Derived Operating Rules for Allocating Recharges and Withdrawals among Unconnected Aquifers

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Abstract: Six balancing rules are derived to inform short-term drawdown and recharge of water in multiple, unconnected aquifers. Management objectives are: (1) minimizing costs; (2) maximizing duration of operation; and (3) maximizing accessibility as a tradeoff between maximizing instantaneous withdrawal rate and the duration to sustain withdrawals. Engineering optimization formulations use either a specified target delivery rate (for withdrawals) or available surface water supply (to recharge). Aquifers are modeled as separate, single-celled basins with lumped parameters representing key physical, institutional, and financial characteristics. Each formulation is solved analytically for the case where constraints are nonbinding. Solutions are explained as operating rules. Two examples confirm the analytical solutions. The results show how cost characteristics, fraction of recharged water available for withdrawal (fractional recovery), initial storage, maximum recharge and pumping rates, and uncertainties regarding the future availability of water for extraction influence recharge and withdrawal decisions.

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Introduction

Water storage for many water supply systems is moving underground. In California, major urban water providers now contract with nearby and distant irrigation and water management districts that overlie large aquifers (Pulido-Velázquez et al. 2004). Although these aquifers can meet urban water demands for several years' duration, they often are far from the urban areas and require extensive water exchanges for delivery. Conjunctive use creates elaborate engineering problems for water supply and drought response. Even when surplus surface water supplies or target deliveries are specified (or recommended by economic analysis), the regional water provider is often challenged with how best to distribute recharges or extractions among the multiple unconnected aquifers given varied physical and non-physical characteristics (Fig. 1).

The spatial aquifer balancing problems presented in Fig. 1 are reminiscent of operating surface water reservoirs configured in parallel (Bower et al. 1966; Lund and Guzman 1999; Sand 1984). However, managing multiple unconnected aquifers differs in several respects. First, aquifer managers can often regulate inflow and withdrawal through choice of recharge and pumping facilities and volumes. Such inflows are generally more certain than reservoir inflows. During droughts, demand is relatively constant, and natural recharge is likely small or trivial. Second, aquifer storage often is refilled and drawn down over several years or decades rather than seasons when anticipating or responding to droughts. Third, recharge and extraction capacity characteristics, storage losses, and legal, institutional, and other nonphysical characteristics of aquifers may constrain aquifer operations. These conditions apply where: (1) drawdowns are small as compared to the saturated thickness of the aquifers; (2) geologic formations (confining layers or lenses) hydraulically isolate aquifers; (3) large distances separate the aquifers; or (4) the hydraulic response time is much longer than the planning horizon so that management for one aquifer does not affect other aquifers. These assumptions reduce stochastic conjunctive use problems (Knapp and Olson 1995; Maddock 1974; Philbrick and Kitanidis 1998; Provencher and Burt 1994; Reichard 1995) into steady, deterministic problems that can use lumped aquifer parameters (representing physical and accounting losses; storage, recharge, and extraction capacities; water quality; cost; and future availability to withdraw water) to specify near optimal engineering management rules.

This paper derives operating rules that allocate steady recharge and withdrawal for multiple independent aquifers with varied hydrogeological, financial, and institutional characteristics. We present six optimization formulations that represent management objectives for: (1) financial performance; (2) duration of operation; and (3) accessibility as a tradeoff between the instantaneous withdrawal rate and the duration to sustain withdrawals. We derive analytical solutions for each case where constraints do not bind and interpret the solutions as operating rules. Two examples verify the analytical solutions, extend them, and show their limitations. Because target withdrawal or recharge quantities are specified exogenously, the paper focuses on spatial rules for short-term operation. Rules for temporal, economic, and multiobjective, dynamic aquifer management are important areas for further work.

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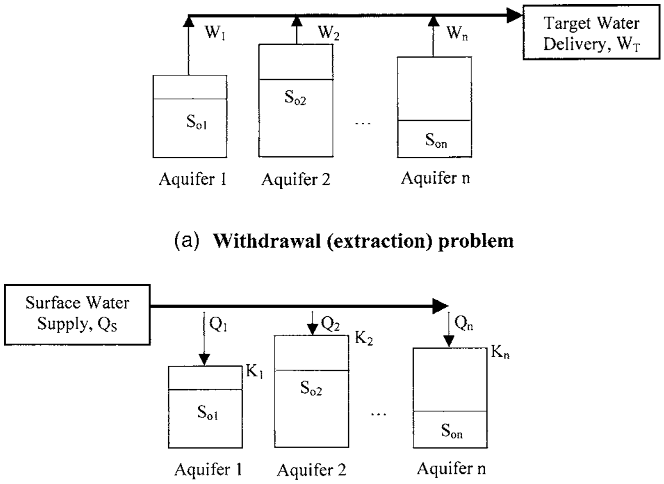

(b) **Recharge problem**

Fig. 1. Water system balancing for n unconnected aquifers considering steady surface water supply (Q_S), steady target delivery rate (W_T), and aquifer characteristics such as initial storage (S_{oi}) and unfilled storage capacity (K_i): (a) withdrawal (extraction) problem; and (b) recharge problem

Optimizing Financial Performance

Financial performance management involves maximizing the benefit from extracting water or minimizing the actual and institutional transactional costs associated with recharging, pumping, treating, conveying, and delivering water to end users. With a downward-sloping demand curve, $v_i(w)$ (\$ volume^{-1}), an upward-sloping supply curve, $c_i(w)$ (\$ volume^{-1}), and decisions on withdrawal rate for each aquifer, W_i(volume time^{-1}), the financial objective is

$$\text{Maximize}\sum_i \int_{W_i} (v_i(W_i) - c_i(W_i)) \cdot dW_i \qquad (1)$$

Formulations are posed, separately, for withdrawals and recharges and are derived as follows.

Minimize Cost of Withdrawals

For withdrawals, Eq. (1) reduces to a linear cost minimization problem when: (1) the amount of water to be delivered from aquifer withdrawals will meet a small portion of a single large drought demand (i.e., the regional water utility uses aquifer withdrawals in conjunction with other water deliveries, urban demand management, and water transfers); and (2) operational storages are small as compared to overall aquifer storages (i.e., withdrawals cause small drawdowns). With these conditions, pumping costs are proportional to the pumping lift, and the end-use value of extracting water becomes fixed and the same for all aquifers. The objective is to identify withdrawal rates from each aquifer, W_i (volume time^{-1}), that minimize the cost of using groundwater. The total cost of withdrawal includes the per-unit costs, c_i (\$ volume^{-1}) of extracting, treating, conveying, and securing the right to access and use the aquifer. Cost characteristics will likely differ among aquifers, because aquifers can differ in hydraulic pumping lifts, extracted water quality, treatment requirements, and conveyance distances. Initial storages are given so prior recharges are sunk costs (literally!) and not considered. The cost minimization objective is expressed by the mathematical formulation:

$$\text{Minimize}\sum_i c_i \cdot W_i \qquad (2)$$

Subject to:

- Withdrawals limited by maximum pumping rates, $p_{\max i}$ (volume time^{-1})

$$W_i \leqslant p_{\max i}, \quad \forall \ \text{i} \qquad (3)$$

- Withdrawal rates for a predetermined and relatively short duration, t (time), limited by initial, operational storages, S_{oi} (volume)

$$W_i \cdot t \leqslant S_{oi}, \quad \forall \ \text{i} \qquad (4)$$

- No negative withdrawals, $W_i \geqslant 0$, $\forall$ i.

This linear program (LP) is solved by the general withdrawal rule: "Unless limited by pumping rates or storage, withdraw water in order of increasing cost, c_i." Take water from aquifers with the smallest costs. This strategy makes water withdrawals more costly as a withdrawal program is sustained, for example, in response to a drought. However, over a population of droughts of uncertain lengths, the rule will generally minimize the cost of drought response.

Maximize Expected Financial Value of Recharge

For recharging, Eq. (1) is modified and expanded to include a discounting factor, $(1+r)^{-t}$ (unitless), with an interest rate, r (unitless), that relates recharge costs, rc_i (\$ volume^{-1}), borne in the present with use benefits, u_i (\$ volume^{-1}), and other costs, c_i (as previously), in the future, t (years), when water is extracted, conveyed, treated, delivered, and used:

$$\text{Maximize}\sum_i [(1+r)^{-t} \cdot (u_i - c_i) - rc_i] \cdot \lambda_i \cdot Q_i \qquad (5)$$

Here, recharge decisions to each aquifer, Q_i (volume), should maximize the value expected from extracting water at a specified, future time t. Because this time is the same for each aquifer, the discount factor is assumed to be constant across aquifers and will often not affect the short-term allocation of recharges. Therefore, the benefit and cost terms can be aggregated into a single, constant, discounted, unit net value of storing water in each aquifer, v_i (\$ volume^{-1}):

$$v_i = (1+r)^{-t} \cdot (u_i - c_i) - rc_i \qquad (6)$$

As in the cost-minimizing withdrawal problem, use benefits, u_i, will be the same for each aquifer when the future withdrawals will meet a small portion of a single, large drought demand.

The fractional recovery term λ_i (unitless) in Eq. (5) describes losses as a fixed fraction of the recharge amount and covers accounting and physical losses. Recoverability will influence the volume of water that can be later extracted and delivered. λ_i will be <1 for aquifers where groundwater flows away from the recharge site. λ_i could equal 1 for recharge by in-lieu exchanges, but may be less with institutional accounting losses. The recovery term can also be a "put-take ratio" or rent on aquifer storage imposed by regulators or overlying landowners.

Eq. (5) omits a scarcity rent on recharged water because the regional authority's prior economic analysis has specified the total quantity of water to be recharged [see Eq. (8)]. The problem is to engineer the financially optimal spatial allocation of recharges. The problem is fully specified with objective Eq. (5) subject to constraint Eqs. (7)–(10):

- Storage capacity on each aquifer, K_i (volume)

$$Q_i \leqslant K_i, \quad \forall \text{ i} \tag{7}$$

- Total recharges limited by surface water supply, Q_S (volume)

$$\sum_i Q_i \leqslant Q_S \tag{8}$$

- Recharges for period t (time) limited by maximum recharge rates, $r_{\max i}$ (volume time^{-1})

$$Q_i \leqslant r_{\max i} \cdot \text{t}, \quad \forall \text{ i} \tag{9}$$

- The probability that a fraction β (unitless) of total recharges are available for future withdrawal must exceed the target reliability α (unitless)

$$P_r\left[\sum_i a_i \cdot Q_i \geqslant \beta \cdot \sum_i Q_i\right] \geqslant \alpha \tag{10}$$

- No negative recharges, $Q_i \geqslant 0$, $\forall$ i.

The random variable, a_i (unitless), incorporates institutional and physical risks for future aquifer withdrawals. Recharged water may be unavailable later for extraction due to unforeseen regulatory, legal, or water quality concerns, or lack of available capacity to convey the water. Aquifers governed by different entities and with different physical-chemical characteristics are likely to differ in these risks. When the distribution of a_i is known, Eq. (10) can be reduced to a deterministic constraint (Tung 1986; Wagner 1969). For example, when a_i takes the Gaussian distribution with expected availability $\bar{a}_i$ (unitless), standard deviation of that availability σ_i (unitless), and standard normal variate Z_α (unitless) for probability α, Eq. (10) becomes

$$\sum_i [(\bar{a}_i - Z_\alpha \cdot \sigma_i - \beta) \cdot Q_i] \geqslant 0 \tag{11}$$

The unfilled storage capacity, K_i, is readily calculated from the unsaturated void space in aquifer i (Rosenberg 2003). Alternatively, regulators, agencies, and local landowners can stipulate unfilled capacity with agreements or by legal precedent.

This problem is also solved as an LP. The following general recharge allocation rule results: "Recharge aquifers in order of $\upsilon_i\lambda_i$, unless limited by recharge or storage capacity or future availability." Recharge water first to basins with the highest discounted net financial value and fraction of recoverable water. As the water available to recharge increases, the marginal value of storing the water will decrease. As high-valued and large fractional recovery aquifers fill, lower-valued and less desirable aquifers remain for use.

Optimizing Duration of Aquifer Operations

Optimizing the time to fill or empty aquifer storage is a second objective for managing a portfolio of aquifers. Duration becomes a relevant operational objective when either the surface water supply (available for recharge) or the target delivery rate (from withdrawals) is known or desired. For blending, regulatory, or operational reasons, we assume steady withdrawal or recharge rates. Formulations for the recharge and extraction problems follow.

Maximize Duration of Withdrawals

The objective is to find the steady withdrawal rates, W_i (volume time^{-1}), to maximize the duration to sustain a specified, steady, total target delivery rate. This objective may be important to sustain operations through a drought. The nonlinear mathematical program is

$$\text{Maximize } WD_{\max} \tag{12}$$

Subject to:

- Withdrawals limited by maximum pumping rates, $p_{\max i}$ (volume time^{-1})

$$W_i \leqslant p_{\max i}, \quad \forall \text{ i} \tag{13}$$

- Total withdrawals must meet or exceed a target delivery rate, W_T (volume time^{-1})

$$\sum_i W_i \geqslant W_T \tag{14}$$

- Withdrawal duration, WD_i (time), for aquifer i defined by initial storage in aquifer i, S_{oi} (volume)

$$WD_i = S_{oi}/W_i, \quad \forall \text{ i} \tag{15}$$

- Definition of maximum feasible duration for withdrawal program, $WD_{\max}$ (time)

$$WD_{\max} \leqslant WD_i, \quad \forall\ i \tag{16}$$

- No negative withdrawals, $W_i \geqslant 0, \forall i$.

This nonlinear program balances withdrawals across all aquifers. When the nonnegativity and pumping capacity constraints do not bind, the program can be solved analytically for a general balancing rule. Under this condition, the set of optimal, duration-maximizing steady withdrawals (W_i^*) will exhaust all aquifers at the same time, so $WD_{\max} = WD_i = S_{oi}/W_i^*, \forall i$. Rearranging gives

$$\frac{S_{oi}}{W_i^*} = \frac{\sum_i S_{oi}}{W_T} \text{ or } W_i^* = \frac{S_{oi} \cdot W_T}{\sum_i S_{oi}} \text{ or } \frac{W_i^*}{W_T} = \frac{S_{oi}}{\sum_i S_{oi}} \tag{17}$$

This rule shows that the duration-maximizing withdrawal from aquifer i is proportional to the fraction of the total system water initially stored in aquifer i. Inversing the withdrawal duration $(1/WD_i)$ transforms the problem into a linear program (Rosenberg 2003).

Minimize Duration of Recharge

Here, the objective is to find recharges that minimize the duration to (1) recharge a specific, total quantity of water, or (2) fill all aquifers. The former objective should apply when the amount of surface water is small as compared to unfilled aquifer storage. The later objective applies when available surface water is significantly more than aquifer storage capacity. These two problems are formulated separately.

Minimize duration to recharge a small volume of water: The objective function is

$$\text{Minimize } RD_{\min} \tag{18}$$

Subject to:

- Storage capacity available in each aquifer, K_i (volume)

$$Q_i \leqslant K_i, \quad \forall \text{ i} \tag{19}$$

- Total recharges must equal surface water supply, Q_S (volume)

$$Q_S = \sum_i Q_i \tag{20}$$

- No negative recharge durations, RD_i (time)

$$RD_i \geq 0, \quad \forall \text{ i} \tag{21}$$

- Recharges limited by maximum recharge rates, $r_{\max i}$ (volume time^{-1})

$$Q_i/RD_i \leq r_{\max i}, \quad \forall \text{ i} \tag{22}$$

- Definition of program recharge duration, $RD_{\min}$ (time)

$$RD_{\min} \geq RD_i, \quad \forall \text{ i} \tag{23}$$

- No negative recharges, $Q_i \geq 0, \quad \forall i$
- The deterministic constraint on future availability [Eq. (11)].

When the nonnegativity, storage capacity, and future availability constraints do not bind, the program can be solved analytically for a general balancing rule. Under this condition, the set of optimal, duration-minimizing steady recharges (Q_i^*) will be related to the largest allowable recharge rate of each aquifer, so $RD_{\max}=RD_i=Q_i^*/r_{\max i}, \ \forall i$. Substituting gives:

$$\frac{Q_i^*}{r_{\max i}} = \frac{Q_S}{\sum_i r_{\max i}} \text{ or } Q_i^* = \frac{r_{\max i} \cdot Q_S}{\sum_i r_{\max i}} \text{ or } \frac{Q_i^*}{Q_S} = \frac{r_{\max i}}{\sum_i r_{\max i}} \tag{24}$$

This rule is that the duration-minimizing recharge to aquifer i should be proportional to the fraction of the total recharge rate capacity aquifer i can handle.

Minimize duration to fill all aquifers: The objective is

$$\text{Minimize } FD_{\min} \tag{25}$$

Subject to:

- Fill duration for aquifer i, FD_i (time), is defined by aquifer storage capacity, K_i (volume); fractional recovery, λ_i (fraction); and recharge rate, R_i (volume time^{-1})

$$FD_i = \frac{K_i}{\lambda_i \cdot R_i} \quad \forall \text{ i} \tag{26}$$

- Total recharges are less than steady surface water available each period, R_S (volume time^{-1})

$$\sum_i R_i \leq R_S \tag{27}$$

- Recharges are limited by maximum recharge rates, $r_{\max i}$ (volume time^{-1})

$$R_i \leq r_{\max i}, \quad \forall \text{ i} \tag{28}$$

- Definition of program fill duration, $FD_{\min}$ (time)

$$FD_{\min} \geq FD_i \tag{29}$$

- No negative recharge rates, $R_i \geq 0, \ \forall i$.

The fill duration for each aquifer is a function of the fractional recovery [Eq. (26)] and assumes that losses occur as recharges are made. This assumption should hold when unfilled capacity is large, recharge rates are small, and expected durations are long.

With nonbinding recharge constraints, optimal steady recharges (R_i^*) should make all aquifers fill at the same time, $FD_{\min}=FD_i=(K_i)/(\lambda_i \cdot R_i^*) \forall i$. Substituting gives

$$\frac{K_i}{\lambda_i \cdot R_i^*} = \frac{\sum_i \left(\frac{K_i}{\lambda_i}\right)}{R_S} \text{ or } R_i^* = \frac{R_S \cdot K_i/\lambda_i}{\sum_i \left(\frac{K_i}{\lambda_i}\right)} \text{ or } \frac{R_i^*}{R_S} = \frac{\frac{K_i}{\lambda_i}}{\sum_i \left(\frac{K_i}{\lambda_i}\right)} \tag{30}$$

To minimize the duration to fill all aquifers, recharge more water into aquifers with larger unfilled capacities or smaller fractional recoveries, i.e., aquifers that are most empty or with the least efficient recharge. Lower fractional recoveries will lengthen the fill duration. Note that the fractional recovery terms (λ_i) drop out when they are equal across all aquifers. Inversing the fill duration ($1/FD_i$) transforms the problem into a linear program (Rosenberg 2003).

Maximizing Accessibility

When filling groundwater storage capacity in wet years, an agency often is unsure about the future demands for water. The agency may want to optimize flexibility to deliver water from a portfolio of groundwater storages at either high withdrawal rates or for a long duration. A formulation is presented to simultaneously address the recharge and withdrawal portions of the problem. Two analytical solutions are derived and the tradeoff between them is presented.

Model Formulation

The biobjective maximizes the total withdrawal rate, W_R (volume time^{-1}) plus the duration of withdrawals, $D_{\max}$ (time) weighted by a tradeoff factor, d (volume time^{-2})

$$\text{Maximize } W_R + d \cdot D_{\max} \tag{31}$$

Subject to:

- Recharges, Q_i (volume), for a specified, short period t (time) are limited by maximum recharge rates, $r_{\max i}$ (volume time^{-1})

$$Q_i \leq r_{\max i} \cdot t, \quad \forall i \tag{32}$$

- Maximum pumping capacities, $p_{\max i}$ (volume time^{-1}) limit withdrawals, W_i (volume time^{-1})

$$W_i \leq p_{\max i}, \quad \forall i \tag{33}$$

- Aquifer duration, D_i (time), is defined by initial storage, S_{oi} (volume), and fractional recovery, λ_i (unitless)

$$D_i = \frac{S_{oi} + \lambda_i \cdot Q_i}{W_i}, \quad \forall i \tag{34}$$

- Recharges are limited by remaining storage capacity, K_i (volume)

$$Q_i \leq K_i, \quad \forall \text{ i} \tag{35}$$

- Total recharges are limited by surface water supply, Q_S (volume)

$$Q_S \geq \sum_i Q_i \tag{36}$$

- Definition of program withdrawal duration, $D_{\max}$ (time)

$$D_{\max} \leq D_i, \quad \forall i \tag{37}$$

- Expected withdrawal rate, W_R (volume time^{-1}), is defined by random variables representing future availability, a_i (unitless)

Table 1. Aquifer Characteristics for Example No. 1

	Physical				Financial			Institutional	
Aquifer	Unfilled storage capacity, K_i (Mm^3)	Maximum pumping rate, $p_{\max i}$ (Mm^3/mon)	Maximum recharge rate, $r_{\max i}$ (Mm^3/mon)	Fractional recovery, λ_i (fraction)	Recharge cost, rc_i (\$ m^{-3})	Use cost, c_i (\$ m^{-3})	Use value, u_i (\$ m^{-3})	Mean expected availability, $\bar{a}_i$ (fraction)	Standard deviation of availability, σ_i (fraction)
A	493	8.6	4.9	0.96	0.02	0.10	0.81	0.9	0.020
B	247	7.4	3.7	0.93	0.01	0.09	0.81	0.9	0.015
C	740	10	4.9	0.90	0.03	0.06	0.81	0.9	0.002
D	987	19	6.2	0.92	0.04	0.05	0.81	0.9	0.001

$$W_R = \sum_i a_i \cdot W_i \tag{38}$$

- Expected withdrawal rate must meet target delivery rate, W_T (volume time^{-1}), with target reliability α (fraction)

$$P_r[W_R \geqslant W_T] \geqslant \alpha \tag{39}$$

- No negative recharges or withdrawals, $Q_i \geqslant 0, W_i \geqslant 0, \ \forall i$.

Future availability is a function of the withdrawal, Eq. (38), rather than recharge [as in Eq. (10)]. Eqs. (38) and (39) are reduced to an equivalent deterministic form as shown previously

$$\sum_i [(\bar{a}_i - Z_\alpha \cdot \sigma_i) \cdot W_i] \geqslant W_T \tag{40}$$

In this nonlinear program, both recharge volumes (Q_i) and withdrawal rates (W_i) are decision variables. The recharge period is fixed to time t, while the withdrawal period ($D_{\max}$) is assumed to start after recharge is completed. Solving the nonlinear program determines the withdrawal duration. Selecting a small value for the tradeoff factor d ($0 \leqslant d \ll 1$) yields recharge and extraction operations that maximize instantaneous withdrawal capacity, giving slight preference to operations that lengthen the duration to sustain withdrawals. Conversely, selecting $d \gg 1$ yields operations that maximize the duration to sustain withdrawals, giving slight preference to operations that increase the rate to withdraw water.

Analytical Solutions

Analytical solutions are derived for cases where the coefficient d is either large or small and the nonnegativity, limited recharge rate, limited extraction rate, aquifer storage capacity, and future availability constraints do not bind.

Maximize Instantaneous Withdrawal Rate (W_R). When the value of d is small, an analytical solution can be derived to maximize the expected instantaneous withdrawal rate (W_R). First, increase aquifer withdrawals to their maximum pumping rates

$$W_i^* = p_{\max i}, \quad \forall \text{ i}. \tag{41}$$

Second, configure recharges so the withdrawal rates are maximally sustained, equalizing withdrawal durations for all aquifers

$$D_{\max} = D_i = \frac{S_{oi} + \lambda_i \cdot Q_i^*}{W_i^*} = \frac{\sum_i (S_{oi} + \lambda_i \cdot Q_i^*)}{\sum_i W_i^*} \tag{42}$$

Here, the asterisk superscript (*) represents the optimal value of a decision variable. Substituting Eq. (41) into Eq. (42) and rearranging gives:

$$\frac{p_{\max i}}{\sum_i p_{\max i}} = \frac{S_{oi} + \lambda_i \cdot Q_i^*}{\sum_i (S_{oi} + \lambda_i \cdot Q_i^*)}, \quad \forall i. \tag{43}$$

Equation (43) is a set of i independent equations that can be solved simultaneously for Q_i^*. The solution suggests recharging water in aquifer i so that the ratio of pumping capacity of aquifer i to total pumping capacity (for all the aquifers) equals the ratio of water recoverable for extraction from aquifer i to the total water recoverable from extraction (from all aquifers). To maximize the instantaneous withdrawal rate, the rule suggests recharging more water into aquifers with highest pumping capacities, lower initial storages, and lower fractional recoveries (i.e., higher losses). Losses are borne to equalize ratios of recoverable water and to maximize the capacity for (but not necessarily the duration of) subsequent withdrawal.

When fractional recoveries and expected availabilities are identical across aquifers, Eq. (43) reduces to the Metropolitan Water District of Southern California's (MWD) aquifer allocation rule. The MWD rule equalizes the ratio of pumping capacity to total water storage in each aquifer, $(p_{\max i}/\Sigma_i p_{\max i}) = S_{oi} + Q_i^* / \Sigma_i (S_{oi} + Q_i^*)]$, $\forall i$ (Tim Blair, personal communication, 1999).

Maximize Duration of Withdrawal ($D_{\max}$). A second analytical solution applies where the tradeoff coefficient d is large. To maximize duration of steady withdrawals, all basins should empty at the same time, so

$$D_{\max} = D_i = \frac{S_{oi} + \lambda_i \cdot Q_i^*}{W_i^*} = \frac{\sum_i (S_{oi} + \lambda_i \cdot Q_i^*)}{\sum_i W_i^*} \tag{44}$$

Recharge and extraction decisions are taken sequentially. First, without knowing the duration-maximizing withdrawal rates for each aquifer (W_i^*), we observe that the duration will be largest when the sum of the withdrawals is smallest. Therefore, minimize withdrawals subject to constraint Eq. (40) on the withdrawal target (W_T) determined exogenously. This substitution gives:

$$D_{\max} = \frac{\sum_i S_{oi} + \sum_i \lambda_i \cdot Q_i^*}{W_T} \tag{45}$$

Second, recognize that duration is maximized when the term $\Sigma\, \lambda_i \cdot Q_i^*$ is maximized. $\Sigma\, \lambda_i \cdot Q_i^*$ represents recharged water recoverable for extraction. To maximize the recoverable amount, recharge into aquifers with the highest fractional recoveries. The duration-maximizing recharge rule is: "Recharge aquifers in order of λ_i, unless limited by recharge or storage capacities."

Table 2. Additional Parameter Values for Example No. 1

Parameter	Value
Withdrawal problem	
Target water delivery rate, W_T, Mm^3/mon	25
Initial storages, S_{oi}, af	K_i
Recharge Problem	
Water available to recharge, Q_S, Mm^3	7
Recharge period, t, mon	1
Initial storages, S_{oi}, Mm^3	0
Steady water available to recharge, R_S, Mm^3/mon	7
Discount factor, b, fraction	0.784
Target availability for withdrawal, β, fraction	0.85
Target reliability, α, fraction	0.9
Standard normal deviate for reliability, Z_α, fraction	1.653

Third, with Q_i^* known, solve Eq. (45) for the duration-maximizing, steady withdrawal rates for each aquifer. Because the initial storages and additional storage generated from recharge were determined in step 2, Eq. (45) takes the same form as the solution for Eq. (17). Rearranging and solving Eq. (45) for the duration-maximizing, steady withdrawal rate gives

$$W_i^* = \frac{W_T \cdot (S_{oi} + \lambda_i \cdot Q_i^*)}{\sum_i (S_{oi} + \lambda_i \cdot Q_i^*)} \qquad (46)$$

This rule makes the duration-maximizing withdrawal from aquifer i proportional to the fraction of total recoverable water stored in aquifer i. The rules for withdrawal [Eq. (46)] and recharge (preceding paragraph) represent sequential solutions for recharge followed by withdrawal decisions. These solutions are similar to those for maximizing the withdrawal duration [Eq. (17)].

Tradeoff between Solutions

The two analytical solutions frame a tradeoff between withdrawal *duration* and *rate*. Operations to maximize the instantaneous withdrawal rate incur higher water losses that diminish the duration over which withdrawals can be sustained, and *vice versa*. The tradeoff will be most apparent when one group of aquifers has high pumping capacities but low fractional recoveries while a second group of aquifers has low pumping capacities but high fractional recoveries. Solving the nonlinear program for a range of values for d can also illustrate the tradeoff.

Example Applications

The first example verifies solutions for the five derived single-objective operation rules. A second example demonstrates solutions for the biobjective accessibility formulation. Examples were set up in Excel and solved with the "Solver" add-in.

Example No. 1 (Single-Objective Programs)

We select a portfolio of four aquifers with different physical, institutional, and financial characteristics (Table 1). Aquifer A has high use costs, while aquifer D has low use costs. We assume aquifers have similar water qualities and use values and that extracted water is delivered to a single location. The parameter values fall within ranges of values the Metropolitan Water District of Southern California and the Santa Clara Valley Water District commonly use for aquifer storage (Pulido-Velázquez et al. 2004). However, values do not represent specific aquifers.

Table 2 summarizes additional parameter values for recharge and extraction. All aquifers are assumed to start full for the withdrawal problems ($S_{oi}=K_i$) and start empty for the recharge problems ($S_{oi}=0$). Withdrawals should meet the target delivery rate of 25 million cubic meters per month [Mm^3/mon; 20,000 acre-feet per month (20 kaf/mon)]. 7 Mm^3 (6 kaf) of surface water is available for recharge. We assume an interest rate of 5% over a planning horizon of five years to calculate a discount factor $b=(1+0.05)^{-5}=0.784$. Furthermore, we require 85% of recharged water be available for withdrawal with 90% reliability.

Numerical solutions to the five single-objective models verify the analytical solutions derived previously (Table 3). To minimize the cost of withdrawals (column 2), aquifers D and C were pumped. These aquifers have the lowest and second-lowest extractive costs (Table 1, column 6). Pumping capacity for aquifer D limited withdrawal to 19 $Mm^3\ mon^{-1}$(15 kaf/mon). Remaining deliveries were met from aquifer C. When the objective was to maximize the duration of meeting target withdrawals (Table 3, column 3), water was withdrawn from each aquifer proportional to the initial storage in each aquifer. Each withdrawal was sustained for 100 months.

When the objective was to maximize the expected value of recharge, water was recharged into both aquifers B and D. These aquifers have the highest and second-highest discounted net financial value of recoverable water (Table 4, column 4). Recharge capacity for aquifer B limited recharge to 3.7 Mm^3/mon (3 kaf/mon). Excess surface water was recharged to aquifer D. To minimize the duration to recharge 7 Mm^3 (6 kaf), recharge each aquifer in proportion to each aquifer's recharge capacity (Table 3, column 5). Recharges were sustained for 0.38

Table 3. Aquifer Balancing Solutions for Five Single Objective Model Formulations

	Withdrawal problems		Recharge problems		
Aquifer	Minimize cost, W_i^*, (Mm^3/mon)	Maximize duration of withdrawal, W_i^* (Mm^3/mon)	Maximize expected value of recharge, Q_i^* (Mm^3)	Minimize duration to recharge small volume, Q_i^* (Mm^3)	Minimize duration to fill all aquifers, R_i^* (Mm^3/mon)
A	0.0	4.9	0.0	1.9	1.4
B	0.0	2.5	3.7	1.4	0.7
C	6.2	7.4	0.0	1.9	2.3
D	19	10	3.7	2.3	3.0

Table 4. Calculated Net Financial Values of Water (\$ m^{-3})

Aquifer	Net extractive use value, $u_i - c_i$	Discounted net financial value, $v_i, b \cdot (u_i - c_i) - rc_i$	Discounted net financial value of recoverable water, $\lambda_i \cdot v_i$
A	0.713	0.535	0.513
B	0.721	0.557	0.518
C	0.746	0.556	0.500
D	0.762	0.561	0.516

months in all aquifers to fully recharge the 7 Mm^3 (6 kaf). To minimize the duration to fill all aquifers with a supply of 7 Mm^3/mon (6 kaf/mon), recharge each aquifer (Table 3, column 6) in proportion to the water needed to fill each aquifer (space available/fractional recovery). Aquifer D took the most water (987 Mm^3/0.92=1,073 Mm^3), while aquifer B took the least (247 Mm^3/0.93=266 Mm^3). 351 months were required to fill all aquifers and reflects large unfilled capacities.

Example No. 2 (Multiobjective Accessibility)

We demonstrate solutions to maximize accessibility using the aquifer portfolio from example no. 1. However, several parameter values were changed so recharge and withdrawal decisions could be examined simultaneously and the availability and recharge constraints were not initially binding (Table 5). The accessibility program was solved 20 times for values of tradeoff coefficient d ranging from 0.1 to 100 Mm^3/mon^2.

For all values of d, solutions converged to one of two solutions. A tipping point between the two solutions was seen at d=1.5 Mm^3 mon^{-2} (Fig. 2). The corner solution that maximized the instantaneous withdrawal rate (Table 6, columns 2 and 3) verified the analytical solution derived for that case [Eqs. (41) and (43)]. The corner solution that maximized duration of withdrawals (Table 6, columns 4 and 5) resembles the analytical solution [Eq. (46)]. However, aquifer B is also recharged because the pumping rate for aquifer A was constrained.

When the tradeoff coefficient was less than 1.5 Mm^3/mon^2, water was withdrawn from each aquifer at maximum pumping rates (Table 6, column 2). Water was recharged to aquifers A, C, and D in proportion to the pumping rates and initial storage (Table 6, column 3). No water was recharged to aquifer B because it had the smallest pumping rate. Aquifer B's preexisting storage

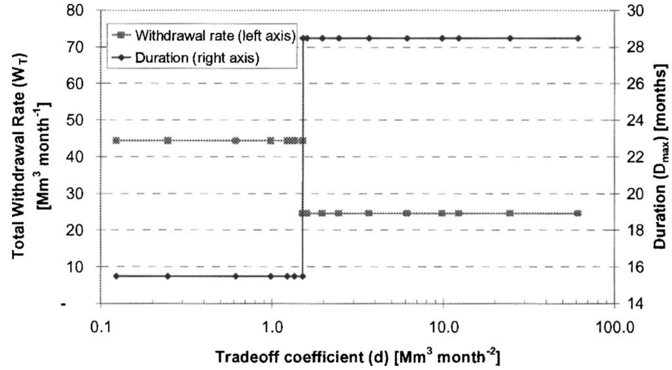

Fig. 2. Tipping point between accessibility solutions that maximize total instantaneous withdrawal rate and duration of withdrawals

could sustain its maximum pumping rate longer than the other aquifers (16.7 periods). Recharges to aquifers A, C, and D allowed the program to sustain the maximum withdrawal rate of 44 Mm^3/mon (36 kaf/mon) for 15.5 months.

For tradeoff coefficient values larger than 1.5 Mm^3/mon^2, recharge was limited to aquifers A and B (Table 6, column 4)—aquifers with the largest and second-largest fractional recoveries. Without limits on pumping rates, the nonlinear program solution would direct all recharge to aquifer A. However, the maximum pumping rate for aquifer A constrained withdrawal to 9 Mm^3/mon (7 kaf/mon), so excess water was recharged to aquifer B. Withdrawals were then made in proportion to the water stored in each aquifer (Table 6, column 4). Aquifer A had the largest withdrawal rate because it had the most stored water. Aquifer B had the second largest withdrawal rate. Aquifers C and D had smaller and identical withdrawal rates, because both aquifers started with 123 Mm^3 (100 kaf) of recoverable storage and no recharge was made to either aquifer. Total withdrawals met the target rate of 25 Mm^3/mon (20 kaf/mon). From the recharges and withdrawals, the program could sustain withdrawals for 28.5 months.

Fig. 2 shows a discontinuous tipping point between the corner solutions, because the objective function is linear with respect to both the withdrawal rate and the duration. Plotting the objective function value against the tradeoff coefficient for several different solutions (including the two corner solutions presented in Table 6) also identifies the tipping point (Fig. 3). For values of d much larger or smaller than 1.5 Mm^3/mon^2, a smooth tradeoff exists between the corner and intermediary solutions. However, for values of d near the tipping point, both corner solutions become superior to the intermediate solutions.

Table 5. Additional Parameter Values for Example No. 2

Parameter	Value
Water available to recharge, Q_S, Mm^3	247
Recharge period, t, mon	200
Initial storage in aquifer A, S_{o1}, Mm^3	99
Initial storages in aquifer B, C, D, $S_{o2,3,4}$, Mm^3	123
Expected availability, $\bar{a}_i$, fraction	1.0
Standard deviation of availability, σ_i, fraction	0
Target water delivery rate, W_T, Mm^3/mon	25
Target reliability, α, fraction	0.5
Standard normal deviate for reliability, Z_α, fraction	0.676
Tradeoff coefficient, d, Mm^3 mon^{-2}	0.1–100

Table 6. Two Numerical Solutions to Accessibility Program in Example No. 2

	Corner solution that maximizes withdrawal rate (d<1.5 Mm^3 mon^{-2})		Corner solution that maximizes withdrawal duration (d>1.5 Mm^3 mon^{-2})	
Aquifer	Withdrawals, W_i^* (Mm^3/mon)	Recharges, Q_i^* (Mm^3)	Withdrawals, W_i^* (Mm^3/mon)	Recharges, Q_i^* (Mm^3)
A	8.6	37	8.6	153
B	7.4	0.0	7.4	93
C	10	33	4.3	0.0
D	19	177	4.3	0.0

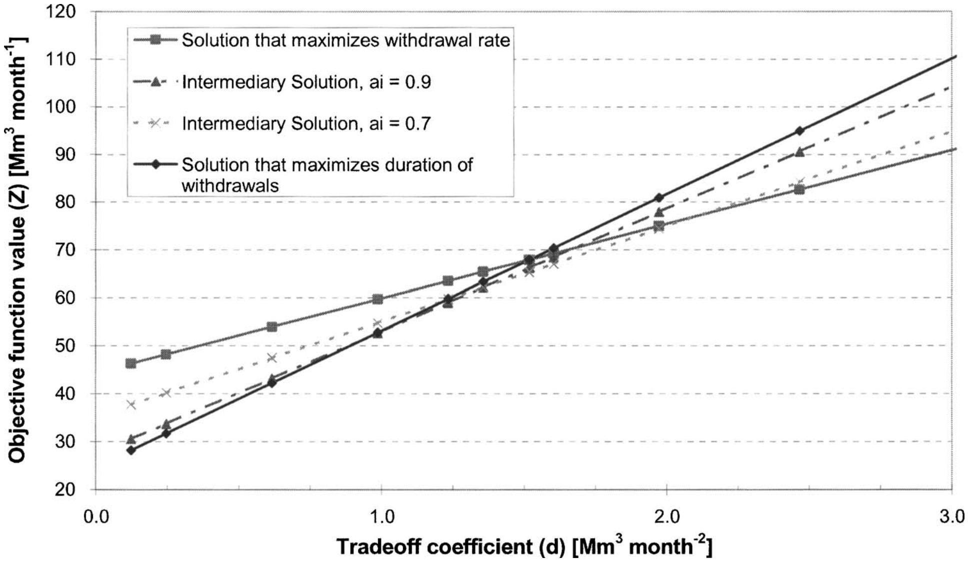

Fig. 3. Accessibility program objective function value plotted versus tradeoff coefficient for four accessibility solutions; intermediate solutions represent cases where expected future availabilities were decreased

The intermediate solutions in Fig. 3 represent duration-maximizing solutions when the program was solved with expected availabilities, $\bar{a}_i$, further constrained to 0.9 and 0.7. Lowering the expected availability raises the expected withdrawal rate required to meet the target delivery. Raising the withdrawal rate lowers the duration. Thus, varying expected availabilities in chance constraint Eq. (40) illustrates a tradeoff between the two corner solutions (Fig. 4). Square markers indicate the corner solutions presented in Table 6 (mean expected aquifer availability $\bar{a}=1.0$ for all aquifers). Other points in Fig. 6 show durations and total withdrawal rates when the program was solved for different expected availabilities ($\bar{a}=0.9$, 0.8. 0.7, 0.6, and 0.57). Each point represents a duration-maximizing solution ($d > 1.5$ Mm^3 mon^{-2}) where each aquifer was assigned the same mean expected availability ($\bar{a}_1=\bar{a}_2=\bar{a}_3=\bar{a}_4$).

As expected availabilities decreased, larger withdrawal rates were required to meet the desired target withdrawal rate (results not shown). Less water was recharged to aquifers A and B and more water was recharged to aquifer D. Water was only recharged to aquifer C when expected availability was less than 0.6. Total pumping rate increases with the largest increases in withdrawals from aquifer D. Aquifer A sustained a maximum pumping rate of 9 Mm^3/mon (7 kaf/mon), and aquifer B reached a maximum pumping rate of 7 Mm^3/mon (6 kaf/mon) for availabilities less than 1.0. As expected availabilities were decreased, optimal recharges and withdrawals approached the solution for maximizing the withdrawal rate. No feasible solutions existed for $\bar{a} < 0.55$ because the program could not increase the total withdrawal rate above a maximum pumping capacity of 44 Mm^3/mon (36 kaf/mon). Recharges to and withdrawals from aquifer D were made to increase the expected reliability of withdrawn water. Because aquifer D had a lower fractional recovery than aquifers A and B, withdrawals from the aquifer could be sustained for a shorter time. This relationship is represented by the negatively sloping tradeoff curve in Fig. 4. Despite the tradeoff, recharges to and maximum pumping rates from aquifer A were sustained over all availabilities, identifying aquifers with large pumping capacities and high fractional recoveries as the most suitable for withdrawals when an aquifer manager seeks to maximize accessibility to stored water (as either duration or rate of withdrawal).

Conclusions

Six operating rules were derived to suggest short-term aquifer recharge and withdrawal decisions to meet financial, duration, and accessibility objectives. The rules are as follows.

Financial Objectives

1. To minimize the cost of withdrawing water, withdraw water first from aquifers with the smallest overall extraction costs.
2. To maximize the future expected financial value, recharge water to the aquifers with the largest discounted net financial value of recoverable water.

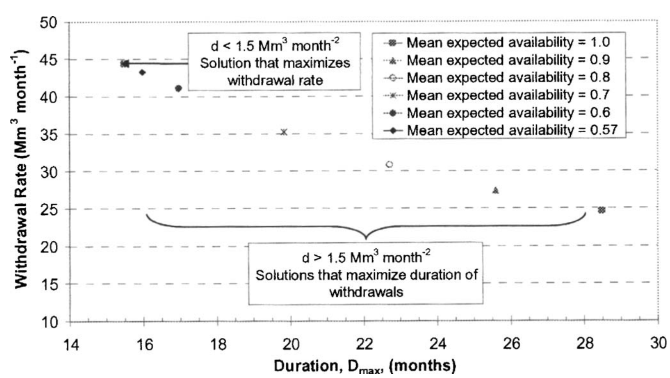

Fig. 4. Tradeoff between duration and instantaneous withdrawal rate by varying aquifer availabilities

Duration Objectives

3. To maximize withdrawal duration, withdraw in proportion to initial storage.
4. To minimize the time to recharge a small quantity of surface water, recharge in proportion to maximum recharge rate.
5. To minimize the duration to fill all aquifers, recharge in proportion to unfilled storage capacity weighted by expected water losses.

Accessibility Objective

6. To maximize flexibility to meet both large future withdrawal rates and durations of withdrawals, preferentially recharge water to aquifers with both high maximum pumping capacities and large fractional recoveries (small storage losses).

The operating rules are based on lumped aquifer characteristics, exogenously determined total recharge or withdrawal amounts, and represent situations where constraints do not bind. The formulations were readily extended and solved numerically to include constraints for more complex systems such as withdrawal capacities, recharge capacities, and uncertainties concerning future availability of banked water for later withdrawal. Further extensions might include aquifers operated in conjunction with a surface water reservoir, multiple reservoirs, and uncertain surface water volumes available for recharge. Additional modifications are required to derive temporal, economic, dynamic, or broader, multiobjective operating rules.

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Tim Blair introduced the aquifer recharge problem. The writers also thank Richard Howitt and Beth Faber for their comments and suggestions.

Notation

The following symbols are used in this paper:

a_i = random variable representing future availability to extract water from aquifer i, fraction;
$\bar{a}_i$ = mean expected availability of aquifer i, fraction;
b = discount factor, unitless;
c_i = sum of unit costs to extract, pump, treat, convey, and cover institutional, legal, and transactional expenses to gain access to aquifer i, \$ volume^{-1};
D_i = duration of withdrawal from aquifer i, time;
$D_{\max}$ = overall duration of withdrawal program, time;
d = tradeoff objective coefficient, volume time^{-2};
FD_i = duration to fill aquifer i, time;
$FD_{\min}$ = overall fill duration for recharge program, time;
i = aquifer index, $1 \ldots n$;
K_i = unfilled, remaining storage capacity of aquifer i, volume;
$p_{\max i}$ = maximum extraction pumping capacity for aquifer i;
Q_i = decision on amount to recharge into aquifer i, volume;
Q_i^* = optimal amount to recharge to aquifer i, volume;
R_i = steady recharge rate into aquifer i, volume time^{-1};
R_i^* = optimal recharge rate into aquifer i, volume time^{-1};
R_S = steady surface water available for recharge in each period, volume time^{-1};
RD_i = recharge duration for aquifer i, time;
$RD_{\min}$ = overall duration for recharge program, time;
$r_{\max i}$ = maximum recharge capacity for aquifer i, volume time^{-1};
rc_i = unit cost to recharge aquifer i, volume time^{-1};
S_{oi} = initial storage in aquifer i available for extraction, volume;
t = predetermined duration of withdrawal/recharge, time;
u_i = unit use value of water extracted from aquifer i, \$ volume^{-1};
v_i = discounted, net financial value of storing water in aquifer i, \$ volume^{-1};
W_i = decision on withdrawal rate from aquifer i, volume time^{-1};
W_i^* = optimal withdrawal rate from aquifer i, volume time^{-1};
W_R = total expected rate of water withdrawal from all aquifers, volume time^{-1};
W_T = total target delivery rate, volume time^{-1};
WD_i = withdrawal duration from aquifer i, time;
$WD_{\max}$ = overall withdrawal duration for program, time;
Z_α = standard normal deviate for probability α, unitless;
α = reliability that water should be available, fraction;
β = required fraction of recharged water to be available in future, unitless;
λ_i = expected fraction of recharge that will be recoverable for extraction, unitless; and
σ_i = standard deviation of expected availability, fraction.

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