price estimate (Jenkins 2001, Appendix B). *UELAST* are California elasticity of demand estimates (Jenkins 2001, Appendix B).

A linear water demand function provides price (*P*) as a function of supply (*X*):

$$P = m + nX \qquad (B6)$$

Parameters *m* and *n*, the parameters of the linear urban demand curve, are derived by beginning with the definition of elasticity of demand, η:

$$\eta = \frac{dX}{dP} * \frac{P}{X}$$

Here we solve for *dP/dX*, the slope of the demand curve (parameter *n*), leading to

$$n = \frac{dP}{dX} = \frac{P}{\eta * X}$$

Using empirical date we can calculate *n*:

$$n = \frac{UPBAR}{UELAST * UXBAR}$$

Solving equation 6 for m allows its estimate:

$$m = UPBAR - n * UXBAR$$

Table B3. Urban water delivery benefit function data

	UXBAR	*UPBAR*	*UELAST*	*m*	*n*
OCT	6.3	334,000	-0.15	2,560,667	-356,267
NOV	4.5	334,000	-0.15	2,560,667	-494,815
DEC	4.1	334,000	-0.15	2,560,667	-544,417
JAN	3.8	334,000	-0.15	2,560,667	-581,375
FEB	4.0	334,000	-0.15	2,560,667	-563,713
MAR	4.3	334,000	-0.15	2,560,667	-523,922
APR	5.6	334,000	-0.35	1,288,286	-171,019
MAY	7.7	334,000	-0.35	1,288,286	-124,580
JUN	10.0	334,000	-0.35	1,288,286	-95,238
JUL	11.9	334,000	-0.35	1,288,286	-80,530
AUG	11.4	334,000	-0.35	1,288,286	-83,636
SEP	8.9	334,000	-0.35	1,288,286	-107,465

Source: Hansen (2007); Jenkins (2001, Appendix B)

Economic benefits from urban deliveries are obtained with the integral of the demand curve, i.e.

$$urbBenefits = mX + 0.5*nX^{2} \qquad (B7)$$

Equation B7 is substituted into the objective function (equation B1).

Ideally, the water demand of the rest of California (south of Redding basin) should also be represented economically, by aggregating all economic benefits of water allocation elsewhere in the state. Here a simpler approach is taken; water demands outside of the modeled area assumed as fixed and represented with a fixed outflow time series boundary condition.

The model is implemented in the GAMS programming language (Brooke et al. 2006) using code and data modified from Hansen (2007). Input data and output are organized in excel sheets and imported/exported automatically using GDX functionality. The Redding basin model run over 30 years solves in approximately 20 seconds using the MINOS nonlinear solver (Murtagh and Saunders 1998).

Model Results

Several types of results emerge from these model runs.

Grid search for carry-over storage value function parameters

Identifying the feasible region is the first step of the search for optimal carry-over storage value function parameters. Given minimum and maximum willingness-to-pay (WTP) of 0 to 500$/AF (500,000$/TAF) and maximum carry-over storage, *K*, for the combined Shasta-Trinity reservoir system of 7,000 TAF, *b* ranges from 0 to 500,000 while a ranges from -500,000/(2*7,000) to 0/(2*7,000), i.e. from –35.7 to 0. Once the feasible space was selected, coarse and then zoomed grid searches were performed (Figure B6). The optimal combination of parameters was WTP_{min} = 0, WTP_{max} = 7.5 $/AF which translates to *a* = -0.536, b = 7,500 ($/TAF). The optimal parameters substituted into equation B3 produce the optimal value function, displayed in Figure B7.

Although the grid search is relatively coarse, it worked acceptably given that only 2 parameters are being searched for. Automated search algorithms must be used for larger problems to enable an efficient solution process. A Nelder-Mead simplex (Nelder and Mead 1965) search routine in GAMS was generalized from Howitt and Msangi (2003) and modified from a 3- to 2-parameter search. The Nelder-Mead algorithm is unconstrained and so the algorithm quickly searched outside the feasible space. The algorithm was modified by Draper (2001) to correct this; but this adaptation was not implemented here. In the case of the Redding Basin application, the telescopic grid search performed satisfactorily.

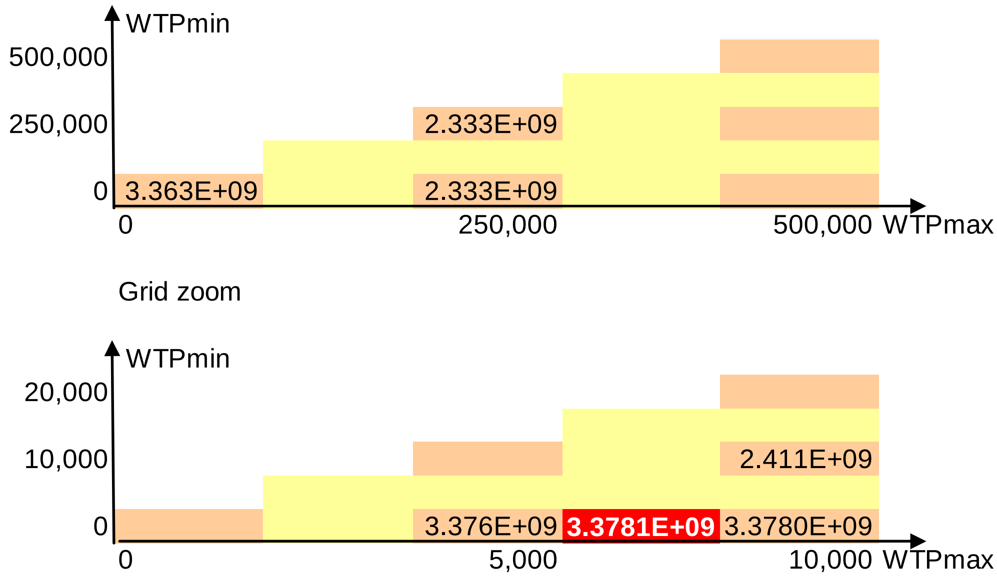

Figure B6. Grid searches for optimal carry-over storage value functions. Each value is the sum of annual net benefits from 1961 to 1977. Upper chart: coarse grid search over entire feasible space; Lower chart: zoomed grid search with optimal net benefits (red box) at WTP_{min} = 0, WTP_{max} = 7.5 $/AF.

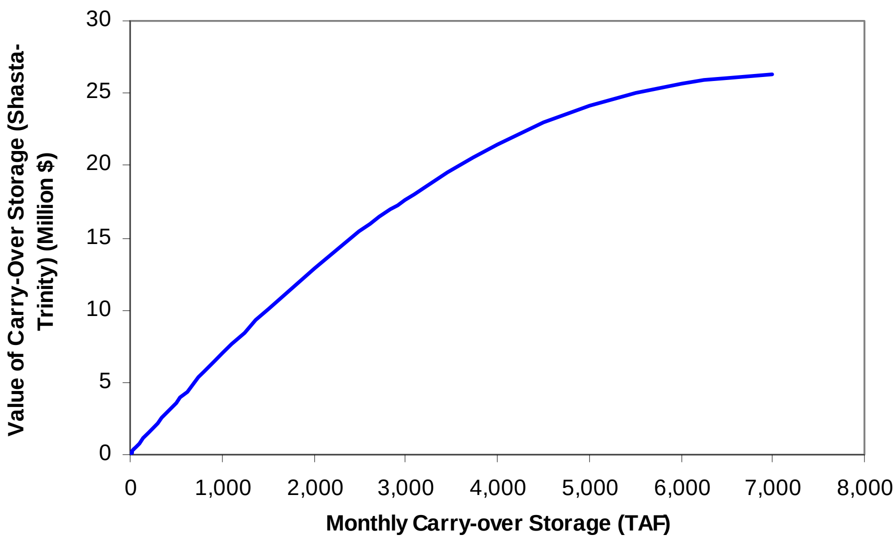

Figure B7. Optimal carry-over storage value function for the Shasta, Trinity and Whiskeytown aggregate storage system.

Flows, storage and costs

Model results include optimal flows and storages throughout the network and at every time step. When links reach constrained levels (either flow or storage), shadow values provide the marginal value on adding a unit of capacity. Shadow values of mass balance constraints at junction nodes provide the economic value of injecting a supplementary unit of flow at the node.

Figure B8 shows how all urban demands and most agricultural demands are met during the 1960 to 1990 modeled period. Agricultural demands are met below their usual levels after the 1977 and 1987 droughts, resulting in significant loss of economic benefits.

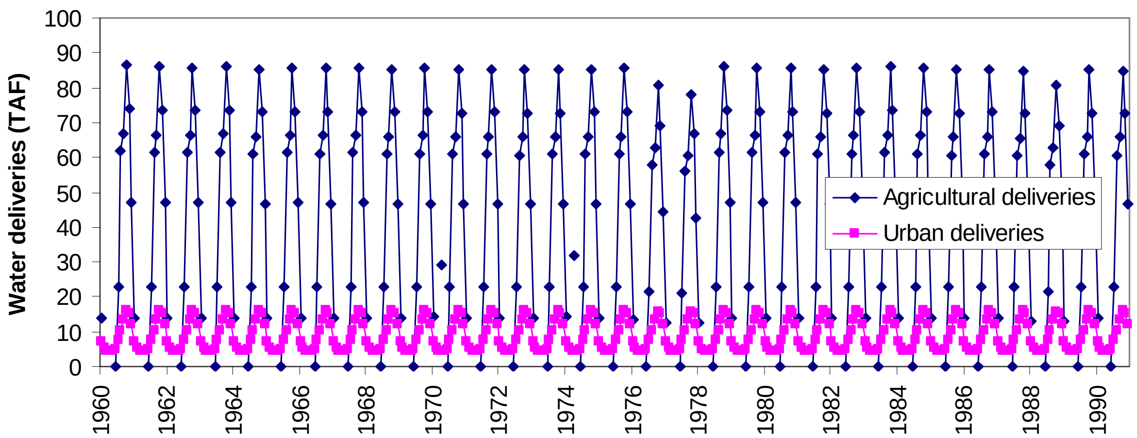

Figure B8. Agricultural and urban water deliveries.

The two dry periods excerpt a strong influence on the calculated optimal storage cycle as seen in Figure B9. In the relatively wet periods, the model avoids pumping costs and relies mostly on surface storage, progressively drawing down both reservoirs. This is accompanied by a rise in piezometric head and decreases in groundwater pumping costs. During the two dry periods, surface water supplies are scarce (Figure B9) and groundwater supplies are intensely used, resulting in strong dips in groundwater levels (Figure B10) and corresponding pumping cost increases (Figure B11). However, unit cost of groundwater pumping only rise significantly during the 1977 drought (Figure B12), when groundwater is most exploited.

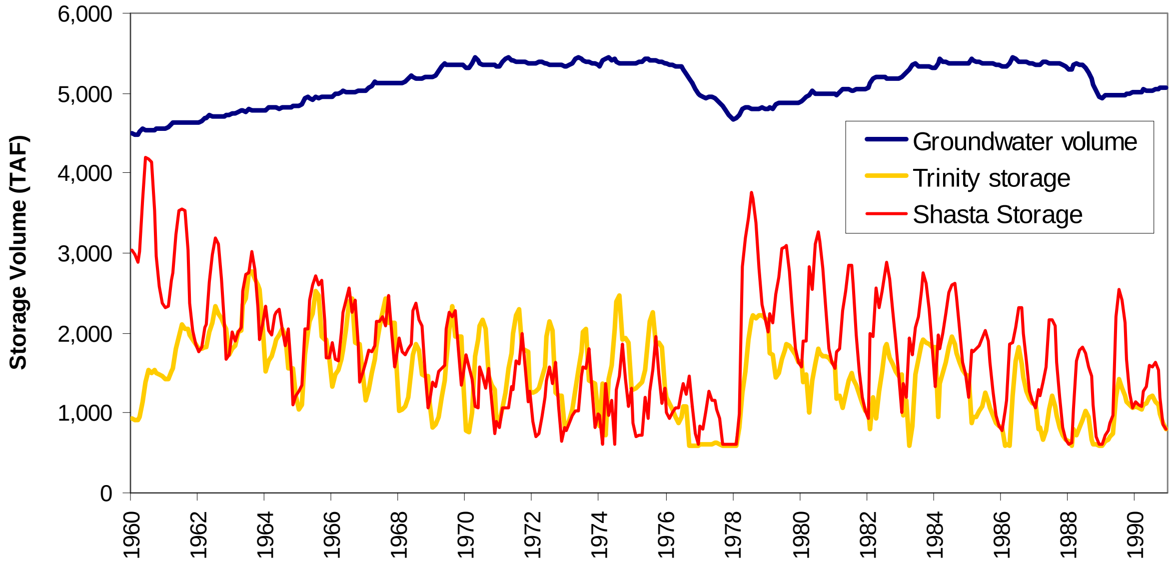

Figure B9. Monthly storage in surface reservoirs and Redding groundwater sub-basin

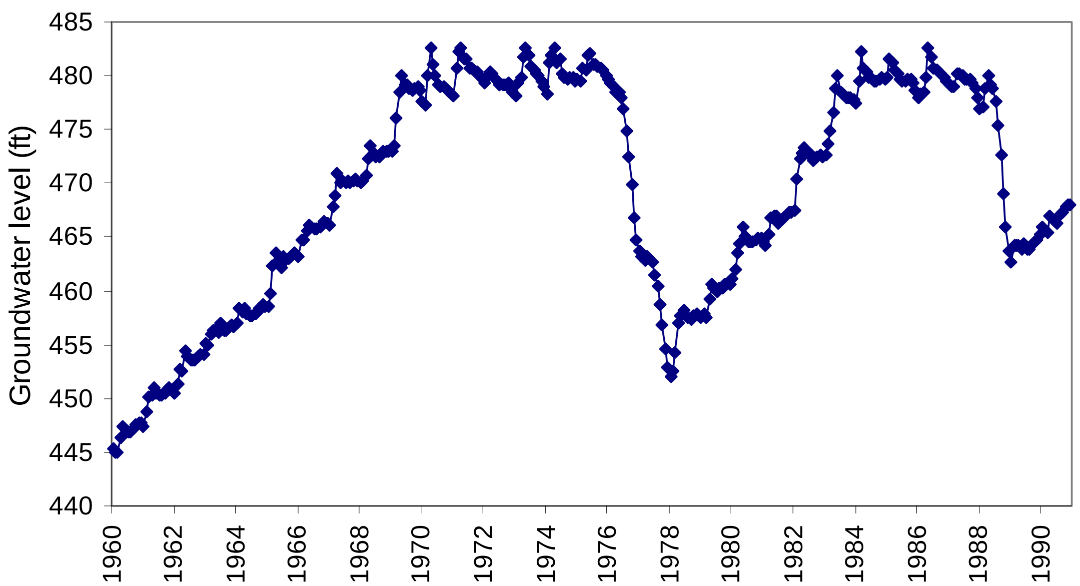

Figure B10. Groundwater head levels (ft) in the Redding Basin

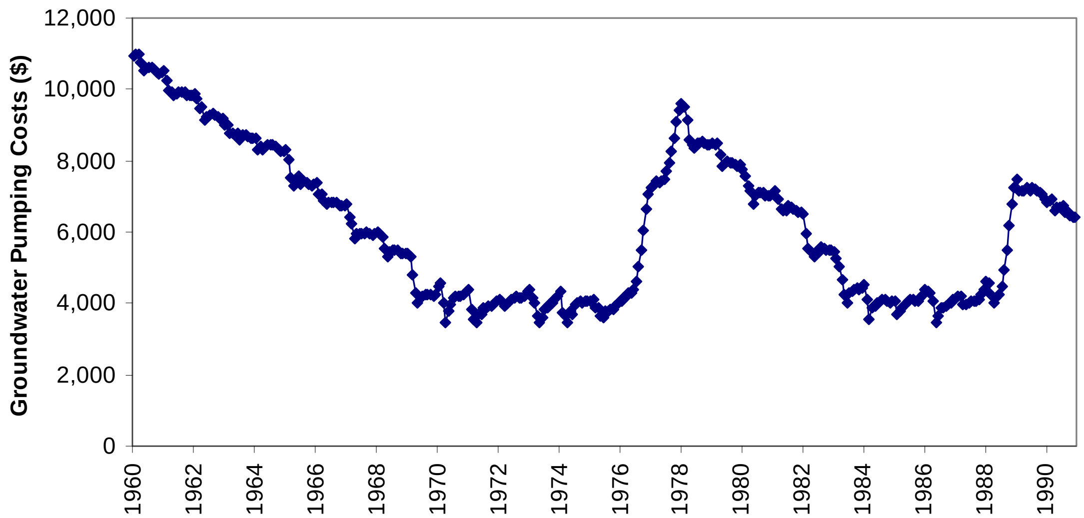

Figure B11. Monthly total groundwater pumping costs

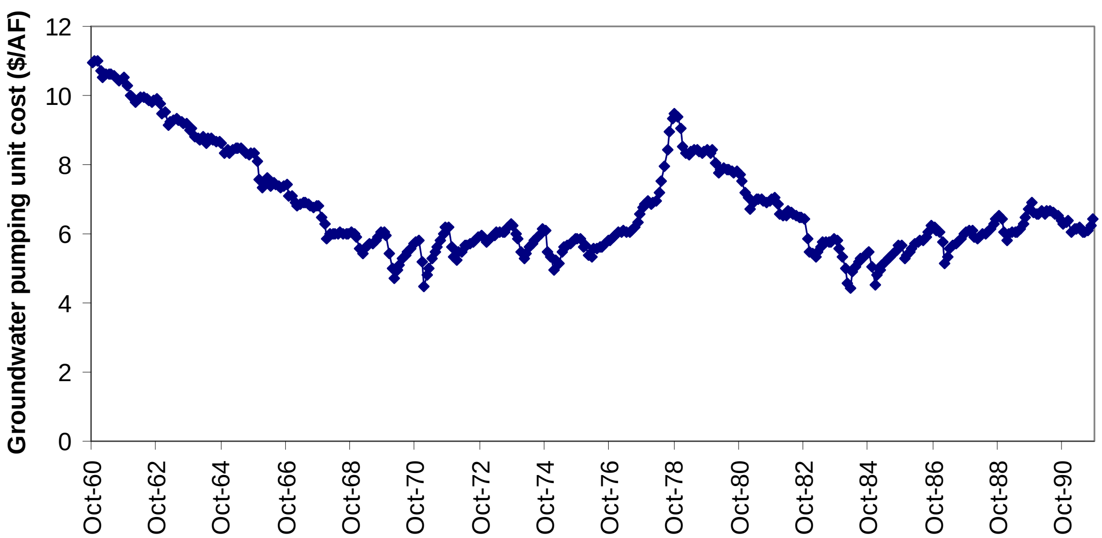

Figure B12. Unit cost of groundwater pumping ($/AF)

The annual value of carry-over storage decreases throughout the modeled time-horizon just as storage decreases, from roughly $25 million to $10 million by the final drought (Figure B13). Carry-over storage value is not considered in Figure B14 which shows the time series of net benefits. Economic benefits from water deliveries in the Redding Basin are only significantly affected by the 1977 drought (25% loss), when agricultural deliveries are below normal levels. Comparing the net benefits with the total groundwater pumping costs (Figure B11) shows that pumping costs, even when temporarily increased due to intensive development, are minor compared with the scarcity cost of lost agricultural productivity in the northern Central Valley.

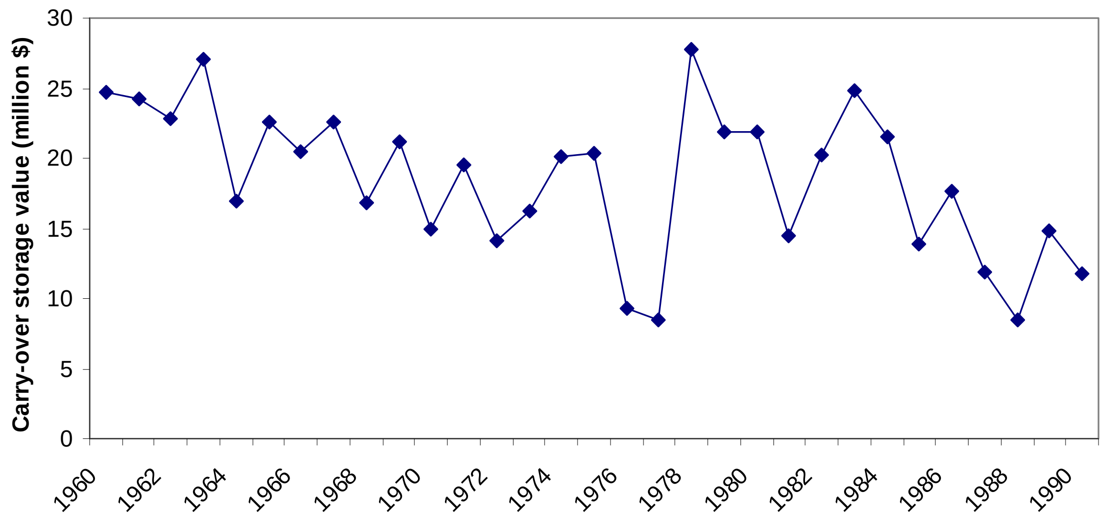

Figure B13. Annual carry-over storage value calculated in September of each year by using the optimal carry-over storage value function. Carry-over storage value decreases because storage decreases.

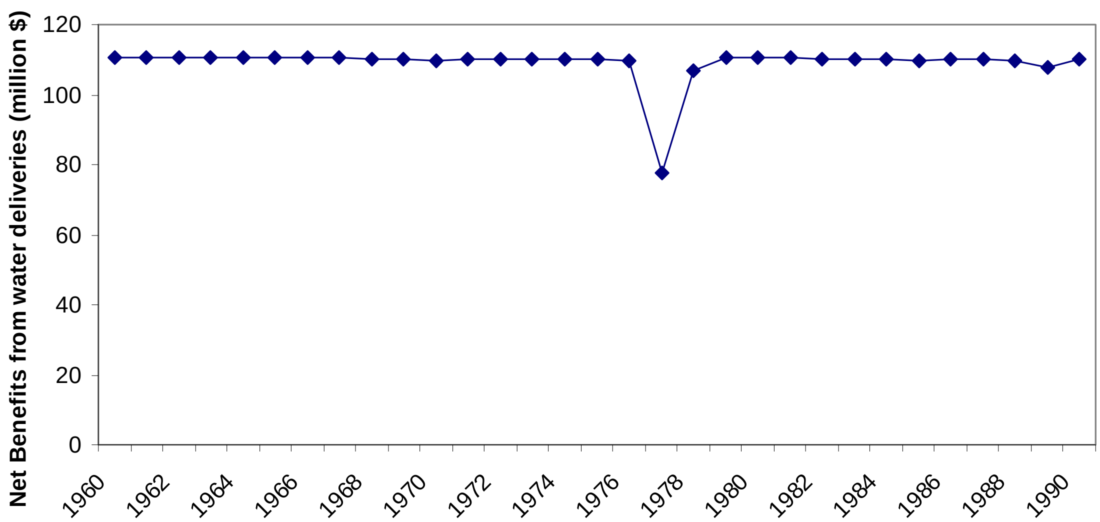

Figure B14. Annual net economic benefits from agricultural and urban water deliveries net operating costs (e.g. groundwater pumping)

Discussion of method and results

Optimal flows and storages produced by hydroeconomic optimization model reflect the net benefit maximization economic objective and the physical models embedded into the constraint set. Results can be scrutinized for insights into better water management; particularly when they can be compared to historical simulations. In the early stages of model building, as is the case here, model results are also useful to assess where the model can be improved.

Although the model has limited (intra-annual) foresight, the time series of groundwater storage seems to indicate the model has inter-annual foresight. This occurs because the economic optimization implicitly performs conjunctive use by keeping off groundwater in wet years (to avoid pumping costs)—thus having it available during dry periods. This exemplifies the buffer value of groundwater, often evoked in the literature on water resource economics and conjunctive use (Tsur and Grahamtomasi 1991).

A major limitation of the method presented here is the exponential computing resources required when the number of carry-over storage value functions is greater than one, as would occur with developing carry-over storage value functions for multiple reservoirs and aquifers. Use of search algorithms may alleviate this problem for cases of 2 or 3 reservoirs but, for larger systems the method will succumb to the curse of dimensionality if a disaggregation scheme is not used. Like dynamic programming models, the number of model runs needed to find optimal carry-over storage value function parameters rises exponentially with the number of parameters searched.

Further limitations of the current implementation include preliminary data, use of fixed rather than economic demands south of the Redding Basin, aggregation of upper unconfined and lower confined aquifers, and use of a coarse grid search for optimal carry-over storage value function parameters (rather than use of an automated search algorithm).

Conclusions

A method for optimizing conjunctive use systems with an economic objective and limited hydrologic insight is described. The work extends existing models (Draper 2001; Draper et al. 2003) by including dynamic groundwater levels (piezometric head) and pumping costs which results in a non-linear formulation. Limited foresight in achieved by dividing the modeled time horizon into annual segments and optimizing them individually using an optimal carry-over storage value function to prevent end of period drainage. Each annual model is linked to the previous one by end of year storage. Optimal carry-over storage value function parameters are derived by repeated evaluation of the model by a search algorithm. A proof of concept application of the methodology to the Redding Basin in California's Northern Central Valley was presented. Model results over the 1960 – 1990 period showed that economic benefits are significantly perturbed during the 1977 drought, even under optimal intra-annual operations, and that agricultural scarcity costs dwarf groundwater pumping costs, even under overexploitation, The modeling method has the potential to help improve conjunctive use water management in California, particularly by revealing the value of over-year storage and by revealing the impact of groundwater pumping costs.

Acknowledgements

The authors thank Kristiana Hansen and Richard Howitt for sharing a GAMS code of the Redding Basin and Richard Howitt and Siwa Msangi for sharing a GAMS Nelder-Mead code.

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