# Up or Out?—Economic-Engineering Theory of Flood Levee Height and Setback

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**Abstract:** Levee setback (location) and height are important issues in flood levee system design and modification. This paper derives an economic-engineering theory of the optimal trade-off of levee setback for height both for original and redesigned flood levees, demonstrating the interconnection of levee setback, height, costs and risks, and economically optimal design. These analyses assume stationary flood hydrology and static ratios among damageable property value, unit construction cost, and land price. The economic trade-off of levee setback for height depends on economic cost and benefit and hydraulic parameters, and only indirectly on flood frequency and economic damage parameters. The redesign rules derived in this paper indicate conditions where existing levees should be raised or moved in response to changes in conditions. Numerical examples illustrate the results. This paper demonstrates several ideas and theory for economic flood levee system planning and policy rather than providing guidelines for direct design practice.

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#### Introduction

Levee systems have long been built for flood protection. Early flood levees usually were designed with scant quantitative analysis, relying primarily on a few observed flood stages and human judgment. The achievements in experimental and theoretical hydraulics since the 18th century (Rouse and Ince 1957), rational estimation of storm discharge in the mid 19th century (Biswas 1970), and early economic-engineering analysis (Humphreys 1861) made possible the modern designs of flood levees. In recent decades, many studies have addressed economic aspects of flood levee design, usually with benefit-cost analysis and optimization techniques (Davis et al. 1972; Wurbs 1983; Goldman 1997; Olsen et al. 1998; Jaffe and Sanders 2001; Lund 2002; USACE 2002, 2006).

Levee construction or rehabilitation often follows flood disaster. There is sometimes considerable controversy over whether flood channel capacity should be obtained more from levee height or from greater levee setbacks. This issue has received increased public attention due to concern for riparian recreation and environmental uses of unprotected floodplain land. Some studies have examined the optimal levee height and setback under static or dynamic hydrologic and economic conditions (Tung and Mays 1981; Zhu et al. 2007) but they only provided the optimal levee design without more general theoretical insight into the trade-off of levee setback for height or flood levee redesign.

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Economic design of a levee system for flood protection involves balancing costs of levee height, losses of land value sacrificed by floodway expansion (setback), and flood damages from inadequate channel capacity. The most common economic objective for floodplain management is minimization of expected annual damages and flood management expenses (Lund 2002; Olsen et al. 2000; USACE 2006). Under static conditions, annual peak flood flows usually fit an independent and identical probability distribution, and economic factors, such as the value of damageable property, construction cost, and floodplain land values, are constant. Optimality conditions can be applied to examine flood levee designs under such static conditions. The approach developed here applies to relatively common cases where the expected net cost function is convex with levee setback and levee height.

Flood levee construction involves large irreversible investments, so rigorous examination is desirable before implementation decisions are made. This paper examines the optimal levee height and setback decisions from a theoretical and analytical perspective. The approach taken here is to examine designs based on overall economic efficiency, considering both flood control action costs and flood damages. Flood warning systems are assumed to allow us to neglect, for now, losses of life in these evaluations. Hypothetical river and levee types are explored to provide conceptual insights, rather than practical levee designs.

# Optimal Static Trade-Off of New Levee Setback for Height

A static model is formulated to minimize the annualized net economic cost, which equals the expected flood damages plus the net annual flood management cost. The net annual flood management cost includes annualized levee construction cost and annualized economic benefit (negative cost) of levee-protected land and unleveed land by the river. The land value usually decreases with levee setback, since a smaller land area is protected. This simple model allows preliminary quantitative examination of the optimal

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economic trade-off between setback and height in designing a new levee. The objective function is

$$\operatorname{Min} \operatorname{EC}(X_{s}, X_{h}) = \int_{Q(X_{s}, X_{h})}^{+\infty} f(q) D(q) dq + C(X_{h}) - B(X_{s}, X_{h})$$
(1)

where EC(.)=expected annual net cost, as a function of levee setback  $X_s$  and height  $X_h$ ; the first term on the right hand side =expected annual flood damage, in which f(.) represents the flood frequency distribution; q=flood flow larger than the levee overtopping flow Q(.), and D(.) denotes the damageable property value (loss when the levee fails) as a function of flood flow q; C(.) refers to the annualized cost to build a levee with height  $X_h$ ; and B(.)=annual value of floodplain lands (both leveed and unleveed floodplain).

Eq. (1) essentially assumes the levee does not fail with flows less than overtopping capacity. In reality, levee failure and flood damages can also be caused by geotechnical failure mechanisms such as slumping, erosion, or water seepage undermining levee foundations. For example, the catastrophic flood damages in New Orleans during Hurricane Katrina were brought by both levee overtopping and structural failure at approximately 50 locations in the city's hurricane protection systems (ASCE-ERP 2007). Levee overtopping may not always lead to levee failure, but the risk is very high since overtopping erodes the levee back as water spills over. For simplicity, in this paper levee breach and flood damage are assumed to occur only when river stage overtops the levee. Geotechnical failures are neglected. This assumption is reflected in the first term of Eq. (1) that only takes into account damages caused by flood flows exceeding the overtopping capacity. This integral term can be approximated with discrete flood frequencies of a number of flood flow intervals and the flood damages associated with those flows.

A constructed flood levee is always expected to function properly without the need for major modification for a period of decades, depending on local economic development, magnitude of floods, and other river characteristics. This is due to the large irreversible investments involved in levee construction and modification. To make the analysis valid for a period of time of interest to floodplain planners, two additional assumptions are made implicitly in Eq. (1). First, the flood hydrology is assumed to be stationary. Second, ratios among the economic terms involved in Eq. (1), damageable property value, unit construction cost, and land price, are assumed to be constant over the planning horizon. These assumptions can often be justified in the real world. For example, the second assumption holds true when damageable property value, construction cost, and land price grow at sufficiently close rates within the planning period. Statistical stationarity is a widely adopted assumption in floodplain planning practice although it is increasingly being challenged by the presence of climate change (Zhu et al. 2007).

The land value function  $B(\cdot)$  depends on levee setback, and also levee height because the bottom width of the levee cross section commonly increases with levee height.  $B(\cdot)$  includes the annual value of land protected from floods by the levee (typically dominated by urban and agricultural use values) and the value of unprotected land on the stream side of the levee (which can have substantial recreational and environmental value).

The first-order condition for minimizing the expected total cost of flood control requires the first partial derivatives of  $EC(X_s, X_h)$  with respect to  $X_s$  and  $X_h$  equal zero

$$\frac{\partial \text{EC}}{\partial X_h} = -f(Q)D(Q)\frac{\partial Q}{\partial X_h} + \frac{\partial (C-B)}{\partial X_h} = 0$$
(2)

$$\frac{\partial \text{EC}}{\partial X_s} = -f(Q)D(Q)\frac{\partial Q}{\partial X_s} - \frac{\partial B}{\partial X_s} = 0$$
(3)

In the above equations,  $\partial Q/\partial X_h > 0$ ,  $\partial Q/\partial X_s > 0$ ,  $\partial C/\partial X_h > 0$ ,  $\partial B/\partial X_h < 0$ , and  $\partial B/\partial X_s < 0$ . Assuming uniform flow in the river channel, the overtopping capacity Q is determined solely by river cross-section geometry, that is,  $X_s$  and  $X_h$  in this case, besides energy slope and channel roughness (Sturm 2001), which are not supposed to be affected by levee modification. Eqs. (2) and (3) lead to

$$\frac{\partial Q}{\partial X_h} \left/ \frac{\partial Q}{\partial X_s} = -\frac{\partial (C-B)}{\partial X_h} \right/ \frac{\partial B}{\partial X_s}$$
(4)

Eq. (4) holds for the optimal levee height  $X_h^*$  and setback  $X_s^*$ . The optimal levee height  $X_h^*$  and setback  $X_s^*$  can be found by numerically solving combined Eqs. (2) and (3) and verifying that a global minimum is attained. In Eq. (4), the left hand side is (in economic parlance) the marginal substitution rate (MSR) of levee setback for height for the optimal overtopping flow  $Q(X_s^*, X_h^*)$ . The right hand side is the ratio of marginal construction and land value costs of levee height  $X_h$  to marginal land value loss due to setback  $X_s$ , that is, the MSR of levee setback for height for the optimal net cost. Note that *B* represents land benefit, a negative cost, therefore a minus sign appears on the right hand side. Eq. (4) implies that optimal economic efficiency occurs where the floodway capacity MSR of levee setback for height equals the cost MSR of levee setback for height.

In Eq. (4), neither flood hydrology nor value of potential flood damages affects the optimal trade-off between levee height and setback, although these factors do affect the optimal flood channel capacity. Thus, changes in flood hydrology (from climate change, human activity impacts on watershed, or structure measures in upstream) and/or damageable property value (due to floodplain zoning, flood warning, or urbanization) do not affect the economic optimal ratio of substitution between levee height and setback, despite changes in optimal flood channel capacity. Rearranging Eq. (4) leads to

$$-\frac{\partial B}{\partial X_s} \left/ \frac{\partial Q}{\partial X_s} = \frac{\partial (C-B)}{\partial X_h} \right/ \frac{\partial Q}{\partial X_h}$$
(5)

If we define  $-\partial B/\partial X_s$  and  $\partial (C-B)/\partial X_h$  as cost efficiencies of setback and height, and  $\partial Q/\partial X_s$  and  $\partial Q/\partial X_h$  as hydraulic efficiencies of setback and height, Eq. (5) means the ratio of cost efficiency to hydraulic efficiency of levee setback should equal that same ratio for levee height. There is an optimal trade-off of levee height for setback and it is affected by the relative economic values of protected versus unprotected lands, construction costs, and hydraulic characteristics, but is not directly affected by flood frequency or damage potential.

Where decreased setback increases damage potential because the larger protected area contains more damageable property,  $D \rightarrow D(q, X_s, X_h)$ . Assuming damageable property value is in proportion to the widths of levee-protected floodplain land, we have

$$D(q, X_s, X_h) = (w - X_s - aX_h)\overline{D}(q)$$
(6)

in which w=total widths of floodplain land behind a levee; a=levee side slope; and  $\overline{D}(q)$ =average damageable property

Setback	Height					
	$\mathbf{X_{h0}}\!<\!\mathbf{X_{h0}}^{c}$	$\mathbf{X_{h0}^{c}} \! < \! \mathbf{X_{h0}} \! < \! \mathbf{X_{h0}^{*}}$	ELSE			
$X_{s0} < X_s^*$	Move levee outward to $(X_s^*, X_h^*)$	Raise current levee to $X^*$	Do nothing if $X_{i,0} > \mathbf{X}^*$			
$X_s^* < X_{s0} < X_s^{c1}$	Move levee inward to $(X_s^*, X_h^*)$					
$\mathbf{X}_{s}^{c1} \! < \! \mathbf{X}_{s0} \! < \! \mathbf{X}_{s}^{c2}$	Move levee inward and	Do nothing if $X_{h0} > \mathbf{X}_{h0}^{c2}$				
$\mathbf{X} \rightarrow \mathbf{X}^{c2}$	M	and racize to $(\mathbf{V}^* \ \mathbf{V}^*)$				

value per unit width of floodplain, for flood flow q. Therefore, Eq. (5) now becomes Eq. (7) and then Eq. (8)

$$-\frac{\partial(B+X_s\bar{D})}{\partial X_s} \left/ \frac{\partial Q}{\partial X_s} = \frac{\partial[C-(B+aX_h\bar{D})]}{\partial X_h} \left/ \frac{\partial Q}{\partial X_h} \right.$$
(7)

where  $\overline{D} = \int_{Q(X_s, X_h)}^{+\infty} f(q) \overline{D}(q) dq$ , the expected flood damage for a unit width of floodplain behind the levee

$$\frac{-\left(\frac{\partial B}{\partial X_{s}}+\bar{D}\right)}{\frac{\partial Q}{\partial X_{s}}} = (X_{s}-aX_{h})f(Q(X_{s},X_{h}))\bar{D}(Q(X_{s},X_{h})) + \frac{\left[\frac{\partial(C-B)}{\partial X_{h}}-a\bar{D}\right]}{\frac{\partial Q}{\partial X_{h}}}$$
(8)

Eq. (8) reflects the situation that decreased setback increases damage potential or vice versa. Note that  $\overline{D}$ =function of overtopping flow Q. For this case, greater damage potential encourages: (1) greater flood control capacity; and (2) greater setbacks, and the probability of flooding and setback's effects on damage potential enters into the optimal trade-off of levee setback for levee height [Eq. (8)].

# Levee Redesign under Static Conditions

This section derives theoretical optimal rules for when levee heights and setbacks should be changed. The previous analysis assumes that no levee currently exists for flood protection. But commonly, levees already exist and one of three decisions must be made: retain the existing levee, raise the existing levee to an optimal height at the current setback, or build a new levee with an optimal height and setback. (We still neglect geotechnical failures.) Let  $G_1(X_{s0}, X_{h0})$ ,  $G_2(X_{s0}, X_{h0})$ , and  $G_3(X_s^*, X_h^*)$  denote the annualized expected costs of flood control under the three decisions, respectively, as given below

$$G_1(X_{s0}, X_{h0}) = \text{EC}(X_{s0}, X_{h0}) - C(X_{h0})$$
(9)

$$G_2(X_{s0}, X_{h0}) = \text{EC}(X_{s0}, X_{h0}^*(X_{s0})) - C(X_{h0})$$
(10)

$$G_3(X_s^*, X_h^*) = \text{EC}(X_s^*, X_h^*)$$
(11)

where EC(.)=function of annualized expected cost of flood control; and C(.)=levee construction cost, as previously defined. Here, levee construction cost is assumed to be linear with levee volume (quadratic with levee height for trapezoidal levees). Values of functions Eqs. (9) and (10) depend on the setback and height of an existing levee,  $X_{s0}$  and  $X_{h0}$ , and Eq. (11) is a function of the optimal levee height  $X_h^*$  and setback  $X_s^*$ . In Eq. (9), the optimal levee height  $X_{h0}^*$  at the current setback  $X_{s0}$  (without a pre-existing levee) can be found from Eq. (2).

These particular formulations neglect any fixed costs to prepare and permit for any levee-raising construction; such fixed costs could be added without loss of generality, and would tend to increase the range of conditions under which no change in height is optimal. These three cost functions allow us to develop rules for raising and relocating levees under fairly general circumstances. The decision criterion here is to minimize expected net economic cost. These rules are summarized after their derivation in Table 1.

If the existing levee height is less than the optimal height at the current location  $(X_{h0} < X_{h0}^*)$ , the optimal levee height at the existing setback leads to the relationship  $EC(X_{s0}, X_{h0})$  $-EC(X_{s0}, X_{h0}^*(X_{s0})) > 0$ , and consequently,  $G_1(X_{s0}, X_{h0})$  $-G_2(X_{s0}, X_{h0}) > 0$ . Therefore, if an existing levee height is less than the optimal height at the current setback, the levee should be raised to the optimal height unless it should be relocated. If the current levee height exceeds the optimal height for the current setback  $(X_{h0} > X_{h0}^*)$ , the levee should not be further raised at the existing setback, given the convex nature of the flood control cost function.

However, it might be economical to relocate the levee. If  $G_1(X_{s0}, X_{h0}) - G_3(X_s^*, X_h^*) < 0$ , the existing levee setback  $X_{s0}$  and height  $X_{h0}$  are preferable to relocating the levee; otherwise, moving the levee to the optimal setback  $X_s^*$  and height  $X_h^*$  is preferable to the existing levee.

Where  $G_2(X_{s0}, X_{h0}) - G_3(X_s^*, X_h^*) = 0$ , the annualized overall cost to raise the existing levee equals that of a new relocated levee. For a fixed levee location  $X_{s0}$ , the levee height where moving and raising the levee are economically equivalent is a critical case. This critical levee height  $X_{h0}^{c1}$  for the current levee setback is a partitioning point for levee redesign options. If the existing levee height is less than this critical levee height (for this location), then the levee should be moved to the optimal setback  $X_s^*$  and height  $X_h^*$ . If the existing levee height exceeds this critical height, but is less than the optimal height, then it should be raised to  $X_{k0}^*$  at the same location.

However, a critical height less than the optimal height for the current setback might not exist when the current setback is too large. For very large setbacks, loss of land value can be so high that no existing levee at optimal height or greater can compensate the loss of land value and it is always optimal to move the levee closer to the stream.

At the optimal setback  $X_s^*$  the critical height  $X_{h0}^{c1}=0$ . The more

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a setback deviates from the optimal setback, the more its corresponding critical height approaches its optimal height until they cross. Beyond a *first critical setback*  $X_s^{c1}$ , where the levee's optimal height coincides with its critical height,  $X_{h0}^* = X_{h0}^{c1}$ , it is never optimal to raise an existing levee. Beyond such a setback, it is either optimal to retain the levee or move the levee to reduce the setback. Such a critical setback  $X_s^{c1}$  is a constant for given hydrologic, hydraulic, and economic conditions, independent of current levee location or height. Beyond this setback, an existing setback would be "too large" owing to the loss of economic value from land on the stream side of the levee. For such a case, the height of an existing levee must exceed the optimal height for current location to avoid being relocated. The decision criterion becomes: If current leve height exceeds a *second critical height*,  $X_{h0}^{c2}$ , where  $G_1(X_{s0}, X_{h0}^{c2}) - G_3(X_s^*, X_h^*) = 0$ , no action should be taken to the existing levee; otherwise, the levee should be relocated to the optimal setback and height,  $X^*$  and  $X_s^*$ .

mal setback and height,  $X_s^*$  and  $X_h^*$ . A second critical setback  $X_s^{c2}$  also exists, beyond the first critical setback, beyond which no pre-existing levee height is preferable to relocating the levee; no  $X_{h0}^{c2}$  exists where  $G_1(X_s^{c2}, X_{h0}^{c2})$   $-G_3(X_s^*, X_h^*)=0$ . When a levee setback exceeds this critical setback, the levee always should be relocated, as no additional height of existing levee can compensate for the land value gained from moving the levee toward the stream. The second critical setback occurs when  $G_1(X_{s0}, X_{h0}^{c2}) - G_3(X_s^*, X_h^*) > 0$ , even for the largest  $X_{h0}^{c2}$ . [Theoretically, the value of the second critical setback should roughly equal the annualized expected economic value of the optimal design  $EC(X_s^*, X_h^*)$  divided by the product of annualized unit value of protected land  $(\partial B / \partial X_s)$  and levee length].

Table 1 summarizes these theoretical decision-making rules for redesign of a levee. For generality, we assume there are two critical setbacks, the small one  $X_s^{c1}$  and the larger one  $X_s^{c2}$ . Numerical examples of these cases are explored later in the paper.

# **Illustrative Examples**

The theory developed is now applied to a common special case.

# Optimal Trade–Off of Flood Wall Setback for Height for Broad Flat Floodplains

Consider a new levee for an ideal prismatic wide shallow rectangular channel (Fig. 1) with width  $X_s$  and levee height  $X_h$  on one side of the channel (the other side being high and fixed), with the overtopping flow given by Manning's equation (Sturm 2001)

$$Q = -\frac{k}{n} S^{1/2} X_s X_h^{5/3}$$
(12)

where n=Manning roughness; S=energy slope, equaling longitudinal channel bed slope for uniform flow cases; and k=1.49 for English units and 1 for metric unit. Where the opposite bank becomes susceptible to flood damage with greater flow stage, these equations become more complex. In Eq. (12), let  $\alpha = k/n \cdot S^{1/2}$  and the partial derivatives with respect to  $X_s$  and  $X_h$  are

$$\frac{\partial Q}{\partial X_h} = \frac{5}{3} \alpha X_s X_h^{2/3} \tag{13}$$

$$\frac{\partial Q}{\partial X_s} = \alpha X_h^{5/3} \tag{14}$$

If levee width does not change with levee height (for example, a flood wall), land value depends on setback only, leading to linear cost functions  $B(X_s) = -\gamma LX_s$  and  $C(X_h) = \beta_0 Lw_0 X_h$ , where  $\gamma$ =unit land protection value per year (the difference between the annual value of protected versus unprotected floodplain land, per unit area);  $\beta_0$ =annualized unit construction cost; *L*=length of levee reach; and  $w_0$ =width of levee. Embedding these cost functions and Eqs. (13) and (14) into Eq. (5) leads to

$$\frac{X_s^*}{X_b^*} = \frac{3\beta_0 w_0}{5\gamma} \tag{15}$$

Eq. (15) shows that the optimal ratio of levee setback to height is a constant for an idealized wide shallow rectangular channel and rectangular levee cross section under static conditions. This ideal ratio of levee setback to height is the ratio of marginal hydraulic effectiveness of levee setback to height in producing channel capacity (3/5 in this case) multiplied by the ratio of the relative economic cost of levee setback and height ( $\beta_0 w_0/\gamma$ ). The optimal ratio of  $X_s$  to  $X_h$  is unaffected by hydrology or the damage potential of protected properties (which do affect optimal channel capacity). In addition, Manning roughness and channel longitudinal slope also do not influence the optimal ratio of  $X_s$  to  $X_h$  (but would affect optimal channel capacity).

The trade-off between levee height and setback for levee planning and design is thus primarily driven by economic factors, related to the unit costs of levee construction and the increased economic value of protected land compared with unprotected floodplain land. As the society comes to value recreational and environmental benefits from unprotected floodplains,  $\gamma$  decreases and the optimal setback increases. However, as urban land values increase,  $\gamma$  increases and the optimal setback decreases. Urbanization, increasing damage potential and protected land values simultaneously, will increase optimal channel capacity and preference for levee height over setback (as a ratio). However, growth of optimal channel capacity sometimes drives increases in absolute values of optimal setback.

### Levee Redesign for Broad Flat Floodplains

The following presents an analysis of the levee redesign decision for the wide shallow rectangular channel and rectangular crosssection levee shown in Fig. 1. Assuming the floods have mean  $\mu$ and standard deviation  $\sigma$ , and fit a lognormal distribution, the mean and standard deviation of this lognormal distribution should be

$$M = \frac{1}{2} \ln \left( \frac{\mu^2}{\left(\frac{\sigma}{\mu}\right)^2 + 1} \right) \tag{16}$$

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Table 2. Hydrologic, Hydraulic, and Economic Parameters, and Resultant Optimal and Critical Values

Parameters							Results				
$\mu$ (ft <sup>3</sup> /s)	$\sigma$ (ft <sup>3</sup> /s)	L (ft)	$\beta_0$ (\$/ft <sup>3</sup> year)	γ (\$/acre year)	$w_0$ (ft)	n	D (\$10 <sup>9</sup> )	$X^*_{s}$ (ft)	$X_h^*$ (ft)	$X_s^{c1}$ (ft)	$\begin{array}{c} X_s^{c2} \\ (\mathrm{ft}) \end{array}$
20,000	20,000	52,800	0.05	10,000	150	0.035	1	519	27	1,440	1,520

$$S = \sqrt{\ln\left(\left(\frac{\sigma}{\mu}\right)^2 + 1\right)} \tag{17}$$

For lognormal distribution, the probability distribution function is

$$f(q) = \frac{1}{qS\sqrt{2\pi}}e^{-[\ln(q) - M]^2/(2S^2)}$$
(18)

Embedding Eqs. (13) and (18), and the land value function  $B(X_s)$  and construction cost function  $C(X_h)$  into Eq. (2), we obtain

$$\frac{e^{-[\ln(\alpha X_s X_h^{5/3}) - M]^2/2S^2}}{X_h S \sqrt{2\pi}} - \frac{3\beta_0 L w_0}{5D} = 0$$
(19)

The parameters in Eq. (19) are the same as previously defined. Combining Eqs. (19) and (15), the optimal levee setback  $X_s^*$  and height  $X_h^*$  can be found for given hydrologic, hydraulic and economic conditions. With an existing levee setback  $X_{s0}$ , the optimal levee height  $X_{h0}^*$  for this existing setback can be solved from Eq. (19). With  $X_{h0}^*$ ,  $X_s^*$ , and  $X_h^*$ , for the existing setback  $X_{s0}$ , the "critical" levee height  $X_{h0}^c$  can be solved by solving  $G_1(.)=G_2(.)$ . Table 2 gives the parameters to calculate optimal and critical levee heights and critical levee setbacks. Results are shown in Fig. 2.

Fig. 2 presents the optimal levee redesign decisions with critical setbacks and the optimal and critical levee heights for various setbacks for a river reach characterized by parameters in Table 2. The solid square represents the optimal levee height and setback for building a new levee for the river reach. The optimal levee height decreases as existing setback increases. The critical height is convex with levee setback with a zero minimum at the optimal setback of the river reach, about 520 ft. If an existing levee is located at its optimal setback it is not economical to move the levee and the levee should be raised to its optimal height. If levee setback deviates from the optimal setback, whether more or less, an existing levee greater than the critical height is needed to avoid



Fig. 2. Redesign rules with optimal and critical levee heights and setbacks

relocating the levee. The first and second critical levee setbacks are also illustrated, showing where distant levee setbacks require more than optimal existing heights to justify their retention (between the first and second critical setbacks) and the setback beyond which even the tallest existing levee cannot justify the land value gained from moving the levee toward the stream (beyond the second critical setback). The optimal levee redesign decisions are indicated in each "zone" defined by the optimal levee height curve (upper curve before critical setback) and critical levee height curves.

# Conclusions

Discussions of levee design and levee setbacks have increased in recent years, particularly in the context of rising public values for recreation and environmental uses of floodplains and rising economic land values and damage potential in leveed floodplains. This paper develops some theoretical aspects of simplified flood levee planning problems, with particular attention paid to levee setbacks and heights, under static hydrologic and economic conditions. While the results developed here are largely theoretical, they illustrate several points of common importance for floodplain planning and policy.

There is an optimal trade-off of levee height for setback, which can be examined analytically. Under static conditions, the optimal trade-off of levee setback for height is determined by levee construction cost and floodplain land values in protected versus nonprotected parts of the floodplain, besides hydraulic factors. Flood frequency and flood damage potential do not influence the optimal substitution between levee setback and height rate (except for some effect when setback affects damage potential), though they affect the optimal overall channel capacity.

Levee redesign decision rules are developed for static conditions where a levee already exists. The *critical heights* for a given setback and *critical setbacks* for making decisions to raise or relocate levees are derived and analyzed for a river reach. The critical setbacks are constant for a given river reach, depending only on hydrologic, hydraulic, and economic factors. Levee redesign decision rules are derived based on these optimal and critical values of a given problem.

As conditions of flood frequency, channel hydraulics, and floodplain economics change, it is likely that levees should be modified in height and/or setback. This too can be examined analytically. As the economic value of unprotected floodplains rises for recreational and environmental purposes, the relative difference between protected and unprotected land values decreases and optimal setbacks should increase (simultaneously decreasing optimal levee heights). However, in growing metropolitan areas, increases in urban land values can outweigh rising values for unprotected floodplains, often tending to decrease the optimal floodway width.

While the results here are unlikely to provide directly useful quantification for actual levee problems, such theoretical formulations and results should help provide qualitative and conceptual insights for levee and floodplain management problems.

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# References

- ASCE Hurricane Katrina External Review Panel (ASCE-ERP). (2007). *The New Orleans hurricane protection system: What went wrong and why*, ASCE, Reston, Va.
- Biswas, A. K. (1970). *History of hydrology*, North-Holland, Amsterdam, The Netherlands and American Elsevier, New York.
- Davis, D. R., Kisiel, C. C., and Duckstein, L. (1972). "Bayesian decision theory applied to design in hydrology." *Water Resour. Res.*, 8(1), 33–41.
- Goldman, D. (1997). "Estimating expected annual damage for levee retrofits." J. Water Resour. Plann. Manage., 123(2), 89–94.
- Humphreys, A. A. (1861). "Report upon the physics and hydraulics of the Mississippi river; upon the protection of the alluvial region against overflow; and upon the deepening of the mouths." *Rep. Submitted to the Bureau of Topographical Engineers, War Department, 1861*, Prepared by Captain A. A. Humphreys and Lieut. H. L. Abbot, J. B. Lippincott & Co., Philadelphia.

- Jaffe, D. A., and Sanders, B. F. (2001). "Engineered levee breaches for flood mitigation." J. Hydraul. Eng., 127(6), 471–479.
- Lund, J. R. (2002). "Floodplain planning with risk-based optimization." J. Water Resour. Plann. Manage., 128(3), 202–207.
- Olsen, J. R., Beling, P. A., and Lambert, J. H. (2000). "Dynamic models for floodplain management." J. Water Resour. Plann. Manage., 126(3), 167–175.
- Olsen, J. R., Beling, P. A., Lambert, J. H., and Haimes, Y. Y. (1998). "Input-output economic evaluation of system of levees." *J. Water Resour. Plann. Manage.*, 124(5), 237–245.
- Rouse, H., and Ince, S. (1957). *History of hydraulics*, Iowa Institute of Hydraulic Research, State University of Iowa Press, Iowa City, Iowa.
- Sturm, T. W. (2001). Open channel hydraulics, McGraw-Hill, New York.
- Tung, Y. Y., and Mays, L. W. (1981). "Optimal risk-based design of flood levee systems." *Water Resour. Res.*, 17(4), 843–852.
- U.S. Army Corps of Engineers (USACE). (2002). Sacramento and San Joaquin River basins comprehensive study: Technical studies documentation, Appendix E, risk analysis, Sacramento, Calif.
- U.S. Army Corps of Engineers (USACE). (2006). "Risk-based analysis for flood damage reduction studies." *Engineering Regulation ER* 11052–2-101, Washington, D.C.
- Wurbs, R. A. (1983). "Economic feasibility of flood control improvements." J. Water Resour. Plann. Manage., 109(1), 29–47.
- Zhu, T., Lund, J. R., Jenkins, M. W., Marques, G. F., and Ritzema, R. S. (2007). "Climate change, urbanization, and optimal long-term floodplain protection." *Water Resour. Res.*, 43, W06421.