# DERIVED POWER PRODUCTION AND ENERGY DRAWDOWN RULES FOR RESERVOIRS

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**ABSTRACT:** Theoretical hydropower operation rules are derived and discussed for reservoirs in parallel, in series, and single reservoirs for cases where reservoirs typically refill before they empty and for parallel reservoirs when reservoirs are expected to draw down to empty. These hydropower rules offer a simplified economic basis for allocating storage and energy in multireservoir hydropower systems. The approach is demonstrated for an illustrative example and should be helpful for making decisions regarding hydropower releases over time, subject to the limited conditions under which these rules hold.

## INTRODUCTION

Dan Sheer (Stedinger, unpublished notes, 1986) provides a rule for drawing down reservoirs in the order of minimum lost potential energy, given reservoirs that will fill before they next empty. Reservoirs for which withdrawal results in the smallest reduction in potential hydropower ($RPH_i$) should be drawn down first, stated by Sheer in 1986 as equation (1), without formal derivation. Conceptually, (1) is proportional to the lost hydropower production before refill with a unit reduction in present storage. This quantity is seen as the sum of losses between the present and refill and additional filled hydropower losses:

$$RPH_i = \frac{\partial \overline{H}_i}{\partial S} R_i + H_i(K_i) \tag{1}$$

where $H_i(K_i)$ = net hydropower head of reservoir $i$ when full (at capacity $K_i$); $\partial \overline{H}_i/\partial S$ = release-weighted average loss of net hydropower head between the present and time of filling per unit reduction in present storage; and $R_i$ = total expected turbine release from the present until refill. The first term estimates the release-weighted average reduction in hydropower head until refill, multiplied by the expected release until refill. The release-weighted average reduction in hydropower head is the sum of release times the reduced head at each time step until refill, divided by the total release over the period until refill. The terms can be estimated if the reservoirs fill each year. The second term is the marginal potential energy lost once the reservoir has filled, with reduced present storage. This term reflects that there will be a shorter period when the reservoir will be operating with full hydropower head if storage is drawn down now. Turbine efficiency and unit weight of water terms for potential hydropower production are neglected.

This technical note offers more complete theoretical development and elaborations on Sheer's conceptually based rule. The theoretical rules developed here apply for single-purpose hydropower operations or hydropower and water-supply operations when reservoirs will fill before they next reach their minimum power storage (pool). These rules are developed for special cases of reservoirs in parallel, reservoirs in series, and single reservoirs, and when parallel reservoirs are expected to draw down to minimum power pool before they next fill. A simple example illustrates use of this rule.

These theoretical rules are intended to help guide more detailed operating rule development studies. For systems with few reservoirs, operated predominantly for hydropower, these results might serve well as starting points for more detailed modeling studies. For larger and more multipurpose systems, additional optimization and simulation studies would clearly be needed. Recent reviews of other derived operating rules appear in Lund and Guzman (1996, 1999).

## DERIVATIONS AND ELABORATIONS FOR PARALLEL RESERVOIRS

Elaborating on the basis of Sheer's rule, the economic value of hydropower from a set of reservoirs in parallel—from the present until the beginning of the next draw-down season, without emptying in between—as a function of the current release volumes from each reservoir $T_i$, is given by

$$z = \sum_{i=1}^{n} e_i[P_0 H_i(S_{i0})T_i + P_r \overline{H}_i(S_{if} = S_{i0} + I_i - T_i)Q_i + P_f H_i(K_i)(F_i - \alpha_i T_i)] \tag{2}$$

where $e_i$ = efficiency of power generation at reservoir $i$; $T_i$ = target release volume from reservoir $i$ in the present time-step; $H_i(S_{i0})$ = present hydropower head at reservoir $i$ with present reservoir storage $S_{i0}$; $\overline{H}_i(S_{if})$ = expected release-weighted hydropower head for reservoir $i$ from the end of the current time-step until refill; $S_{if} = S_{i0} + I_i - T_i$, storage in reservoir $i$ at the end of the current time-step; $Q_i$ = expected turbine release volume from the end of the current time-step until reservoir refill; $P_0$ = present price of energy; $P_r$ = release-weighted average price of energy expected from the end of the current time-step until the reservoir refills; $P_f$ = expected price of energy when the reservoir is filled; $I_i$ = inflow expected for reservoir $i$ in the present period; $F_i$ = volume of turbine release for reservoir $i$ expected between refill and the beginning of drawdown; and $\alpha_i$ = marginal proportion of additional storage in the present, which would not be spilled during the refill season for reservoir $i$ (if = 0.9, 10% of any additional storage now is expected to be spilled). This accounts for the ability to capture and generate hydropower from additional inflows (spilling less), if more storage is empty now.

In (2), the two periods in (1) are expanded to three periods. The first term represents the value of present energy production (the current hourly, daily, or weekly release decision). The second term represents hydropower value during the remainder of the refill period. The last term represents the value of hydropower production between refill and the next drawdown, all as a function of the present release decision. (The unit weight of water has been omitted.) This formulation assumes the reservoirs do not empty the power pool before they refill. Release-weighted averages are used in the second term to account for correlations between release, price, and hydropower head (such as seasonal and peak versus off-peak changes). Eq.

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(2) is used as a basis to formalize and extend the original Sheer rule concept by considering the full economic value of hydropower, as opposed to potential energy (meaning efficiencies and prices now are included) and explicitly incorporating the hydropower value of present releases [reflected in the first term of (2)].

## Water Volume Drawdown Rule While Maximizing Hydropower Value

The most efficient drawdown of water volume from the system would minimize total reduction in annual hydropower revenue. The marginal hydropower value of present release from reservoir $i$ is

$$\partial z/\partial T_i = e_i(P_0 H_i(S_{i0}) + P_r Q_i \partial \bar{H}_i(S_{if})/\partial T_i - P_f H_i(K_i)\alpha_i) \quad (3)$$

The rule, then, is to draw down reservoirs with the greatest values of $\partial z/\partial T_i$ first, and refill them in the reverse order. (Note that the last two terms are negative and a positive $\partial z/\partial T_i$ indicates increased overall hydropower revenue with increased release.) This rule is particularly applicable where the withdrawals are being made to supply some downstream water demand.

## Energy Drawdown Rule While Maximizing Hydropower Value

If the current drawdown is intended to supply an energy demand or contract, then the rule is modified somewhat, preserving the earlier assumptions. The most efficient drawdown of potential energy from the system would minimize total reduction in annual hydropower revenues. Energy would be withdrawn first from reservoirs with the highest net marginal value of energy withdrawal, $\partial z/\partial E_i$, where $E_i = H_i(S_{i0})e_i T_i$. The net marginal value of energy withdrawal from reservoir $i$ is

$$\partial z/\partial E_i = P_0 + \frac{P_r Q_i}{H_i(S_{i0})}\frac{\partial \bar{H}_i(S_{if})}{\partial T_i} - \frac{P_f H_i(K_i)}{H_i(S_{i0})}\alpha_i \quad (4)$$

The same rule would be used to refill reservoirs, refilling based on $\partial z/\partial E_i$.

Both hydropower production rules should apply well where the reservoirs refill in most years and do not empty. Under these circumstances, energy spills might be common unless sufficient turbine flow capacity exists to pass common high refill-season flows. Thus, the coefficient $\alpha_i$ can be important. [Where refill is uncertain, spills represent wasted energy, and a more conventional energy space rule might be desirable (Lund and Guzman 1999).] Since the value of hydropower production often varies seasonally, these formulations also allow consideration of relative energy prices in different periods.

# ELABORATIONS FOR RESERVOIRS IN SERIES

Because reservoirs in series can allow the downstream recapture of upstream releases, this form of power generation rule often should take on a more complex form. When water is released from the entire system of reservoirs in series, as when water is released to meet some water demand downstream of all reservoirs, then the rule is very similar. However, when releases are made to meet energy demands, releases from a higher reservoir often can be captured in a lower reservoir, raising the number of release decisions, particularly early in the drawdown season.

For $m$ reservoirs in series, where reservoir $m + 1$ is out the bottom of the system, the overall benefit equation becomes

$$z = \sum_{i=1}^{m} e_i \left\{ P_0 H_i(S_{i0}) \sum_{j=i+1}^{m+1} T_{ij} + P_r \bar{H}_i(S_{if}) Q_i + P_f H_i(K_i) \cdot \left[ F_i - \alpha_i \left( \sum_{j=i+1}^{m+1} T_{ij} - \sum_{k=1}^{i-1} T_{ki} \right) \right] \right\} + \sum_{i=1}^{m} \sum_{j=2}^{m+1} \left( \sum_{k=i+1}^{j-1} e_k P_0 H_k(S_{k0}) \right) T_{ij} \quad (5)$$

where

$$S_{if} = S_{i0} + I_i - \sum_{j=i+1}^{m+1} T_{ij} + \sum_{k=1}^{i-1} T_{ki}$$

Here, the first sum is similar as that for parallel reservoirs [(2)], except for replacing releases $T_i$ with the sum of all releases to downstream locations in the first term and, in the third term, accounting for net release from the reservoir. Releases $T_{ij}$ originate in reservoir $i$ and are recaptured before the end of the time-step in reservoir $j$. $S_{if}$ has a somewhat different definition as well, accounting for net change in storage from both releases to downstream locations ($T_{ij}$) and recapture of upstream releases ($T_{ki}$). The new triple sum term accounts for additional hydropower benefits in the present time-step from flow through lower reservoirs ($i + 1$ through $j - 1$) before recapture of releases in some downstream reservoir $j$.

## Releases for Water Drawdown

The net benefit of a marginal decision to release from reservoir $i$ and recapture in reservoir $j$ is $\partial z/\partial T_{ij}$. There are $0.5m(m + 1)$ of these possible decisions, as shown in Table 1, including releases from the entire system. Decisions in the upper-right triangle generally are not possible, releasing from a lower reservoir to a higher one, unless pumped storage facilities exist. (Pumped storage operating decisions would have negative $P_0$ and pump, rather than turbine, efficiency terms.) The term $\partial z/\partial T_{ij}$ for reservoirs in series is very similar to that for reservoirs in parallel:

$$\partial z/\partial T_{ij} = \sum_{k=1}^{j-1} e_k P_0 H_k(S_{k0}) - P_r e_i Q_i \frac{\partial \bar{H}_i(S_{if})}{\partial S_{if}} - P_f e_i H_i(K_i)\alpha_i + P_r e_j \frac{\partial \bar{H}_j(S_{jf})}{\partial S_{jf}} Q_j + P_f e_j H_j(K_j)\alpha_j \quad (6)$$

Note that $\partial \bar{H}_i/\partial T_{ij} = -\partial \bar{H}_i/\partial S_{if}$ and $\partial \bar{H}_j(S_{jf})/\partial T_{ij} = \partial \bar{H}_j(S_{jf})/\partial S_{jf}$. For releases out of the system, $j = m + 1$ and the last two terms vanish. For recapture of water lower in the system, both latter terms will always be positive.

The operation rule would be to calculate the marginal decisions for all reservoir release decisions in Table 1 and make release and recapture decisions for the most favorable combinations of reservoirs first until downstream water supply targets are accomplished.

**TABLE 1. Matrix of Net Marginal Benefit of Possible Hydropower Release Decisions for Reservoirs in Series**

| Destination reservoir $j$ (1) | Origin Reservoir $i$: 1 (2) | 2 (3) | . . . (4) | $m$ (5) |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 |
| 2 | $\partial z/\partial T_{12}$ | 0 | 0 | 0 |
| . . . | $\partial z/\partial T_{1j}$ | $\partial z/\partial T_{2j}$ | 0 | 0 |
| $m$ | $\partial z/\partial T_{1m}$ | $\partial z/\partial T_{2m}$ | $\partial z/\partial T_{im}$ | 0 |
| Release from system, $m + 1$ | $\partial z/\partial T_{1,m+1}$ | $\partial z/\partial T_{2,m+1}$ | $\partial z/\partial T_{i,m+1}$ | $\partial z/\partial T_{m,m+1}$ |

## Releases for System Energy Drawdown

For reservoirs in series, the present energy produced from releasing $T_{ij}$ from reservoir $i$ with recapture of the release in reservoir $j$ is

$$E_{ij} = \sum_{k=i}^{j-1} e_k H_k(S_{k0}) T_{ij} \quad (7)$$

or

$$\partial E_{ij}/\partial T_{ij} = \sum_{k=i}^{j-1} e_k H_k(S_{k0}) \quad (8)$$

a constant. When releases for the current time-step are to satisfy current energy demands, a slightly different rule results. The rule is to make energy production/release decisions such that the marginal net benefit is greatest.

Using the chain rule, (6) becomes

$$\partial z/\partial E_{ij} = P_0 - \frac{P_r e_j Q_i \partial \bar{H}_i(S_{if})/\partial S_{if} + P_f e_j H_i(K_i)\alpha_i}{\partial E_{ij}/\partial T_{ij}} + \frac{P_r e_j \dfrac{\partial \bar{H}_j(S_{jf})}{\partial S_{jf}} Q_j + P_f e_j H_j(K_j)\alpha_j}{\partial E_{ij}/\partial T_{ij}} \quad (9)$$

The last term (for destination reservoir $j$) is always nonnegative. Energy releases should be made from/to the reservoirs with the highest $\partial z/\partial E_{ij}$ until the energy production target is met.

The same number of possible release and recapture decisions are possible as in the previous case and Table 1, except the equivalent Table 1 is filled with terms $\partial z/\partial E_{ij}$. When releases of energy are required to meet energy demands, water recapture in lower reservoirs becomes more feasible—and more optimal, especially early in the drawdown season when lower reservoirs need filling to increase long-term power production.

## ILLUSTRATIVE EXAMPLE FOR RESERVOIRS IN SERIES

Reservoirs in series with recapture of flows are probably the most complex application of this operating rule. Consider a hypothetical system of three reservoirs in series, described in Table 2 below. Let the relative prices of energy be $P_0$ = \$0.04/kW·h, $P_r$ = \$0.02/kW·h, and $P_f$ = \$0.01/kW·h. The decision must be made of which reservoir to withdraw energy from to meet a current contractual power demand, and should the release be recaptured at a reservoir downstream.

The terms $e_i H_i(S_{if})$ and $e_i H_i(K_i)$ are fairly easy to estimate based on the physical characteristics of the reservoir system. $Q_i$ and $\alpha_i$ can be estimated from historical performance, since current operations is probably an elaboration of current operating policies, for which historical or simulation results should be available. The term $e_i \partial \bar{H}_i(S_{if})/\partial S_{if}$, representing the efficiency weighted and release-averaged change in hydropower head over the refill period with a marginal change in storage now, must be estimated based on simulation or optimization studies.

**TABLE 2. Description of Illustrative Example of Three Reservoirs in Series**

| Parameter (1) | Reservoir 1 (2) | Reservoir 2 (3) | Reservoir 3 (4) |
|---|---|---|---|
| $e_i H_i(S_{i0})$ | 150 | 100 | 60 |
| $Q_i$ | 200 | 300 | 500 |
| $e_i \partial \bar{H}_i(S_{if})/\partial S_{if}$ | 1 | 3 | 2 |
| $e_i H_i(K_i)\alpha_i$ | 300 | 200 | 100 |

**TABLE 3. Net Marginal Economic Values of Energy Production Decisions, $\partial z/\partial E_{ij}$**

| Destination reservior $j$ (1) | Origin Reservoir $i$ 1 (2) | Origin Reservoir $i$ 2 (3) | Origin Reservoir $i$ 3 (4) |
|---|---|---|---|
| 1 | 0 | 0 | 0 |
| 2 | 0.13 | 0 | 0 |
| 3 | 0.10 | 0.05 | 0 |
| Release from system, $m + 1$ | 0.02 | −0.09 | −0.31 |

Performing the calculations for (9) for each case of marginal energy withdrawal from each reservoir gives the relative marginal benefits in Table 3. For this case, it is optimal to satisfy marginal energy demands by withdrawing water first from Reservoir 1 and recapturing the water in Reservoir 2. If this release/recapture decision were limited by the storage capacity of Reservoir 2, then additional releases would be made from Reservoir 1 and recaptured at Reservoir 3 (the second highest marginal value in the table).

Beyond being the most economical decision to meet energy contracts, since $\partial z/\partial E_{ij}$ is positive for these decisions, these calculations indicate that this energy withdrawal and recapture decision actually increases overall revenue for this system over the period until the next refill, raising revenue in the present time-step while increasing downstream head and power production later.

## OPERATIONS FOR SINGLE RESERVOIR

Eq. (2) also can be reformulated for operation of a single reservoir over time, substituting the time-step subscript $t$, the particular time a single reservoir release decision is being made:

$$z = P_t H_t(S_t) T_t + P_r \bar{H}_t(S_{t+1}) Q_t + P_f H(K)(F - \alpha_t T_t) \quad (10)$$

where $S_{t+1} = S_t + I_t - T_t$. The efficiency term is dropped, because it does not vary with time. The derivative $\partial z/\partial T_t$ represents the change in seasonal economic value of hydropower with a change in current release, $T_t$:

$$\partial z/\partial T_t = P_t H_t(S_t) + P_r(\partial \bar{H}_t/\partial T_t)Q_t - P_f H(K)\alpha_t \quad (11)$$

The decision to release water for hydropower in this case is driven by relative price and energy production circumstances. If current prices $P_t$ are high enough, $\partial z/\partial T_t$ is positive; it becomes economical to produce energy now rather than later during refill (the second two terms). To estimate the optimal amount of present release, responding to market prices for power, assuming the reservoir does not empty or face any other binding constraint on operations (such as turbine capacity), the above equation is solved for $\partial z/\partial T_t = 0$, with optimal release $T_t^*$, as shown in Fig. 1.

This single-reservoir operation approach also illustrates how this hydropower rule form would apply to dynamic operation of multiple reservoirs. The prices for the various time periods (present, refill, and refilled) are used in this rule to estimate the optimal balance of present versus future hydropower releases. This principle applies as well to multiple reservoir systems as to single reservoirs.

The rules derived above apply to pure hydropower production systems where the reservoirs are unlikely to empty before they refill. The method applies more easily when the system's reservoirs typically refill each year (simplifying estimation of many of the rule parameters, particularly $\partial \bar{H}_i/\partial T_i$).

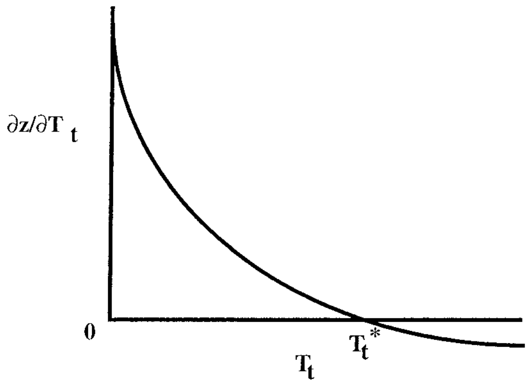


**FIG. 1. Optimal Hydropower Release**

## WHEN RESERVOIRS WILL EMPTY BEFORE REFILL

Where reservoirs are expected to empty (in terms of hydropower storage) before they refill, (2) can be modified to (12), for the case of reservoirs in parallel:

$$z' = \sum_{i=1}^{n} e_i[P_0 H_i(S_{i0})T_i + P'_r \bar{H}'_i(S_{if})(Q'_i - T_i)] \qquad (12)$$

where the primed terms indicate prices, average heads, and total releases expected between the present time and the emptying of the reservoir's power pool.

The marginal hydropower revenue impact of water releases from each reservoir can be estimated by

$$\partial z'/\partial T_i = e_i \left\{ P_0 H_i(S_{i0}) - P'_r \left[ \bar{H}'_i(S_{if}) - \frac{\partial \bar{H}'_i(S_{if})}{\partial T_i}(Q'_i - T_i) \right] \right\} \qquad (13)$$

where the first term represents the hydropower value of current release, the second term represents the later loss of hydropower due to decreased later release volume, and the last term represents hydropower revenue due to lost hydropower head.

For energy releases from the reservoirs, the companion to (4) for reservoir systems expected to draw down completely with respect to hydropower is

$$\partial z'/\partial E_i = P_0 - \frac{P'_r}{H_i(S_{i0})} \left[ \bar{H}'_i(S_{if}) - \frac{\partial \bar{H}'_i(S_{if})}{\partial T_i}(Q'_i - T_i) \right] \qquad (14)$$

Reservoirs would be emptied in the order of $\partial z'/\partial T_i$ if emptied for downstream water supply purposes and in order of $\partial z'/\partial E_i$ if emptied for energy production purposes.

## PARAMETER ESTIMATION

Estimation of most parameters is fairly simple: $P_0$, $P_r$, and $P_f$ by present and expected market prices for energy, and $e_i$, $H_i(S_{i0})$, and $H_i(K_i)$ by plant physical characteristics. Estimation of other parameters, $\alpha_i$, $\partial H_i/\partial T_i$, and $Q_i$, is more difficult and would typically require simulation modeling studies, either in real time or prior to rule specification. These modeling studies would reflect the likely changes in spill, flow-averaged head, and expected turbine flow volumes during the refill period. They would probably use existing reservoir operating procedures to estimate changes in hydropower production, given different initial starting storages, $S_{if} = S_{i0} + I_i - T_i$. Thus, the terms $\partial \bar{H}_i/\partial T_i$, and $Q_i$ could be estimated together. For large reservoirs with overyear storage, long expected periods to refill can make estimates of these parameters more uncertain.

## CONCLUSIONS

A relatively general single and multireservoir operating rule for hydropower production is derived, elaborated, and illustrated based on a concept by Dan Sheer (1986). The rule incorporates the tradeoffs of energy, head, production efficiency, and time-varying price into an economic rule for making storage and release decisions for hydropower revenue maximization. Variants of the rule are developed for cases where the reservoirs are expected to refill before they empty and where they are expected to empty before they refill. The results presented here should be helpful for both understanding optimal operations and developing more precise optimal operating rules for hydropower systems, as part of more general system modeling studies.

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## APPENDIX. REFERENCES

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