Derived Operating Rules for Storage and Recovery in Multiple, Unconnected Aquifers

By

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Fall 2002 did not seem like the optimal time to study cooperative water management in the Jordan River. While attending a free concert in Golden Gate Park, Nina inspired the idea to instead use aquifers in Jordan as an example to demonstrate the theoretical formulations. However, by December, with no response from aquifer managers in Jordan but the theoretical formulations most finished, I was left conjuring two, simple, numerical examples to demonstrate potential solutions.

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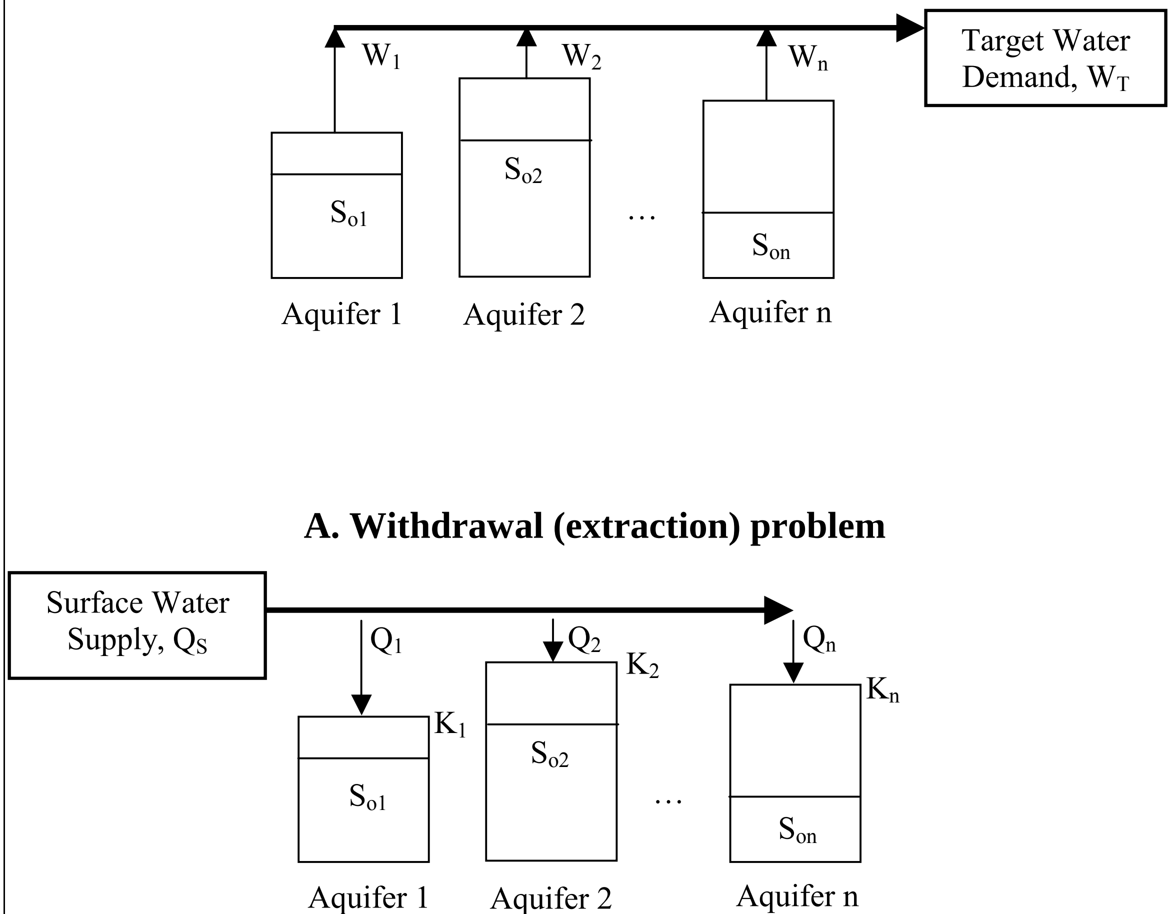
Abstract

Six optimization formulations and balancing rules are presented to inform management for the drawdown and recharge of storage in multiple, unconnected aquifers. Program objectives relate to (i) economic performance, (ii) duration of operation, and (iii) accessibility as a tradeoff between maximizing instantaneous withdrawal rate and the duration that withdrawals can be sustained. Aquifers are modeled as separate, single-celled basins with lumped parameters representing key physical, institutional, and economic characteristics. Recharges and withdrawals are treated separately for the first two program objectives and sequentially for the third objective. Each program is solved analytically with resulting operating rules for the case where constraints are non-binding. Solutions are also demonstrated for two simple numerical examples. Results show how economic characteristics, fraction of recharged water available for withdrawal (fractional recovery), initial storage, maximum recharge and pumping rates guide optimal recharge and withdrawal decisions. Uncertainties regarding future availability of water for extraction also influence decisions. Where higher reliability is desired, managers should recharge and pump aquifers with higher expected availabilities. However, increased reliability will decrease either economic performance or duration that withdrawals can be sustained. Lastly, to maximize flexibility and accessibility, managers should preferentially recharge aquifer(s) with both large maximum pumping capacities and high fractional recoveries.

1. Introduction

Water storage for many water supply systems is moving underground. As surface and ground waters have come to be more conjunctively managed, more elaborate problems in aquifer management for water supply and drought response arise. In California, major urban areas in Southern California (Metropolitan Water District) and the San Francisco Bay region (Santa Clara Valley Water Agency) now contract with irrigation and water management districts that overlie large aquifers (Pulido-Velazquez 2002). These aquifers can provide water storage to meet demands for several years duration. However, they are often far from the urban water demand area and require extensive water exchanges to deliver water during drought. As illustrated in Figure 1, the water provider may be challenged with how best to recharge water into and extract water out of multiple aquifers, while accounting for various physical and non-physical characteristics among aquifers.



A. Withdrawal (extraction) problem

B. Recharge problem

Figure 1: Water System Balancing for Multiple, Unconnected Aquifers

The aquifer balancing problems presented in Figure 1 are somewhat similar to the classical problem of operating surface water storage reservoirs configured in parallel (Bower et al. 1966; Lund 1996; Lund and Guzman 1999; Sand 1984). But managing multiple, unconnected aquifers differs in several key regards. First, aquifer managers often can regulate aquifer inflow and withdrawal through choice of recharge and pumping facilities and volumes. Aquifer operators are subject to less uncertainty in inflow than operators of surface water reservoirs. Especially in times of drought, demand is relatively constant and natural recharge is likely small or trivial. Second, aquifer storage often is refilled and drawn down over several years or decades rather than seasons when anticipating or responding to droughts. Third, recharge and extraction capacity characteristics, storage losses, legal, institutional, and other non-physical characteristics of aquifers may constrain aquifer operations. This is especially applicable where (i) draw-downs are small compared to the saturated thickness of the aquifers, (ii) geologic formations (confining layers or lenses) hydraulically isolate aquifers, (iii) large distances separate the aquifers, or (iv) the hydraulic response time is much longer than the planning horizon so that operations in one aquifer do not operationally affect other aquifers.

The above assumptions transform a stochastic conjunctive use problem (Philbrick and Kitanidis 1998) into a steady-state one and narrow the scope to a simple case where lumped parameters can represent the different hydro-geological, institutional, and economic properties of multiple, unconnected aquifers. Lumped aquifer characteristics include physical and accounting losses, storage, recharge, and extraction capacities, water quality, cost, and future availability to withdraw water.

This thesis derives operating rules to specify steady recharge to and withdrawal from multiple independent aquifers based on lumped individual aquifer characteristics. Six optimization formulations are presented for management objectives based on (i) economic performance, (ii) duration of operation, and (iii) water accessibility (as a tradeoff between instantaneous withdrawal rate and the duration that withdrawals can be sustained). For each program, analytical solutions are derived for cases where constraints are non-binding. Analytical solutions are restated as operating rules. Solutions are also demonstrated for two simple numerical examples. In the examples, a chance constraint specifying future availability is tightened to highlight tradeoffs between reliability and economic performance or the duration that withdrawals can be sustained. The goal of the work is to suggest how aquifer characteristics should influence the optimal recharge and withdrawal of multiple, unconnected aquifers.

2. Management to Maximize Net Expected Economic Performance

A general management objective is to maximize the net expected economic value of operations. This valuation incorporates the economic value of extracted water, costs associated with recharging, pumping, treatment, conveyance to end users, and institutional transaction costs associated with the above activities. Economic management formulations can be posed, separately, for withdrawal or recharge problems. Withdrawal and recharge rules for maximizing net economic performance are derived as follows.

2A. Maximize the benefit of withdrawals

The objective is to identify withdrawal rates from each aquifer that maximize the benefit of using groundwater. Initial storages are taken as given. For withdrawal, prior recharge costs are sunk (literally!) and not considered. The net economic benefit of withdrawal is the economic use value of withdrawn water minus costs of withdrawing, treating, conveying, and securing right to access and use the aquifer. Because aquifers can differ in extracted water quality and use to which that aquifer's water can be applied, the net use value of water can differ among aquifers. Hydraulic pumping lifts, treatment requirements, and conveyance distances also will differ among aquifers. The following mathematical program expresses this net economic objective:

$$\text{Maximize } Z_{2A} = \sum_i (u_i - c_i) \cdot W_i \qquad (1)$$

Subject to:

- No negative withdrawals

$$W_i \geq 0, \forall \text{ i} \qquad (2)$$

- Withdrawals limited by maximum pumping rates

$$W_i \leq p_{maxi}, \ \forall \text{ i} \qquad (3)$$

- Volume of withdrawals limited by initial storages,

$$W_i \cdot t \leq S_{oi}, \ \forall \text{ iI} \qquad (4)$$

- Withdrawals must meet the exogenous water demand rate

$$\sum_i W_i = W_T \qquad (5)$$

Where W_i is the withdrawal rate decision for aquifer i [volume time^{-1}], u_i is the unit economic use value of water extracted from aquifer i [$ volume^{-1}], c_i is the sum of unit costs to extract, pump, treat to the desired quality standard, convey, and cover other institutional, legal, and transactional expenses related to gaining access to aquifer i [$ volume^{-1}], p_{maxi} is the maximum withdrawal rate for aquifer i [volume time^{-1}], S_{oi} is the amount stored in aquifer i available for extraction [volume], t is a predetermined, relevant duration of withdrawal [time], and W_T is the total target demand rate [volume time^{-1}]. These demand rate and duration are fixed exogenously. Here we only want to find the most profitable allocation of withdrawals among aquifers. The assumption that unit-pumping costs are fixed for each aquifer allows the problem to be solved as a linear program.

The following general withdraw rule results, "Unless limited by pumping rates or storage, withdraw water in order of decreasing net value, u_i - c_i." Take water from aquifers with the largest, positive differences between use value and costs. Because high-valued water is used first, water withdrawals become increasingly costly (lower net value) as a withdrawal program is sustained, for example in response to a drought. Over a population of droughts of uncertain lengths, this rule will generally maximize the expected net benefit of drought response.

2B. Maximize the expected economic value of recharge

For recharging, the objective is to identify volumes to recharge aquifers that will maximize the net, expected economic value of withdrawing water at a future date. Equation 1 is modified slightly to consider the cost of recharge borne in the present, and that benefits and other associated costs enumerated for the withdrawal problem occur in the future. Also, recoverability and future availability of recharged water must be explicitly considered since these characteristics will likely influence the amount of water that can be later extracted and used. This problem formulation is:

$$\text{Maximize}\, Z_{2B} = \sum_i v_i \cdot \lambda_i \cdot Q_i \qquad (6)$$

Subject to:

- No negative recharges

$$Q_i \geq 0,\ \forall\ \text{i} \qquad (7)$$

- Storage capacity on each aquifer

$$Q_i \leq K_i,\ \forall\ \text{i} \qquad (8)$$

- Total recharges limited by surface water supply

$$\sum_i Q_i \leq Q_S \qquad (9)$$

- Recharges limited by maximum recharge rates

$$Q_i \leq r_{maxi} \cdot \text{t},\ \forall\ \text{i} \qquad (10)$$

- A fraction β of total recharges must be available for future withdrawal with target reliability α

$$P_r\left[\sum_i a_i \cdot Q_i \geq \beta \cdot \sum_i Q_i\right] \geq \alpha \tag{11}$$

Where Q_i is the decision on the amount to recharge into aquifer i [volume], λ_i is the expected fraction of recharge recoverable during extraction [unitless], K_i is the unfilled, remaining storage capacity of basin i [volume], r_{maxi} is the maximum recharge rate for basin i [volume time^{-1}], a_i is a random variable representing future availability to extract water from aquifer i [fraction], β is the fraction of recharged water which is desired to be recovered in the future [unitless], and α is the target reliability that the water should be available [fraction]. t (duration) and S_{oi} (initial storage) are as defined previously. v_i is the discounted, unit net economic value of storing water in aquifer i [\$ volume^{-1}] and can be calculated as:

$$v_i = b \cdot (u_i - c_i) - rc_i \tag{12}$$

Where, u_i and c_i are values and costs as specified previously, rc_i is unit cost to recharge aquifer i [\$ volume^{-1}], and b is a factor that relates the value of money when water is recharged to the future when water is extracted, conveyed, treated, and sold [unitless]. Since it is typical for withdrawal from different aquifers to occur at similar times, the parameter b is assumed to be constant across aquifers. As such, b does not affect the distribution of recharges and v_i can be treated as a constant.

The fractional recovery term λ_i lumps accounting and physical losses into a single factor and expresses losses as a fixed fraction of the recharge amount. λ_i will be < 1 for aquifers where groundwater flow directs recharge water away from the recharge site. λ_i will likely equal 1 for recharge by in-lieu exchanges, but may be less if institutional accounting losses are significant. The institutional losses can also be though of a “put / take ratio” representing rent on groundwater storage.

The random variable a_i allows the institutional and physical risks of each aquifer to be represented explicitly in the formulation (equation 11). Recharged water might not be available later for extraction due to unforeseen regulatory, legal, or water quality concerns, or lack of available conveyance capacity to move the water. Aquifers governed by different entities and with different physical-chemical characteristics are likely to differ in these risks. When the distribution of a_i is known, the chance constraint can be reduced to a deterministic constraint (Tung 1986; Wagner 1969). For example, when a_i takes the Gaussian distribution, equation (11) becomes

$$\sum_i \left[\left(\overline{a}_i - Z_\alpha \cdot \sigma_i - \beta \right) \cdot Q_i \right] \geq 0 \tag{13}$$

where $\bar{a}_i$ is the expected availability of aquifer i [fraction], σ_i is the standard deviation of that availability, and Z_α is the standard normal variate for probability α.

The unfilled storage capacity, K_i, is limited by the unsaturated void space in aquifer i, typically below the root zone of overlying vegetation. K_i can be readily calculated from the aquifer porosity parameter using:

$$K_i \leq \iint_{x_i, y_i} \left(\int_{z=e_{i,xy,ls}}^{z=e_{i,xy,us}} \rho_{i,\,xyz} dz \right) dy \quad dx \;\; , \quad \forall i \tag{14}$$

Where x_i and y_i are the longitudinal and latitudinal domains of the recharge area for aquifer i, $\rho_{i,xyz}$ is the porosity of aquifer i at location x,y,z , and $e_{i\ xy\ us}$ and $e_{i\ xy\ ls}$ are, respectively, the upper (i.e., ground surface) and lower (i.e., groundwater) surface elevations of the unsaturated depth profile available for storing water at location x,y in aquifer i [length]. The unfilled available storage capacity also can be limited by institutional arrangements made with local or regional aquifer owners or regulators.

This problem also can be solved as a linear program. The following general recharge allocation rule results, "Recharge aquifers in order of $v_i\lambda_i$, unless limited by recharge or storage capacity or future availability." Water is recharged first to basins with the highest discounted net economic value and fraction of recoverable water. As the amount of water available to recharge increases, the marginal value of storing the water will decrease. As high valued and large fractional recovery aquifers fill, lower valued and less desirable aquifers remain for use.

3. Optimizing Duration of Aquifer Operations

Optimizing the time to fill or empty aquifer storage is another objective for managing a portfolio of aquifers. Duration can be described as the recharge period or the duration of steady aquifer withdrawal. Objectives and constraints to optimize these durations are specific to recharge and extraction problems as follows. For blending, regulatory, or operational reasons, we assume steady withdrawal or recharge rates.

3A. Maximize duration of withdrawals

For the withdrawal problem, the objective is to find the steady withdrawals that maximize the duration over which a specified steady target demand rate can be met. This might represent the duration of a drought over which the portfolio can supply a given delivery rate. The following non-linear mathematical program results:

Maximize WD_{max} (15)

Subject to:

- No negative withdrawals

  $W_i \geq 0, \forall$ i (16)

- Withdrawals limited by maximum pumping rates

  $W_i \leq p_{max\ i}, \forall$ i (17)

- Withdrawals must meet or exceed a target demand rate

  $$\sum_i W_i \geq W_T \quad (18)$$

- Definitions of withdrawal duration for each aquifer

$$WD_i = S_{oi} / W_i,\ \forall\ i \qquad (19)$$

- Definition of maximum feasible duration for program

$$WD_{max} \le WD_i,\ \forall\ i \qquad (20)$$

Where WD_{max} is the duration to maintain the withdrawal target [time] and WD_i is the withdrawal duration from aquifer i [time]. W_i (withdrawal rate), $p_{max\ i}$ (maximum pumping rate), S_{oi} (initial storage), and W_T (target demand rate) are as defined previously.

Since each aquifer withdrawal contributes to the target withdrawal rate, the duration for the program of withdrawals will be constrained by the aquifer with the smallest duration of withdrawal. This nonlinear program balances withdrawals across all aquifers.

When the non-negativity and pumping capacity constraints do not bind, the program can be solved analytically for a general balancing rule. Under this condition, the set of optimal, duration-maximizing steady withdrawals (W_i^*) will exhaust all aquifers at the same time, so $WD_{max} = WD_i = S_{oi} / W_i^*,\ \forall$ i. Algebraically, this gives the following withdrawal rule:

$$\frac{S_{oi}}{W_i^*} = \frac{\sum_i S_{oi}}{W_T} \quad \text{or} \quad W_i^* = \frac{S_{oi} \cdot W_T}{\sum_i S_{oi}} \quad \text{or} \quad \frac{W_i^*}{W_T} = \frac{S_{oi}}{\sum_i S_{oi}} \qquad (21)$$

This rule says that the duration-maximizing withdrawal from aquifer i should be proportional to the fraction of the total system water initially stored in aquifer i.

Program 3A can be transformed into a linear program by taking the inverse of the decision variable ($1/W_i$) as derived in Appendix A.

Minimize Duration of Recharge

For the recharge problem, the objective is to find the recharges that minimize the duration to recharge a specific quantity of water (3B) or fill all aquifers (3C). The former objective should apply when the amount of surface water is small compared to unfilled aquifer storage. The later objective applies when available surface water is significantly more than aquifer storage capacity. Formulations for each problem are:

3B. Minimize duration to recharge a small volume of water

Minimize RD_{min} (22)

Subject to:

- No negative recharges

  $Q_i \geq 0, \forall$ I (23)

- Storage capacity available in each aquifer

  $Q_i \leq K_i, \forall$ I (24)

- Total recharges must equal surface water supply

  $$Q_S = \sum_i Q_i \quad (25)$$

- No negative recharge durations

  $RD_i \geq 0, \forall$ I (26)

- Recharges limited by maximum recharge rates

  $Q_i / D_i \leq r_{maxi}, \ \forall$ i (27)

- Definition of program duration

  $RD_{min} \geq RD_i, \forall$ i (28)

- A fraction β of total recharges must be available for future withdrawal with at least reliability α

$$\sum_i \left[(\bar{a}_i - Z_\alpha \cdot \sigma_i - \beta) \cdot Q_i \right] \geq 0 \tag{29}$$

Where RD_i is the recharge duration for aquifer i [time], RD_{min} is the overall recharge duration for the program [time], and S_{oi} (initial storage), Q_i (recharge volume), K_i (unfilled capacity), Q_S (available surface water), $r_{max\ I}$ (maximum recharge rate), $\bar{a}_i$ (future expected availability), σ_i (standard deviation of future availability), α (target reliability), and β (target fraction of recovery) are as defined previously.

When the non-negativity, storage capacity, and future availability constraints do not bind, the program can be solved analytically for a general balancing rule. Under this condition, the set of optimal, duration-minimizing steady recharges (Q_i^*) will be related to the largest allowable recharge rate of each aquifer, so $RD_{max} = RD_i = Q_i^*/r_{max\ i}$, ∀ i. Algebraically, this gives the following withdrawal rule:

$$\frac{Q_i^*}{r_{\max i}} = \frac{Q_S}{\sum_i r_{\max i}} \quad \text{or} \quad Q_i^* = \frac{r_{\max i} \cdot Q_S}{\sum_i r_{\max i}} \quad \text{or} \quad \frac{Q_i^*}{Q_S} = \frac{r_{\max i}}{\sum_i r_{\max i}} \tag{30}$$

This rule says that the duration-minimizing recharge to aquifer i should be proportional to the fraction of the total recharge rate capacity aquifer i can handle.

3C. Minimize duration to fill all aquifers

Minimize FD_{min} (31)

Subject to:

- No negative recharges

  $R_i \geq 0$, ∀ i (32)

- Definition of fill durations for each aquifer

$$FD_i = \frac{K_i}{\lambda_i \cdot R_i}, \ \forall \text{ i} \tag{33}$$

- Total recharges less than steady surface water available in each period

$$\sum_i R_i \le R_S \tag{34}$$

- Recharges limited by maximum recharge rates

$$R_i \le r_{maxi}, \ \forall \text{ i} \tag{35}$$

- Definition of program fill duration

$$FD_{min} \ge FD_i, \ \forall \text{ i} \tag{36}$$

Where FD_i is the recharge duration to fill aquifer i [time], FD_{min} is the program duration to completely fill all aquifers [time], R_i is the rate at which to recharge water into aquifer i in each time period [volume time^{-1}], and R_S is the steady surface water available for recharge in each period [volume time^{-1}]. λ_i (fractional recovery), K_i,(unfilled capacity), and $r_{max\ i}$ (maximum recharge rate) are as defined previously.

The fill duration for each aquifer is a function of the fractional recovery (equation 33). This statement assumes that losses occur as recharges are made. This assumption should hold when unfilled capacity is large, recharge rates are small, and long durations are expected.

If aquifer recharge constraints do not bind, the steady recharge allocation (R_i^*) will equalize durations across aquifers, so that all aquifers fill at the same time $FD_{min} = FD_i = (K_i)/(\lambda_i \cdot R_i^*)\ \forall$ i. This gives the following recharge allocation rule:

$$\frac{K_i}{\lambda_i \cdot R_i^*} = \frac{\sum_i \left(\frac{K_i}{\lambda_i}\right)}{R_S} \quad \text{or} \quad R_i^* = \frac{R_S \cdot K_i / \lambda_i}{\sum_i \left(\frac{K_i}{\lambda_i}\right)} \quad \text{or} \quad \frac{R_i^*}{R_S} = \frac{\frac{K_i}{\lambda_i}}{\sum_i \left(\frac{K_i}{\lambda_i}\right)} \tag{37}$$

To minimize the duration of recharge across aquifers, more water should be recharged into aquifers with larger unfilled capacities or smaller fractional recoveries, i.e., aquifers that are most empty and with the least efficient recharge. Lower fractional recoveries will lengthen the duration to fill all aquifers. The term λ_i drops out when expected fractional recoveries of recharging water are equal across all aquifers or the fill period is short.

4. Maximizing Accessibility

When filling groundwater storage capacity in wet years, an agency often is unsure about the future demands for instantaneous withdraw or the duration over which aquifer withdrawals must persist. The agency may want to optimize flexibility to take water from a portfolio of groundwater storages at high withdrawal rates or sustain withdrawals over a long duration. A formulation is presented to simultaneously address the recharge and withdrawal portions of the problem. Two analytical solutions are derived and the tradeoff between them is presented.

4.1 Model formulation

The multiple objective problem is to maximize the total instantaneous withdrawal rate plus a weighted duration of withdrawals:

$$\text{Maximize } W_R + d \cdot D_{max} \tag{38}$$

Subject to:

- No negative recharges

$$Q_i \geq 0, \ \forall \text{ i} \tag{39}$$

- No negative withdrawals

$$W_i \geq 0, \ \forall \text{ i} \tag{40}$$

- Recharges limited by maximum recharge rates

$$Q_i \leq r_{\max i} \cdot t, \ \forall \text{ i} \tag{41}$$

- Withdrawals limited by maximum pumping capacities

$$W_i \leq p_{\max\, i}, \ \forall \text{ i} \tag{42}$$

- Definition of aquifer duration

$$D_i = \frac{S_{oi} + \lambda_i \cdot Q_i}{W_i}, \ \forall i \tag{43}$$

- Recharges limited by remaining storage capacity

$$Q_i \leq K_i \tag{44}$$

- Recharges limited by surface water supply

$$Q_S \geq \sum_i Q_i \tag{45}$$

- Definition of program withdrawal duration

$$D_{\max} \leq D_i, \ \forall i \tag{46}$$

- Definition of total expected withdrawal rate

$$W_R = \sum_i a_i \cdot W_i \tag{47}$$

- Total expected withdrawal rate must meet target demand rate with target reliability α

$$P_r[W_R \geq W_T] \geq \alpha \tag{48}$$

Where W_R is the total expected rate of water withdrawal from all aquifers [volume time^{-1}], D_i is the duration that withdrawal W_i can be sustained in aquifer i [time], D_{max} is the maximum duration of time the operation program can be sustained [time], d is a user-selected factor weighting the relative importance of the two components of the objective [volume time^{-2}], and S_{oi} (initial storage), λ_i (fractional recovery), a_i (future availability), Q_i (recharge volume), W_i (withdrawal rate), W_T (target demand rate), K_i (unfilled storage capacity), Q_S (available surface water), $r_{max\ i}$ (maximum recharge rate), $p_{max\ i}$ (maximum pumping rate), α (target reliability), and t (period) are as defined previously.

Future availability is stated as function of the withdrawal (47) rather than recharge (11). Equations (47) and (48) can be combined and reduced to an equivalent deterministic constraint as shown previously:

$$\sum_i \left[\left(\bar{a}_i - Z_\alpha \cdot \sigma_i \right) \cdot W_i \right] \geq W_T \tag{49}$$

Where $\bar{a}_i$ (future expected availability), Z_α (standard normal deviate for target reliability α), and σ_i (standard deviation of availability) are as defined in equation (13). In the model, both recharge volumes (Q_i) and withdrawal rates (W_i) are decision variables, with the recharge period fixed to time t, but the withdrawal period (D_{max}) determined by the program and assumed to start after recharge is completed. If the user selects a small value for d ($d \ll 1$), the program will identify recharge and extraction operations that maximize instantaneous withdrawal capacity giving slight preference to operations that lengthen the duration the withdrawal can be sustained. Conversely, for large values of d, the program will identify operations that maximize the duration over which water can be withdrawn giving slight preference to operations that expand the rate at which water can be withdrawn.

4.2 Analytical solutions

Analytical solutions are derived for cases where the coefficient d is either large or small.

4.2.1 Maximize instantaneous withdrawal rate (W_R)

When the non-negativity, limited recharge rate, aquifer storage capacity, and future availability constraints do not bind and the value of d is small, an analytical solution can be derived to maximize the expected instantaneous withdrawal rate (W_R). First, increase aquifer withdrawals to their maximum pumping rates:

$$W_i^* = p_{\max i}, \ \forall \text{ i} \tag{50}$$

Second, the program will configure recharges so maximum expected withdrawal rates can be sustained for as long as possible, equalizing withdrawal durations for all aquifers:

$$D_{\max} = D_i = \frac{S_{oi} + \lambda_i \cdot Q_i^*}{W_i^*} = \frac{\sum_i \left(S_{oi} + \lambda_i \cdot Q_i^*\right)}{\sum_i W_i^*} \qquad (51)$$

Where the asterisk superscript (*) represents the optimal values of the decision variables. Substituting (50) into (51) and rearranging gives:

$$\frac{p_{i\max}}{\sum_i p_{i\max}} = \frac{S_{oi} + \lambda_i \cdot Q_i^*}{\sum_i \left(S_{oi} + \lambda_i \cdot Q_i^*\right)}, \forall i \qquad (52)$$

Equation (52) is a set of i independent equations that can be solved simultaneously for Q_i^*. The form of the solution suggests the decision maker should recharge water in aquifer i so that the water recoverable for extraction from aquifer i compared to the total water recoverable from extraction (from all aquifers) is proportional to the pumping capacity of aquifer i compared to the total pumping capacity of all aquifers. To maximize the total withdrawal rate and duration that rate can be sustained, the rule suggests that decision makers should recharge more water into aquifers with higher pumping capacities, lower initial storages, and lower fractional recoveries (i.e., higher losses).

When fractional recoveries and expected availabilities are identical across all aquifers, equation (52) reduces to the Metropolitan Water District of Southern California's (MWD) rule for allocating water into multiple aquifers. The MWD rule equalizes the ratio of total water storage to pumping capacity in each aquifer, $(S_{oi} + Q_i)/P_{max\,i} = (S_{oj} + Q_j)/P_{max\,j}$ (Tim Blair, personal communication, 2002).

4.2.2 Maximize duration of withdrawal (D_{max})

A second analytical solution maximizes the duration of withdrawal (D_{max}) for the case where the value of d is large and the non-negativity, limited extraction rate, and future availability constraints do not bind. To maximize duration of steady withdrawals, all basins should empty at the same time, so

$$D_{\max} = D_i = \frac{S_{oi} + \lambda_i \cdot Q_i^*}{W_i^*} = \frac{\sum_i \left(S_{oi} + \lambda_i \cdot Q_i^*\right)}{\sum_i W_i^*} \quad (53)$$

Recharge and extraction decisions are taken sequentially. First, without knowing the duration-maximizing withdrawal rates for each aquifer (W_i^*), we observe that the duration will be largest when the sum of the withdrawals is smallest. Therefore, the program will minimize withdrawals subject to constraint (49) on withdrawal target (W_T). This allows the substitution:

$$D_{\max} = \frac{\sum_i S_{oi} + \sum_i \lambda_i \cdot Q_i^*}{W_T} \quad (54)$$

Duration will also be maximized when the term $\Sigma\ \lambda_i \cdot Q_i^*$ is maximized. This term represents recharged water recoverable for extraction. To maximize this amount, recharge should be into aquifers with the highest fractional recoveries. The duration-maximizing recharge rule is “recharge aquifers in order of λ_i, unless limited by recharge or storage capacities.”

With Q_i^* known, solve (53) for the duration-maximizing, steady withdrawal rates for each aquifer. Because the initial storages and additional storage generated from recharge are now determined, equation (53) takes the same form as the solution for problem 3A (equation 21). Equation (54) be rearranged and solved for the duration-maximizing, steady withdrawal rate:

$$W_i^* = \frac{W_T \cdot \left(S_{oi} + \lambda_i \cdot Q_i^*\right)}{\sum_i \left(S_{oi} + \lambda_i \cdot Q_i^*\right)} \quad (55)$$

This rule says that the duration-maximizing withdrawal from aquifer i should be proportional to the fraction of total recoverable water stored in aquifer i. The rules for withdrawal (equation 55) and recharge (paragraph above) represent sequential solutions for recharge followed by withdrawal decisions. These solutions take forms similar to the solution for the withdrawal decision alone (model 3A).

4.3 Tradeoff between solutions

The two analytical solutions frame a tradeoff between withdrawal *duration* and *rate*. The tradeoff can be stated as follows: higher water losses incurred by an operation plan to maximize the instantaneous withdrawal rate will diminish the duration over which those operations can be sustained. The tradeoff will be most apparent when one group of aquifers has high pumping capacities but low fractional recoveries while a second group of aquifers has low pumping capacities but high fractional recoveries. Solving the mathematical program for a range of values for d can also illustrate the tradeoff.

5. Example Applications

Two numerical examples demonstrate the six aquifer balancing rules derived above. The first example demonstrates solutions for each of the five single-objective rules. The second example demonstrates solutions for the multi-objective accessibility formulation. Examples were and solved within Excel worksheets.

5.1 Example #1 (Single objective programs)

A portfolio of 4 aquifers was selected with different physical, institutional, and economic characteristics (Table 1). Aquifer A is located farthest from the demand area and Aquifer D was located closest to the demand area. Aquifer D has the largest storage capacity and second-to-smallest fractional recovery (i.e., large losses associated with recharge). Aquifer storage capacities range from 200,000 to 800,000 acre-feet (af). Maximum recharge capacities and pumping rates vary between 3,000 and 5,000 af/month and 6,000 and 15,000 af/month, respectively. The fraction of recharge available for extraction

Table 1. Aquifer characteristics

Aquifer	Physical				Institutional		Economic		
	Maximum storage capacity	Maximum pumping rate	Maximum recharge rate	Fractional recovery	Mean expected availability	Standard deviation of expected availability	Recharge cost	Use cost (to pump, treat, convey)	Use value
	K_i	$p_{max\ i}$	$r_{max\ i}$	λ_i	$\bar{a}_i$	σ_i	rc_i	c_i	u_i
	[af]	[af/mo]	[af/mo]	[fraction]	[fraction]	[fraction]	[$/af]	[$/af]	[$/af]
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
A	400,000	7,000	4,000	0.96	0.9	0.020	30	120	1000
B	200,000	6,000	3,000	0.93	0.9	0.015	10	110	1000
C	600,000	8,000	4,000	0.90	0.9	0.002	35	80	1000
D	800,000	15,000	5,000	0.92	0.9	0.001	45	60	1000

ranges between 90 and 96%. All aquifers have the same expected availability to withdraw water (90%), however, availability is more certain for aquifers C and D and than for aquifers A and B. Extraction and conveyance costs vary between aquifers. All aquifers are assumed to have similar water qualities. Economic use values are identical since extracted water supplies a single demand location.

Table 2 summarizes additional parameter values related to the recharge and extraction. All aquifers are assumed to start full for the withdrawal problems ($S_{oi} = K_i$) and empty for the recharge problems ($S_{oi} = 0$). Withdrawals should meet the target demand rate of 20,000 af/month. 6,000 af of surface water is available for recharge. I assume an interest rate of 5% over a planning horizon of five years to calculate a discount factor of b = 0.784 = $(1+ 0.05)^{-5}$. Furthermore, 85% of recharged water was desired to be available for withdrawal with 90% reliability.

Table 2. Additional parameter values for extraction and recharge problems in Example #1

Parameter (1)	**Symbol** (2)	**Value** (3)
(a) Extraction problem		
Target water demand rate [af/mon]	W_T	20,000
Initial storages	S_{oi}	K_i
(b) Recharge problems		
Water available to recharge [af]	Q_S	6,000
Recharge period [mon]	t	1
Initial storages [af]	S_{oi}	0
Steady water available to recharge [af/mon]	R_S	6,000
Discount factor [fraction]	b	0.784
Water available for withdrawal [fraction of amount recharged]	β	0.85
Target reliability [fraction]	α	0.9
Standard normal deviate for reliability α [fraction]	Z_α	1.653

The parameter values described in Tables 1 and 2 fall within ranges of values common for aquifers and drought water management in the San Joaquin Valley and Southern California. However, the aquifers do not represent specific aquifers.

Using the above example data, the aquifer withdrawal and recharge programs were solved for the 2 economic and 3 duration-based objectives (Table 3). Numerical solutions for each objective matched the analytical solutions derived previously. To maximize the economic value of withdrawals (column 2), aquifers D and C were pumped. Aquifer D had the highest extractive value of water (Table 4, column 11); therefore, the full pumping capacity of 15,000 af/month was utilized. Remaining demand was met by withdrawing water from the aquifer with second highest extractive value. When the objective is to maximize the duration of meeting target withdrawals (Table 3, column 3), water was withdrawn from each aquifer proportional to the amount of initial storage in each aquifer. Withdrawals were sustained for 100 periods from each aquifer (months).

Table 3. Aquifer balancing solutions for economic and duration-based objectives

	Withdrawal Problem		**Recharge Problem**		
Aquifer	**Maximize economic value** (Model 2A) W_i^* [af/mon]	**Maximize duration of withdrawal** (Model3A) W_i^* [af/mon]	**Maximize expected value of recharge** (Model 2B) Q_i^* [af]	**Minimize duration to recharge small volume** (Model 3B) Q_i^* [af]	**Minimize duration to fill all aquifers** (Model 3C) R_i^* [af/mon]
(1)	(2)	(3)	(4)	(5)	(6)
A	0	4,000	0	1,500	1,153
B	0	2,000	3,000	1,125	595
C	5,000	6,000	0	1,500	1,845
D	15,000	8,000	3,000	1,875	2,407

Table 4. Economic calculations

Aquifer	**Net extractive use value** $u_i - c_i$ [$/af]	**Discounted net economic value** $v_i = b*(u_i - c_i) - rc_i$ [$/af]	**Discounted net economic value of recoverable water** $\lambda_i \cdot v_i$ [$/af]
(1)	(2)	(3)	(4)
A	880	660	633
B	890	687	639
C	920	686	617
D	940	692	636

For the recharge problems, water is recharged into both aquifers B and D when the objective is to maximize the expected value of recharge. Aquifer B has the highest discounted net economic value of recoverable water (Table 4, column 4); so, Aquifer B's full recharge capacity of 3,000 af/month is utilized. Remaining surface water is recharged to aquifer D, the aquifer with the second highest discounted net economic value. To minimize the duration to recharge 6,000 af, recharge each aquifer in proportion to each aquifer's recharge capacity (Table 3, column 5). Recharges are sustained for 0.38 months in all aquifers to fully recharge the 6,000 af. To minimize the duration to fill all aquifers with a supply of 6,000 af/month, recharge each aquifer (Table 3, column 6) in proportion to the water needed to fill each aquifer (space available / fractional recovery). Aquifer D takes the most water (800,000 af / 0.92 = 870,000 af) while aquifer B takes the least (200,000 af / 0.93 = 215,000 af). Given the large unfilled capacities in all aquifers, 351 months were required to fill all aquifers. The numerical solutions discussed above verify the analytical solutions.

The availability constraint for Program 2B was changed to demonstrate how increasing desired availability could influence the net expected value and configuration of optimal recharges. The parameter β (required fraction of recharged water to be available for extraction) was increased sequentially from 0.85 to 0.898. For each value, the program was re-solved. The shadow value associated with the constraint described by equation (13) was recorded (Figure 2). As the value of β increased from 0.887 to 0.892, the program decreased recharge to Aquifer B and increased recharge to Aquifer D. When β = 0.892, the recharge capacity of Aquifer D was reached. For larger β, the program increased recharge to Aquifer C and continued to decrease recharge to Aquifer B. Aquifers D and C have more narrowly bound expected reliabilities than Aquifer B but lower discounted, net economic values of recoverable water. Figure 2 shows the step-wise costs incurred to recharge water into Aquifers C and D and improve future availability. In this example, it was not feasible to configure recharges to make more than 90% of the amount recharged was available for extraction.

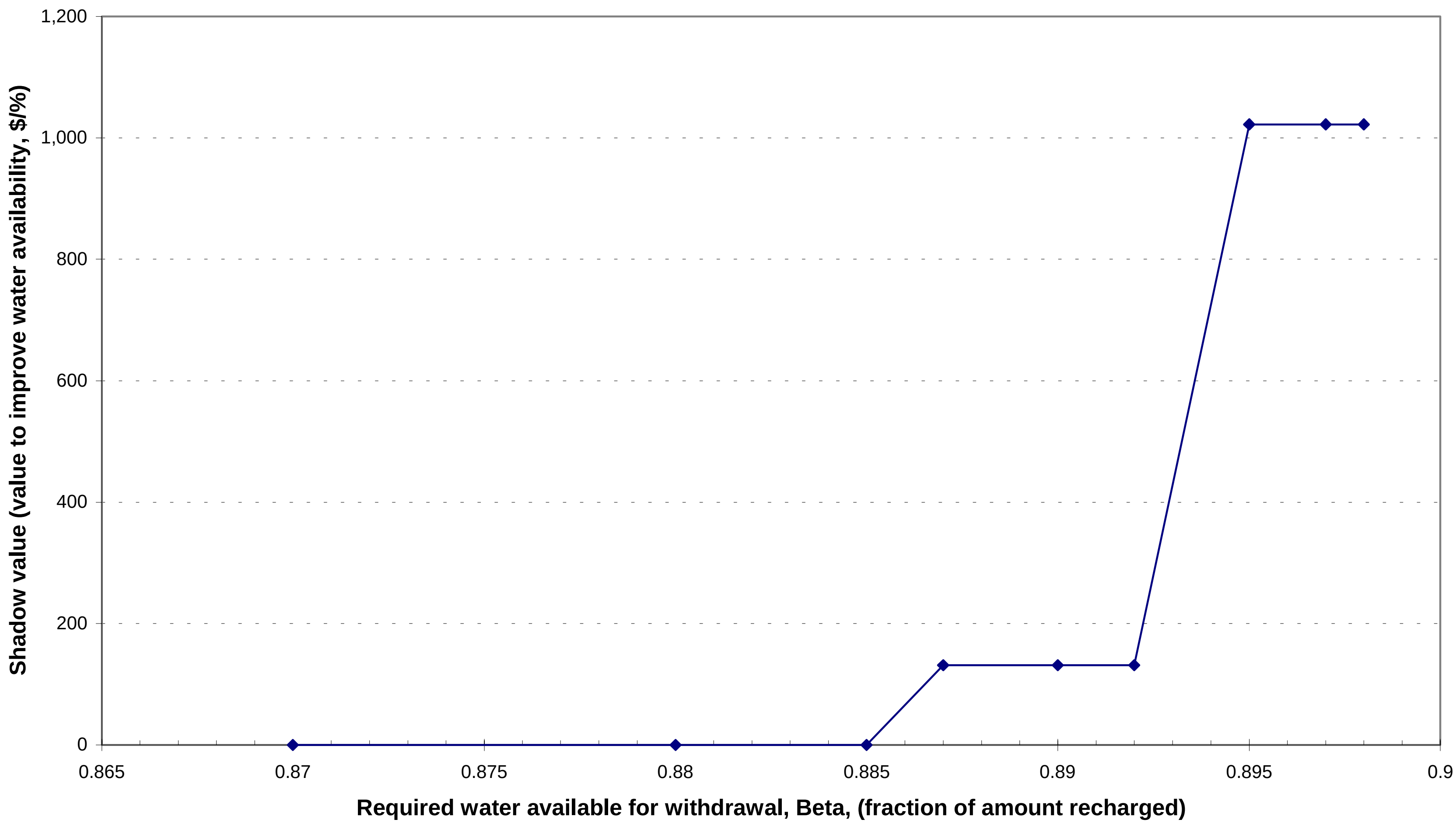


Figure 2. Shadow prices to improve availability of recharges for future withdrawal (Program 2B)

5.2 Example #2 (Multi-objective accessibility)

The portfolio of 4 aquifers from Example #1 was also used to demonstrate solutions for the tradeoff objective. However, problem parameter values were changed (Table 5). A surface water supply of 200,000 af was available for recharge over a time period sufficient to recharge any single aquifer. Initial storages were 80,000 af for aquifer A and 100,000 af for aquifers B, C, and D. For each aquifer, expected availability ($\bar{a}_i$) was increased to 1 and standard deviation of availability (σ_i) was reduced to zero. Changes were made so recharge and withdrawal decisions could be examined simultaneously and the availability and recharge constraints were not binding initially.

Table 5. Parameter values for Example #2

Parameter (1)	Symbol (2)	Value (3)
Water available to recharge [af]	Q_S	200,000
Recharge period [mon]	t	200
Initial Storage, Aquifer A	S_{o1}	80,000
Initial Storages, Aquifer B,C,D	$S_{o2,3,4}$	100,000
Expected availability	$\bar{a}_i$	1.0
Standard deviation of availability	σ_i	0
Target water demand rate [af/mon]	W_T	20,000
Target reliability [fraction]	α	0.5
Standard normal deviate for reliability α [fraction]	z_α	0.676
Tradeoff coefficient [af/mon^2]	d	10^{-3} to 10^4

The tradeoff objective formulation was solved 20 times for discrete values of the tradeoff coefficient (*d*) ranging from 10^{-3} to 10^5 withdrawal rate per duration squared [af/mon^2]. For all values of d, solutions converged to one of two solutions. A tipping point between the two solutions was seen at d = 1.23 10^3 af/mon^2 (Figure 3). The corner solution that maximized the instantaneous withdrawal rate (Table 6, columns 2 and 3) coincided with the analytical solution derived for that case (equations 50 and 52). The

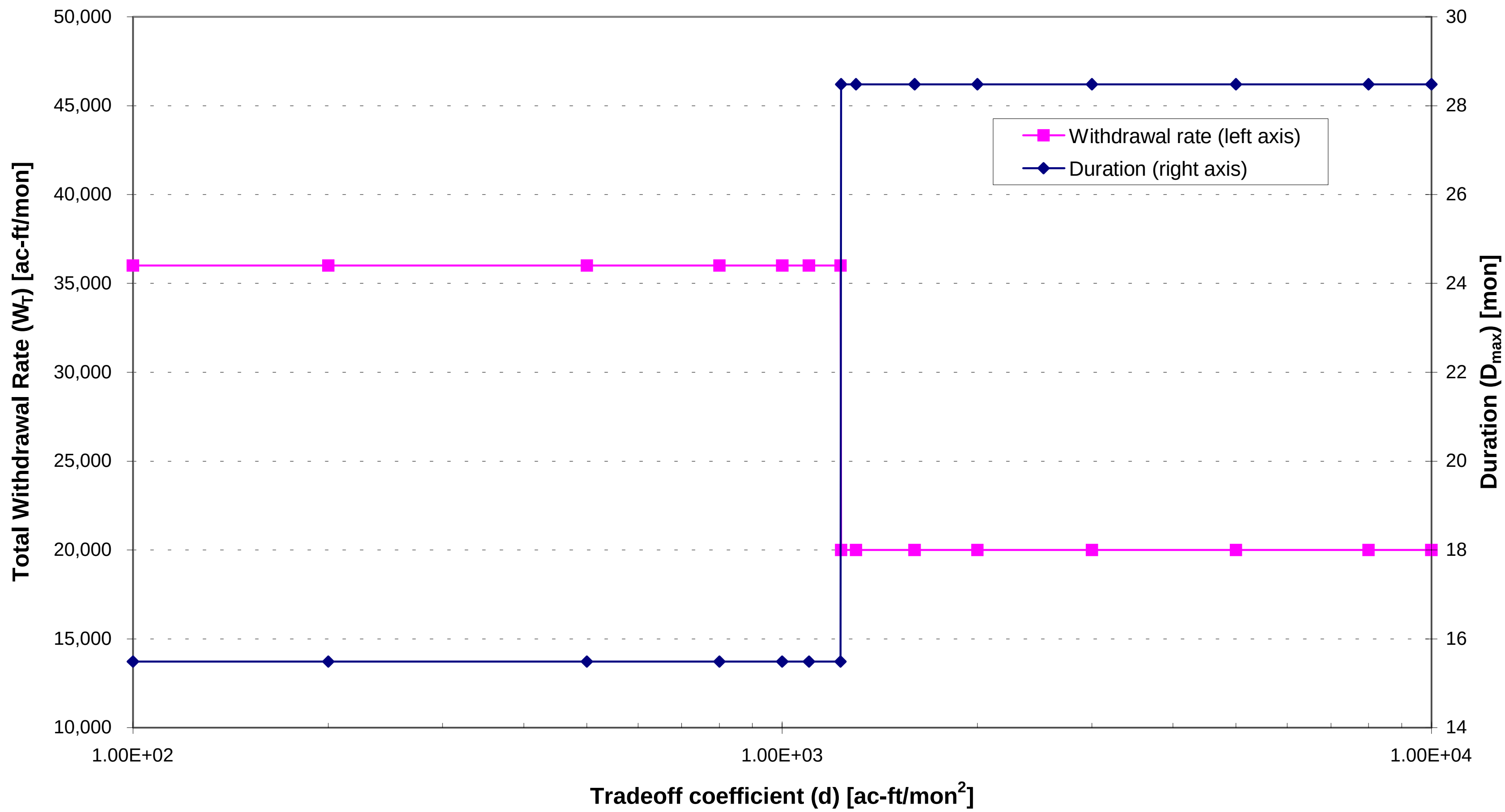


Figure 3. Tipping point between solutions that maximize total withdrawal rate and duration of withdrawal

corner solution that maximized duration of withdrawals (Table 6, columns 4 and 5) also agreed with the analytical solution (equation 55). However, recharges were made to both Aquifers A and B because the maximum pumping rate for Aquifer A was constraining.

Table 6. Two numerical solutions to accessibility program in Example #2

Aquifer	Corner solution that maximizes withdrawal rate ($d < 1.23*10^3$)		Corner solution that maximizes duration ($d > 1.23*10^3$)	
	W_i^* [af/mon]	Q_i^* [af]	W_i^* [af/mon]	Q_i^* [af]
(1)	(2)	(3)	(4)	(5)
A	7,000	29,603	7,000	124,381
B	6,000	-	5,979	75,619
C	8,000	26,564	3,510	-
D	15,000	143,833	3,510	-

When the tradeoff coefficient was less than $1.23*10^3$, water was withdrawn from each aquifer at maximum pumping rates (Table 6, column 2). Water was recharged to aquifers A, C, and D in proportion to the pumping rates and initial storage (Table 6, column 3). No water was recharged to Aquifer B because it had the smallest pumping rate. Based on its initial storage, Aquifer B could sustain its maximum pumping rate longer than the other aquifers (16.7 periods). Recharges to Aquifers A, C, and D allowed the program to sustain the maximum rate of withdrawal of 36,000 af/month for 15.5 months.

When the tradeoff coefficient was larger than $1.23*10^3$, recharge was limited to aquifers A and B (Table 6, column 4), aquifers with the largest fractional recoveries. The program would have directed all recharge to Aquifer A, but the maximum pumping rate for Aquifer A constrained withdrawal to 7,000 af/month. Therefore, water was also recharged to Aquifer B, the aquifer with the second largest fractional recovery. Withdrawals were then made in proportion to the water stored in each aquifer (Table 6, column 4). Aquifer A had the largest withdrawal rate because it had the most stored water. Aquifer B had the second largest withdrawal rate. Aquifers C and D had smaller and identical withdrawal rates because both aquifers started with 100,000 af of recoverable storage and no recharge was made to either aquifer. Total withdrawals met the target rate of 20,000 af/month. From the recharge and withdrawal decisions, the program could sustain withdrawals for 28.5 months.

The program shows a discontinuous tipping-point between the corner solutions because the objective function is linear with respect to both the withdrawal rate and the duration. Plotting the objective function value against the tradeoff coefficient for several different solutions (including the two corner solutions presented in Table 6) again identifies the tipping point (Figure 4). For values of *d* much larger or smaller than $1.23\ 10^3$, a smooth tradeoff exists between the corner and intermediary solutions. However, for values of *d* near the tipping point, both corner solutions become superior to the intermediary solutions.

The intermediate solutions in Figure 4 represent duration maximizing solutions achieved when expected availabilities, $\bar{a}_i$, were further constrained (to 0.9 and 0.7) and the program was resolved. Lowering the expected availability raises the expected withdrawal rate required to meet the target demand. Raising the withdrawal rate lowers the duration. Thus, varying expected availabilities in the chance constraint illustrates a tradeoff between the two corner solutions (Figure 5). Square markers indicate the corner solutions presented in Table 6 (when the mean expected aquifer availability, $\bar{a}$, was 1.0 for all aquifers). Other points in Figure 6 show durations and total withdrawal rates when the program was solved for different expected availabilities ($\bar{a} = 0.9$, 0.8. 0.7, 0.6, and 0.57). Each point represents a duration-maximizing solution ($d > 1.23*10^3$) where each aquifer was assigned the same mean expected availability ($\bar{a}_1 = \bar{a}_2 = \bar{a}_3 = \bar{a}_4$).

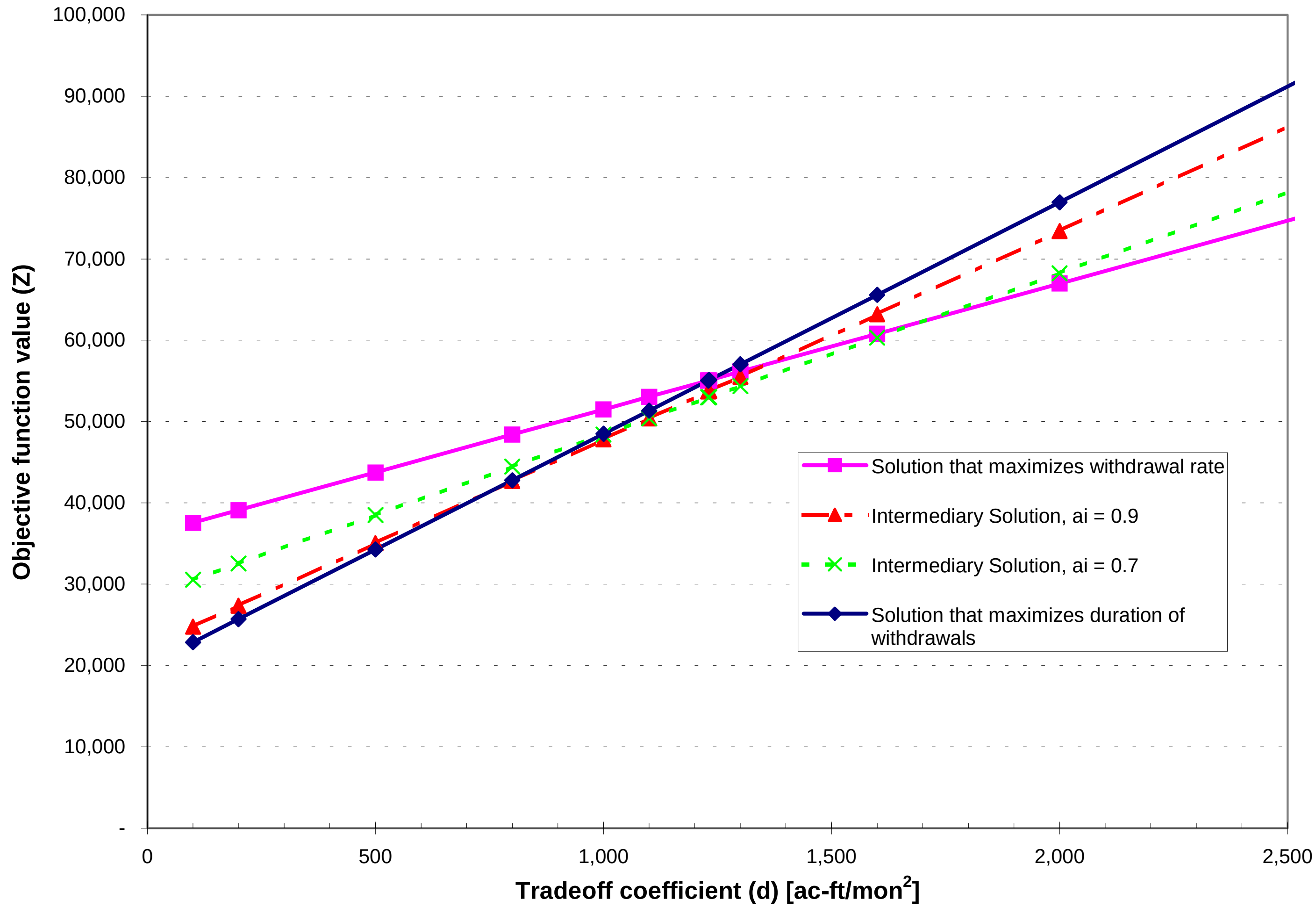


Figure 4. Accessibility program objective function value versus tradeoff coefficient for 4 solutions

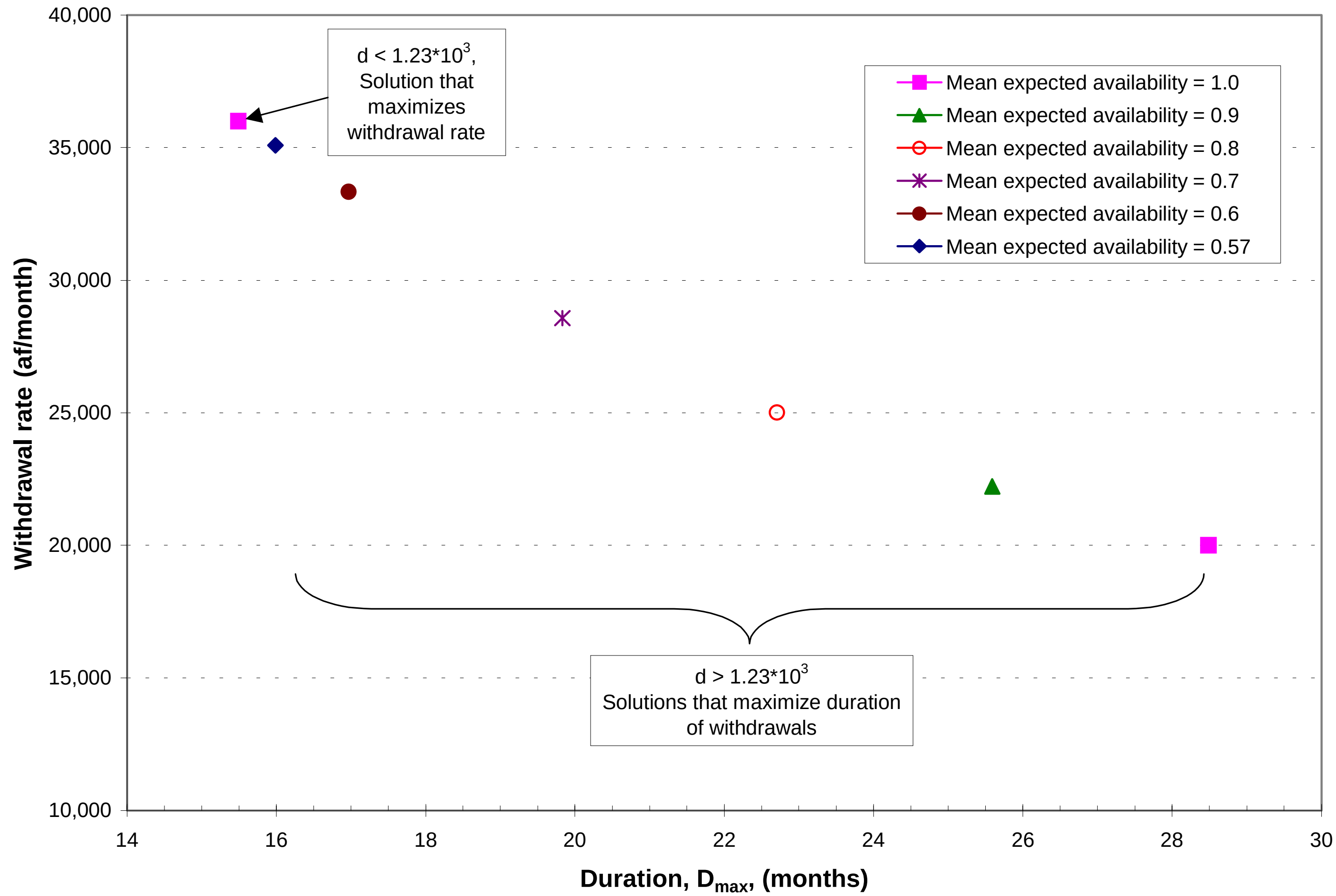


Figure 5. Tradeoff between duration and instantaneous withdrawal rate by varying aquifer availabilities

As expected availabilities were decreased, larger withdrawal rates were required to meet the desired target withdrawal rate (Appendix B). Less water was recharged to aquifer A and B and more water recharged to aquifer D. Water was only recharged to aquifer C when expected availability was less than 0.6. Total pumping rate increases with the largest increases in withdrawals from Aquifer D. Aquifer A sustained a maximum pumping rate of 7,000 af/month and Aquifer B reached a maximum pumping rate of 6,000 af/month for availabilities less than 1.0. As expected availabilities were decreased, optimal recharges and withdrawals approached the solution for maximizing the withdrawal rate. Note that no feasible solutions existed for $\bar{a} < 0.55$ because the program could not increase the total withdrawal rate above a maximum pumping capacity of 36,000 af/month. Recharges to and withdrawals from Aquifer D were made to increase the expected reliability of withdrawn water. Because Aquifer D had a lower fractional recovery than Aquifers A and B, withdraws from the aquifer could be sustained for a shorter time. This is represented by the negatively sloping tradeoff curve in Figure 5. Despite the tradeoff, recharges to and maximum pumping rates from Aquifer A were sustained over all availabilities, identifying aquifers with large pumping capacities and high fractional recoveries as the most suitable for taking water from when an aquifer manager seeks to maximize accessibility to stored water (as either duration or rate of withdrawal).

6. Conclusions

Six operating rules were derived to suggest optimal aquifer management decisions for three types of objectives based on lumped aquifer characteristics. The rules are:

Economic Objectives

1. To maximize the economic value of withdrawing water, withdraw water from aquifers with the largest differences between use value and extraction costs.

2. To maximize the future expected value of water, recharge water to the aquifers with the largest discounted net, economic value of recoverable water.

Duration Objectives

3. To maximize the duration of withdrawals, withdrawal in proportion to initial storage.
4. To minimize the duration to recharge a small quantity of surface water, recharge in proportion to maximum recharge rate.
5. To minimize the duration to fill all aquifers, recharge in proportion to unfilled storage capacity weighted by expected water losses.

Accessibility Objective

6. To maximize flexibility to meet both large future withdrawal rates or durations of withdrawals, preferentially recharge water to aquifers with both high maximum pumping capacities and large fractional recoveries (small storage losses).

Operating rules were demonstrated and verified for two simple, numerical examples. When uncertainties concerning future availability of banked water for later withdrawal and target reliability were incorporated into problem statements, results highlight cost and performance tradeoffs and changes to recommended allocations. The operating rules and examples represent situations where constraints were non-binding. However, the formulations can readily be extended and solved numerically to include constraints for more complex systems. Examples might include multiple, unconnected aquifers operated in conjunction with a surface water reservoir, multiple reservoirs, and uncertain surface water volumes available for recharge.

Appendix A. Derivation of Linear Programs to Optimize Duration of Steady Water Supply

Two of the duration-based programs presented in section 2 can be transformed into equivalent linear programs. The transformation requires taking the inverse of duration (1/duration). The resulting linear programs are presented for maximizing the duration of withdrawal and minimizing the duration to fill all aquifers.

A1. Maximize duration of withdrawal (linear program)

The objective of program 2A was to maximize the duration of withdrawal. When duration is transformed and inverted (1/duration), the objective must likewise be inverted. Therefore we need to minimize the inverse duration to maximize the duration (smaller inverse-durations correspond to larger durations). The decision variables are still the withdrawal rates (W_i). The 5 constraints presented for model 2A are included, however, the definitions of inverse duration for the program (ID_{min} [$time^{-1}$]) and for each aquifer (ID_i [$time^{-1}$]) must also be inverted. The resulting linear program is:

Minimize ID_{min} (A1)

Subject to:

- No negative withdrawals

$$W_i \geq 0, \ \forall \text{ i} \quad \text{(A2)}$$

- Withdrawals limited by maximum pumping rates

$$W_i \leq p_{max\ i}, \ \forall \text{ i} \quad \text{(A3)}$$

- Withdrawals must meet or exceed a target demand rate

$$\sum_i W_i \geq W_T \quad \text{(A4)}$$

- Definitions of inverse duration for each aquifer withdrawal

$$ID_i = W_i/S_{oi}, \ \forall \text{ i} \quad \text{(A5)}$$

- Definition of inverse duration for program

$ID_{min} \geq ID_i, \ \forall$ i (A6)

A3. Minimize duration to fill all aquifers (linear program)

The objective of program 2C was to minimize the duration to fill all aquifers. When duration is transformed and inverted (1/duration), the objective must likewise be inverted. Therefore we need to maximize the inverse duration to minimize the duration (larger inverse-durations correspond to smaller durations). The decision variables are still the recharge rates (R_i). The 5 constraints presented for model 2C are included, however, the definitions of inverse duration for the program (ID_{max} [time^{-1}]) and for each aquifer (ID_i [time^{-1}]) must also be inverted. The resulting linear program is:

Max ID_{max} (A7)

Subject to:

- No negative recharges

$R_i \geq 0, \ \forall$ i (A8)

- Definition of inverse durations to fill each aquifer

$$ID_i = \frac{\lambda_i \cdot R_i}{(K_i - S_{oi})}, \forall i \quad \text{(A9)}$$

- Total recharges less than steady surface water available in each period

$$\sum_i R_i \leq R_S \quad \text{(A10)}$$

- Recharges limited by maximum recharge rates

$R_i \leq r_{maxi}, \ \forall$ i (A11)

- Definition of inverse program duration

$ID_{max} \leq ID_i, \ \forall$ i (A12)

Appendix B. Solutions to Accessibility Program with binding constraints on future availability

Aquifer	$(d < 1.23*10^{6})$		$(d > 1.23*10^{6})$											
	Corner solution that maximizes withdrawal rate		**Intermediatry solution**		**Intermediatry solution**		**Intermediatry solution**		**Intermediatry solution**		**Intermediatry solution**		**Corner solution that maximizes duration**	
	$(a_i = 1)$		$(a_i = 0.57)$		$(a_i = 0.6)$		$(a_i = 0.7)$		$(a_i = 0.8)$		$(a_i = 0.9)$		$(a_i = 1)$	
	W_i^*	Q_i^*	W_i^*	Q_i^*	W_i^*	Q_i^*	W_i^*	Q_i^*	W_i^*	Q_i^*	W_i^*	Q_i^*	W_i^*	Q_i^*
	[af/mon]	[af]	[af/mon]	[af]	[af/mon]	[af]	[af/mon]	[af]	[af/mon]	[af]	[af/mon]	[af]	[af/mon]	[af]
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
A	7,000	29,603	7,000	33,241	7,000	40,399	7,000	61,282	7,000	82,240	7,000	103,274	7,000	124,381
B	6,000	-	6,000	-	6,000	1,951	6,000	20,428	6,000	38,971	6,000	57,582	5,979	75,619
C	8,000	26,564	7,088	14,793	5,893	-	5,042	-	4,404	-	3,908	0	3,510	-
D	15,000	143,833	15,000	151,967	14,440	157,650	10,529	118,291	7,596	78,789	5,315	39,145	3,510	-

Appendix C. Definition of Terms

λ_i	Expected fraction of recharge that will be recoverable for extraction, unitless
a_i	Random variable representing future availability to extract water from aquifer i, fraction
α	Reliability that the water should be available, fraction
$\bar{a}_i$	Mean expected availability of aquifer i, fraction
b	Discount factor, unitless
β	Required fraction of recharged water to be available in the future, unitless
c_i	Sum of unit costs to extract, pump, treat, convey, and cover institutional, legal, and transactional expenses to gain access to aquifer i, \$ volume^{-1}
d	Tradeoff objective coefficient, volume time^{-2}
D_i	Duration of withdraw from aquifer i, time
D_{max}	Overall duration of withdrawal program, time
$e_{i\ xy\ ls}$	Lower (i.e., groundwater surface) elevation of unsaturated area available to store water at location x,y in aquifer i, length
$e_{i\ xy\ us}$	Upper (i.e., ground surface) elevation of the unsaturated area available to store water at location x,y in aquifer i, length
i	Aquifer index, 1..n
FD_i	Duration to fill aquifer i, time
FD_{min}	Overall fill duration for recharge program, time
K_i	Unfilled, remaining storage capacity of aquifer i, volume
p_{maxi}	Maximum extraction pumping capacity for aquifer i,
Q_i	Decision on the amount to recharge into aquifer i, volume
Q_i^*	Optimal amount to recharge to aquifer i, volume
rc_i	Unit cost to recharge aquifer i, volume time^{-1}
R_i	Steady recharge rate into aquifer i, volume time^{-1}
RD_i	Recharge duration for aquifer i, time
RD_{min}	Overall duration for recharge program, time
R_i^*	Optimal recharge rate into aquifer i, volume time^{-1}
r_{maxi}	Maximum recharge capacity for aquifer i, volume time^{-1}
R_S	Steady surface water available for recharge in each period, volume time^{-1}
$\rho_{i,xyz}$	Porosity of aquifer i at location x,y,z, fraction

S_{oi} Initial storage in aquifer i available for extraction, volume

σ_i Standard deviation of expected availability, unitless

t Predetermined duration of withdrawal / recharge, time

u_i Unit use value of water extracted from aquifer i, \$ volume^{-1}

v_i Discounted, net economic value of storing water in aquifer i, \$ volume^{-1}

W_i Decision on withdrawal rate from aquifer i, volume time^{-1}

W_i^* Optimal withdrawal rate from aquifer i, volume time^{-1}

W_R Total expected rate of water withdrawal from all aquifers, volume time^{-1}

WD_i Withdraw duration from aquifer i, time

WD_{max} Overall withdraw duration for program, time

W_T Total target demand rate, volume time^{-1}

x_i Latitudinal domain of recharge area i, length

y_i Longitudinal domain of the recharge area for aquifer i, length

Z_α Standard normal deviate for probability α, unitless

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