Representing Energy Price Variability in Long- and Medium-term Hydropower Optimization

Marcelo A. Olivares[1], Jay R. Lund[2]

[1]Department of Civil Engineering, University of Chile, Santiago, Chile
[2]Department of Civil and Environmental Engineering, University of California, Davis, USA

Journal of Water Resources Planning and Management, 2012, in press

ABSTRACT
Representing peak and off-peak energy prices is often difficult in hydropower modeling because the time scale of price variability (hours or less) is much shorter than that needed for many operations planning models (days to months). This work extends and examines the reliability of an existing approximate method to incorporate hourly energy price information into revenue functions used in hydropower reservoir optimization models with larger time steps (weekly, monthly, etc). The method assumes constant head, an exogenously known frequency distribution for hourly prices during each modeled time period (day, week, month) and a revenue-maximizing operational strategy that allocates hydropower releases in order of decreasing hourly price. The method is extended to the case with minimum instream flows requirements. The reliability of the method was tested for the cases with and without minimum instream flow requirements. Revenue estimates for a hypothetical hydropower site were compared with the exact optimal revenue from solving the hourly optimization problem within one week, showing less than 1% error using a finely discretized price frequency curve.

INTRODUCTION
Optimization models are commonly used for hydropower reservoir operations planning, with time horizons ranging from weeks to years and time steps ranging from days to months. In this modeling, it is often important to represent energy price variability (peak vs. off-peak prices) which occurs on short time scales (minutes to hours) in models using longer (daily to monthly) time steps. These models are almost always based on maximization of hydropower revenues, with essentially fixed operational costs for hydropower production (Labadie, 2004; Pereira & Pinto, 1991; Kelman et al., 1990; Faber & Stedinger, 2001; Fleten & Kristoffersen, 2008). Some models also include coordination of hydropower and thermal power assets (Jacobs and Schultz, 2002). The output of each hydropower plant is usually small relative to the overall energy market and so typically has almost no effect on energy prices, allowing energy prices to be considered exogenous from the perspective of individual hydropower facilities.

Because energy prices usually vary on short (e.g., hourly) time scales, much hydropower modeling on longer (daily to monthly) time steps can be misleading if it relies on a single representative price for each time step (e.g. week, month), especially if an average energy price is used. Using an average price in most cases underestimates revenues for a given level of power production, because reservoir releases will be allocated preferentially for generation during hours when energy prices are high (Bushnell, 2003). In hydrothermal systems, peaking operations are most common for hydropower plants. However, as hydropower's share of total system installed capacity increases, some hydropower plants can provide base load instead. The use of an average energy price disregards the usual peaking nature of most hydropower operations. Efforts to improve this simplification have involved the use of block pricing schemes (peak and off-peak), with an upper bound on the number of hours of generation at peak energy price (e.g. Grygier and Stedinger, 1985; Trezos and Yeh, 1987). California's PG&E scheduling system SOCRATES (Jacobs *et al*. 1995), for example, divides each weekly or monthly period into 4 to 12 sub-periods, which distinguishes between peak and off-peak generation during weekdays and weekends.

Research on hydropower reservoir operations has focused on developing efficient algorithms for multi-reservoir systems (e.g. Turgeon and Charbonneau, 1998), and incorporating hydrologic uncertainty (Kelman *et al*., 1990; Tejada-Guibert *et al*., 1995). Little consideration has been given to short-term variability of energy prices, which drive operational decisions in decentralized energy systems. In contrast to this simple common energy scheduling problem, the energy systems literature focuses on electricity markets modeling and electricity price forecasting (Nogales *et al.*, 2002; Pritchard *et al*., 2005; Fleten, 2008; Scott and Read, 1996; Bushell, 2003; Fleten, 2007; Hobbs and Pang, 2007). On the other hand, Tesser *et al*. (2009) show a method for solving the medium-term problem using the load duration curve and relating it to average prices.

Recent studies on hydropower operations in California (Vicuna *et al*., 2008; Madani & Lund, 2009) have recognized the need to incorporate short-term price variability information into long-term models. In particular, hourly energy price variability often emerges from the time frame of market clearing prices developed by an Independent System Operator (ISO). Better representation of shorter-time variations of prices and operations in longer-time-step models provides great improvements for model solution times and post-processing, and improve problem representation where option values depend to a great extent on price variability.

Madani and Lund (2009) proposed a method to incorporate hourly price variability information into revenue functions at coarser (e.g. weekly, monthly) modeling time steps for hydropower systems with constant head. The method assumes the reservoir operator knows the frequency distribution of hourly prices for each coarse modeled time period, can forecast energy prices one hour ahead for operating purposes, and operates primarily for hydropower revenue maximization. This paper extends the work by Madani and Lund (2009) to hydropower systems with minimum instream flows requirements and tests the reliability of the method compared to exact solution of the short-term, perfect-foresight optimization problem.

Next, we discuss the method proposed by Madani and Lund (2009) and explicitly develop the relationship between revenues and total volume of water allocated to the period. Then, we provide a discussion on the constant-head assumption and the error it would introduce. The method is then extended to the case with minimum instream flow requirements. The reliability of the approximation is assessed for a hypothetical hydropower site by comparison with the exact optimal revenue from solving the hourly optimization problem with a weekly horizon. Finally, we present conclusions and possible extensions to this work.

PRICE VARIABILITY AND OPTIMAL OPERATING RULES

A common objective for hydropower operations planning is to maximize the total revenue from generation during a time horizon T , typically discretized into smaller decision periods of length ΔT . Time horizons of several months with daily to monthly decisions are common. As proposed by Madani and Lund (2009), hourly price variability within decision periods can be represented by a price duration curve relating the percent of a time period that energy price exceeds various levels. This requires only that releases within a decision period be allocated in order of decreasing hourly energy price, requiring only hourly forecasts of energy prices, given the full price distribution.

Following the approach by Madani and Lund (2009), the total revenue from energy sales during a decision period of length ΔT (e.g. one week) can be given by:

$$B = G_{cap} \cdot \int_0^g P(f)df \qquad (1)$$

where $P(f)$ is the price corresponding to a frequency f from the price duration curve, $Gcap$ is the plants' weekly energy generation capacity, and $g = G/G_{cap}$ is the proportion of generation capacity used during the week.

Discretizing Eq. (1) and defining $n_g = g/\Delta f$ as the number of discrete sub-intervals we obtain:

$$B = \frac{G}{n_g}\sum_{i=1}^{n_g} P_i \qquad (2)$$

Defining $\overline{P}(g)$ as the average of all prices exceeding $P(g)$, Eq. (2) becomes :

$$B = G \cdot \overline{P}(g) \qquad (3)$$

Eq. (3) represents the revenues obtained from producing an amount of energy G given a maximum generation capacity G_{cap}.

Assuming constant head, energy generation can be calculated as

$$G = \varepsilon \cdot \gamma \cdot Q \cdot h \cdot \Delta T = \varepsilon \cdot \gamma \cdot V \cdot h$$

where ε is generation efficiency and γ is the unit weight of water. Q and h are the release flow and head, respectively. V represents the total volume of water allocated during the period (i.e. week).

Plant efficiency ε is affected by discharge and, for plants with several turbines, by how generation is allocated among units. Given our assumption of operation at turbine capacity, discharge is fixed. Also, we assume a total plant discharge will always be allocated among units in the same manner. So, efficiency can be considered constant.

Defining C as the plant's flow capacity, generation capacity is given by

$$G_{cap} = \varepsilon \cdot \gamma \cdot C \cdot h \cdot \Delta T$$

Therefore, the proportion of generation capacity used can be rewritten as:

$$g = \frac{G}{G_{cap}} = \frac{\varepsilon \cdot \gamma \cdot V \cdot h}{\varepsilon \cdot \gamma \cdot C \cdot h \cdot \Delta T} = \frac{V}{C \cdot \Delta T} = f_V \qquad (4)$$

Where f_V represents the proportion of hours of operation (at full capacity) over the period ΔT. Given $P(f)$ and a good 1-hour lead price forecast (or suitable approximations), the optimal operating rule is to generate at turbine capacity during all hours when the price $P \geq P(f_V)$.

This allows actual revenue per unit output to depend on the plant's total generation or proportion of hours generated. The operator cannot influence the market price, so hourly prices are exogenous to the optimization. The realized average price depends on the portion of the price duration curve covered by operations, beginning with the highest-priced hours.

Introducing the dependence on f_V, C, and h explicitly, Eq. (3) can be expressed as:

$$B(C, f_V, h) = E_V \cdot \overline{P}(f_V) \qquad (5)$$

where E_V is the total energy that can be generated with a volume V of releases at constant head h and $\overline{P}(f_V)$ is the average of all prices exceeding $P(f_V)$.

As described above, the method proposed by Madani and Lund (2009) assumes constant head. Here we analyze the implications of this assumption. A relatively constant head on the turbine is common for many reservoirs when V is small or when most head is produced by a long penstock, as is common in mountain regions.

In principle, when total head depends markedly on reservoir storage level, the method by Madani and Lund (2009) cannot be used. Estimation of optimal revenue becomes then more complicated, because each hour of operation generates a different quantity of energy depending on the reservoir level during that period. However, to a first-order approximation the operating rule remains valid, only the estimation of total revenue becomes more complicated as it depends on the price-storage pairs realized during operations.
To be completely accurate, the "head effect" creates nonlinearities in the objective function that modify the optimal strategy somewhat from our simple operating rule, because a release decision made at time t will affect storage, and therefore head and revenue at all subsequent periods until an upper or lower storage bound is reached. The resulting non-separability of the objective function makes the revenue optimization problem harder, which is why many hydropower scheduling systems assume a constant head (Jacobs *et al*. 1995).

The head sequence is determined by initial storage conditions and the balance between inflows and outflows. Thus, reservoir storage, and therefore head, will increase or decrease depending on both net reservoir inflows and release decisions during the operational period. The hourly storage sequence is determined by water balance:

$$S_{i+1} = S_i + \left(I_i - Q_i\right) \cdot \Delta t - e_t \quad (6)$$

where I_t and e_t are the inflow to and evaporation from the reservoir during the ith hour of the week, respectively. Assuming the operational scheme previously described, during hours of generation (6) becomes: $S_{i+1} = S_i + \left(I_i - C\right) \cdot \Delta t - e_t$. During hours without generation (6) becomes: $S_{i+1} = S_i + I_i \cdot \Delta t - e_t$. The size of Δt must be small enough to avoid significant changes in head.

Therefore, this approximation's accuracy will depend on the particular conditions of the hydropower system under study. For example, in California's Sierra Nevada, spring has large inflows from snowmelt and moderate evaporation, so reservoirs tend to refill overall, increasing head over most time steps. During summer, evaporation increases and inflows decrease, so refill during non-generation hours is small and storage and heads decrease over most time steps.

On the other hand, as shown in Fig.1, in the California market the sequence of price values during generation hours tends to be periodic, for any capacity utilization level. This implies that higher contributions to total revenue coincide with the earliest releases during the drawdown season and coincide with the later releases during the refill season. The sequences in Fig. 1 are obtained from the truncated total time series obtained for each generation frequency level. In this particular series, lower peak prices occur at the beginning of the sequence. This is explained because the considered 7-day sequence started on a Saturday, which along with Sunday is an off-peak period. Other than that, no marked bias on price levels seems to exist during weekdays.
However, a simple estimate can be obtained using Eq. (5) with the average between the initial and final head during the period, which requires only the initial and final storage for each period ΔT. Final storage can be calculated from the initial storage and net inflow over the entire period. The error of this approach will be quantified for typical reservoir conditions as part of the numerical example.

ENVIRONMENTAL CONSTRAINTS ON RELEASES

Often, environmental constraints take the form of minimum instream flows (MIFs) and maximum ramping rates (MRRs). Such restrictions can change the optimal operational strategy from the simple policies described above. With a MIF alone, the proposed approach can be slightly modified so the total volume V cannot be freely allocated by the operator. However, the described procedure remains valid if the "effective" volume available for discretionary release is considered. Discretionary releases would be the volume that can be allocated by the operator during hours of high price.

Given a total volume V available for the entire period and a minimum flow $Q_{\min}$, the percentage of the time f_{EFF} that operation at full capacity can take place is given by:

$$V = V_{EFF} + V_{MIN} = C \cdot \Delta T \cdot f_{EFF} + Q_{\min} \cdot \Delta T \cdot (1 - f_{EFF}) \quad (7)$$

Solving for f_{EFF}, with $Q_{\min} = \alpha \cdot C$ we obtain: $f_{EFF} = \dfrac{(f_V - \alpha)}{(1-\alpha)}$ (8)

Eq. (9) shows that total revenue can be separated in two terms: i) the revenue that comes from operation at peak hours and ii) the revenue associated to hours of operation at MIF.

$$B(C, f_V, h, \alpha) = \varepsilon \cdot \gamma \cdot h \cdot \Delta t \cdot \left\{ C \cdot \sum_{i \in I(V_{eff})} P_i + \alpha \cdot C \cdot \sum_{i \notin I(V_{eff})} P_i \right\} \tag{9}$$

where $I(V_{eff})$ represents the set of hours when energy prices are equal or exceed $P\left(f_{Veff}\right)$. Defining the excess turbine capacity over the MIF as $Q_{ex} = C \cdot (1-\alpha)$, after some algebra Eq. (9) can be rewritten in terms of the revenue associated to the MIF (which is at least released every hour) and the extra revenue that comes from additional release (above MIF) in some hours:

$$B(C, f_V, h, \alpha) = \varepsilon \cdot \gamma \cdot h \cdot \Delta t \cdot \left\{ C \cdot (1-\alpha) \cdot \sum_{i \in I(V_{eff})} P_i + \alpha \cdot C \cdot \sum_{i=1}^{N} P_i \right\}$$

In terms of the average prices:

$$B(C, f_V, h, \alpha) = \varepsilon \cdot \gamma \cdot h \cdot \Delta t \cdot \left(C \cdot (1-\alpha) \cdot N \cdot f_{EFF} \cdot \overline{P}_{EFF} + \alpha \cdot C \cdot N \cdot \overline{P} \right) \tag{10}$$

where $\overline{P}_{EFF} = \overline{P}(f_{EFF})$ and $\overline{P}$ is the average price for the entire period of interest.

Substituting (8) in (10) we obtain:

$$B(C, f_V, h, \alpha) = \varepsilon \cdot \gamma \cdot h \cdot C \cdot \Delta t \cdot N \cdot \left(\overline{P}_{EFF} \cdot (f_V - \alpha) + \overline{P} \cdot \alpha \right) \tag{11}$$

RELIABILITY OF THE PROPOSED METHOD

In this section we test the reliability of the approach proposed by Madani & Lund (2009) to approximate the exact solution of the intra-period optimization problem for a hypothetical hydropower plant, using hydrological and energy price data from California. A precise representation of the intra-decision period problem would involve maximizing total revenue subject to operational constraints on release flows at each hour. The problem also would be subject to an initial storage (S_{ini}) and total water availability or release target (V) for the operational period ΔT. The release target for the intraperiod problem is a decision variable in the longer-term multiperiod problem. The proposed method approximates the solution to the following deterministic intraperiod (smaller time-step) optimization problem with a known price sequence and variable head:

$$\begin{aligned}
&\underset{Q_i}{Max} \quad B(P_i, Q_i, h_i) = \sum_{i=1}^{N} P_i \cdot \varepsilon \cdot \gamma \cdot Q_i \cdot h_i \cdot \Delta t \\
&s.t. \quad \sum_{i=1}^{N} (Q_i - I_i) \cdot \Delta t = V \\
&\qquad h_i = h(S_i) \quad i = 1, \ldots, N \\
&\qquad S_1 = S_{ini} \\
&\qquad Q_i \geq 0
\end{aligned}$$

where I_i and S_i are the net inflow and the reservoir storage, respectively in period i; N is the number of time steps in the intraperiod problem, Δt is the time step within the intraperiod problem, and $h(S)$ represents the head-storage curve of the reservoir.

A numerical example illustrates the reliability of the method. Revenue estimates obtained with the proposed method are compared with the exact solution to the intraperiod problem, with an hourly time step. The proposed method is compared against the solution of an intraperiod problem with two versions of two-block price approximations. The exact intraperiod problem assumes a know price sequence, whereas the proposed method employs a price frequency curve and assumes a 1-hour ahead price forecast is available to the operator.

The proposed method is applied to a small reservoir with a capacity of 92.5 million m^3 and the storage-head curve shown in Fig.2. The curve can be approximated analytically. At very low storage values the relationship is linear. For the rest of the storage range, the curve can be approximated by a quadratic polynomial. The power house is an additional 15 m below the lowest reservoir level. Installed capacity is about 25 MW.

Two weeks are considered to test the method: one in late winter (10^{th} week of the calendar year) and the other in summer (35^{th} week of the year). Table 1 summarizes the information used in this example. Based on historical records for the California Sierra Nevada, for the summer week the net inflow (inflow minus losses) is set to zero, so losses, mainly evaporation, offset small natural inflows to the reservoir. Initial storages are about 13% and 47% of storage capacity for weeks 10 and 35, respectively.

Hourly energy prices for the 10^{th} and 35^{th} week of the year 1999 were obtained from the California PX data (available at http://www.ucei.berkeley.edu/). The corresponding raw Price Duration (PD) and operating Moving Average (MA) curves for each week are shown in Fig. 3 at 5% exceedance frequency intervals. The "exact" MA for each percentile was calculated from the actual sample of prices. The "approximated" MA is obtained directly from the PD curve by averaging the corresponding price percentiles at 5% intervals. As seen in Fig. 3, this approximation can result in errors of about 10% of the exact MA in week 35. Better integration of the price curves substantially reduces error. The error in the price MA will propagate by Eq. (5), resulting in more error in the revenue for plants with larger installed capacity.

Error due to average head assumption

We now examine errors of the method for the case with variable head using the average between initial and final head in equation (5). The error in revenue estimation can be illustrated by an example calculation under reservoir drawdown and refill conditions. The revenue obtained with average head can be contrasted against the exact solution to the nonlinear optimization problem which accounts for head changes.

A drawdown condition is represented with zero net inflow to the reservoir. The filling condition examined has the same average between initial and final head as in the drawdown condition. This allows for comparison of different storage sequences without worrying about the average head effect. Our method does not distinguish between drawdown and filling conditions as long as the average head is the same.

Results of this comparison, for energy prices in week 35, appear in Fig. 4. The net inflow defining the filling condition is set to 42 m^3/s, 50% more than turbine flow capacity. This ensures a filling condition even when total release equals turbine flow capacity. The average head will be set at 32 m, corresponding to a storage of 50% of reservoir capacity. Optimal total weekly revenues are not very different under the two conditions. The proposed approximation introduces little error over the entire range of total releases for these conditions of variable head.

Approximation without minimum instream flows

The purpose is to estimate the optimal revenue during each week as a function of the initial storage, net inflow and total volume of water allocated for generation during the week. Storage information is needed for head calculation.

For both example weeks, the exact nonlinear optimization problem was solved using the CONOPT (GAMS, 2010b) solver in GAMS (GAMS, 2010a) assuming perfect foresight of energy price and net inflow to the reservoir. For the proposed approximation, the total revenue for each week was calculated. The total revenues for each method are shown in Fig. 5 for the entire range of weekly release relative to turbine capacity. The proposed method results using the actual sample MA, matches the optimal results almost perfectly. With the approximated MA, the quality of the approximation depends on the week. As shown in Fig.3, for week 10 the approximated MA is nearly exact and revenue approximations nearly coincide. For week 35, errors from approximating the MA are worse. The difference between optimal and estimated revenues increases with the total weekly release, reaching as high as 10%. This is due to the greater price variability during week 35, having a much larger price range than week 10, as shown in the price duration curve in Fig.3. More variable energy prices require more exact integration of the price frequency curve, requiring little additional effort.

Comparison with peak/off-peak price schemes

For comparison, we optimized a two-block (peak/off-peak) price scheme so corresponding revenues have the least squared deviations from the optimal revenues over the range of total release. The design consists in finding the parameters P_{PEAK} , P_{OFF} , and f_{PEAK} as shown in Fig.6. The corresponding optimization problem is

$$\underset{P_{PEAK},P_{OFF},f_{PEAK}}{Min} \sum_{f_V} \left(B_{f_V}^{OPT} - B_{f_V}^{P/O} \right)^2$$

where the revenue calculated with the peak/off-peak structure is given by:

$$B_{f_V}^{P/O} = \begin{cases} \varepsilon \cdot \gamma \cdot \overline{h} \cdot C \cdot N \cdot P_{PEAK} \cdot f_V & f_V \le f_{PEAK} \\ \varepsilon \cdot \gamma \cdot \overline{h} \cdot C \cdot N \cdot \left(P_{PEAK} \cdot f_{PEAK} + P_{OFF} \cdot (f_V - f_{PEAK}) \right) & f_V > f_{PEAK} \end{cases} \tag{12}$$

This problem requires a three-dimensional search. However, a condition can be imposed that relates the peak price P_{PEAK} and the frequency f_V, namely that the peak price matches the moving average price at that frequency, i.e. $P_{PEAK} = \overline{P}\left(f_{PEAK} \right)$. With this condition, the search becomes two-dimensional. The optimal values found for each week are presented in Table 2. For both prices, the corresponding exceedance percentile in the price duration curve is included in parenthesis. For week 10, given the flatness of the duration curve, the obtained percentiles differ somewhat from the common definition of peak/off-peak prices. The percentiles seem higher than expected for both peak and off-peak price. This shows that the common definition used in this kind of pricing scheme approximation does not necessarily result in revenues closest to the actual ones.

The revenues were calculated using Eq. (12) with the parameters from Table 2. Fig. 7 shows how the estimation based on the optimal peak/off-peak scheme compares to the optimal revenue and to the estimation obtained with our method. The results for an alternative two-block pricing, where the peak price is defined as the 5% exceedance percentile, the off-peak as the 50% percentile, and a frequency of 20%, are presented for comparison. The optimal two-block pricing scheme used the exact MA for estimating the peak price for each f_{PEAK}.

The results in Fig.7 show that the new method, when applied with the exact MA, outperforms the peak/off-peak price approximation. Also, the optimal two-block pricing scheme better approximates revenues compared to the common 16 hr on / 8 hr off two-block pricing scheme. The optimal two-block design minimizes the deviation from optimal revenues.

A summary of the relative error results for the case without MIF appears in Table 3. The proposed method has the least error (less than 1%) when applied with the exact MA. The performance of the proposed method with the approximated MA depends on the week. For week 10, where the approximated MA is close to the actual MA, the proposed method still outperforms the two-block pricing scheme. Interestingly, for week 35, when applied with the MA approximated from the price duration curve at 5% intervals, this inexact implementation of the proposed method has more error than the two-block pricing. This is because two-block pricing schemes tend to first underestimate and then overestimate revenues. As seen in Fig.7, for week 35 the optimized peak/off-peak scheme underestimates revenues for total releases under 75% of capacity and overestimates revenues for releases exceeding 75% of turbine capacity. At 75%, the error is almost zero. Therefore, errors for percentiles around 75% are small and then have a small contribution to the relative error. In contrast, as seen in Fig.5, when the proposed method uses an approximated MA, the resulting revenues overestimate those optimal over the entire range of releases. Therefore, since the error is never close to zero, the relative error is higher. Both two-block pricing schemes use the exact MA; where the exact MA estimation is used directly, the proposed method has much less error.

Approximation results with minimum stream flows

In general, MIFs decrease revenues by allocating more releases to hours with lower energy prices. We test our approach to cases with MIFs from 5% to 50% of turbine capacity. The results are presented as the ratio between the exact and approximated weekly revenue. In Fig. 8, each point corresponds to an average over the range of MIF requirements. Results are consistent with those without MIF. When applied with the exact MA, the proposed method approximates the optimal revenues within 1%. When a coarse approximation of the MA is used, errors can reach 7% for a week with high price variability such as week 35.

Fig.9 shows the average ratio of approximated to optimal revenue over total weekly water allocation for different levels of MIF, as a percent of turbine capacity. When the exact MA is used, our method underestimates the revenues by less than 1%. With MA approximated from the price duration curve at 5% intervals, the magnitude of the error varies. Again, the approximation for week 35 can be as large as 9% for small MIFs. In week 10 the proposed method with approximated MA gives errors slightly more than 1%. As the MIF requirement increases, the error of the approximated MA decreases, reaching 3% and 0.5% for weeks 35 and 10, respectively. This can be explained from (12), where the benefit estimation is proportional to a weighted sum of the weekly average price and the MA price for the effective frequency. As the MIF increases, the weight on the MA price decreases and so its error has less influence on revenue estimation.

CONCLUSIONS

This study extends and examines the reliability of a simple method proposed by Madani and Lund (2009) to use hourly price information as a basis for hydropower revenue functions for longer decision periods. The method is based on the hourly price distributions and allows efficient use of hourly peak price information within longer-period scheduling or operational planning models. A key element of the method is the availability of a good estimate of moving average price obtained from a duration curve and perfect price forecasts one hour ahead. The method was extended for the case with environmental constraints in the form of minimum instream flows and applied to the case with storage-dependent head and storage varying over the operational period using the average storage over the time period.

The original and extended methods were applied to a hypothetical example to estimate the weekly revenues for one week in summer and another week in winter in California. With the exact MA, our method has errors less than 1% for both weeks. The exact MA can be closely matched with an approximation based on the duration curve at 1% intervals.

Our approach was compared with the traditional two-block hydropower modeling price structure approach. An optimal peak/off-peak price scheme was designed to minimize deviation from the optimal revenues. This optimized approximation results in relative errors of 2.5% and 4.2%, much worse than the 0.4% and 0.7% obtained with our method using the exact MA. Similar results were obtained for the case with minimum instream flow requirements.

Further extensions to this work might include possible options representation of energy prices for cases where energy and generation capacity options are traded as part of the electricity market.

Acknowledgements

The authors thank Jan Grigier, Tung Van Do, and Charles Howard for comments to an early version of this manuscript. Funding was provided by the Resources Legacy Fund through the UC Davis Center for Watershed Sciences.

REFERENCES

Bushnell, J.: A mixed-complementarity model of hydrothermal electricity competition in the Western. United States. Operations Research, 51, 80–93 (2003)

Faber, B. A. and J. R. Stedinger (2001). "Reservoir optimization using sampling SDP with ensemble streamflow prediction (ESP) forecasts." Journal of Hydrology 249(1-4): 113-133.

Fleten, S.E. and T.K. Kristoffersen (2007). "Stochastic programming for optimizing bidding strategies of a Nordic hydropower producer", European Journal of Operational Research 181(2): 916-928.

Fleten, S. E. and T. K. Kristoffersen (2008). "Short-term hydropower production planning by stochastic programming." Computers & Operations Research **35**(8): 2656-2671.

GAMS Development Corporation (2010a). “GAMS- A user’s guide”. Washington, DC, USA. Tutorial by Richard E. Rosenthal. (http://www.gams.com/dd/docs/bigdocs/GAMSUsersGuide.pdf)

GAMS Development Corporation (2010b). “GAMS- The solver manuals”. Washington, DC, USA. (http://www.gams.com/dd/docs/solvers/allsolvers.pdf)

Grygier, J. C. and J. R. Stedinger (1985). "Algorithms for Optimizing Hydropower System Operation." Water Resources Research **21**(1): 1-10.

Hobbs, B. F. and J. S. Pang (2007). "Nash-Cournot equilibria in electric power markets with piecewise linear demand functions and joint constraints." Operations Research **55**(1): 113-127.

Jacobs, J., G. Freeman, J. Grygier, D. Morton, G. Schultz, K. Staschus and J. Stedinger. (1995). "Socrates - a System for Scheduling Hydroelectric Generation under Uncertainty." Annals of Operations Research **59**: 99-133.

Jacobs, J. M. and G. L. Schultz (2002). "Opportunities for Stochastic and Probabilistic Modeling in the Deregulated Electricity Industry. " Decision Making under Uncertainty. Energy and Power. C. Greengard and A. Ruszczynski (eds.), Springer.

Kelman, J., J. R. Stedinger, L. A. Cooper, E. Hsu, and S. Yuan . (1990). "Sampling Stochastic Dynamic-Programming Applied to Reservoir Operation." Water Resources Research **26**(3): 447-454.

Labadie, J. W. (2004). "Optimal operation of multireservoir systems: State-of-the-art review." Journal of Water Resources Planning and Management-Asce 130(2): 93-111.

Madani, K. and J.R. Lund (2009), “Modeling California's High-Elevation Hydropower Systems in Energy Units.” Water Resources Research, 45, W09413, doi:10.1029/2008WR007206.

Nogales, F. J., J. Contreras, A. J. Conejo, and R. Espínola. (2002). "Forecasting next-day electricity prices by time series models." IEEE Transactions on Power Systems **17**(2): 342-348.

Olivares, M.A. (2008), “Optimal Hydropower Reservoir Operation with Environmental Requirements,” PhD dissertation in Civil and Environmental Engineering, University of California – Davis, Davis, CA.

Pereira, M. V. F. and L. Pinto (1985). "Stochastic Optimization of a Multireservoir Hydroelectric System - a Decomposition Approach." Water Resources Research **21**(6): 779-792.

Pritchard, G., A. B. Philpott, and P.J. Neame. (2005). "Hydroelectric reservoir optimization in a pool market." Mathematical Programming **103**(3): 445-461.

Scott, T. J. and E. G. Read (1996). "Modelling Hydro Reservoir Operation in a Deregulated Electricity Market." International Transactions in Operational Research **3**(3): 243.

Tejada-Guibert, J. A., S. A. Johnson, and J. R. Stedinger (1995). "The Value of Hydrologic Information in Stochastic Dynamic-Programming Models of a Multireservoir System." Water Resources Research **31**(10): 2571-2579.

Tesser, M., A. Pagès, and N. Nabona (2009). "An Oligopoly Model for Medium-Term Power Planning in a Liberalized Electricity Market." IEEE Transactions on Power Systems **24**(1): 67-77.

Trezos, T. and W. W. G. Yeh (1987). "Use of Stochastic Dynamic-Programming for Reservoir Management." Water Resources Research **23**(6): 983-996.

Turgeon, A. and R. Charbonneau (1998). "An aggregation-disaggregation approach to long-term reservoir management." Water Resources Research **34**(12): 3585-3594.

Vicuna, S., R. Leonardson, M. W. Hanemann, L. L. Dale, and J. A. Dracup . (2008). "Climate change impacts on high elevation hydropower generation in California's Sierra Nevada: a case study in the Upper American River." Climatic Change **87**: S123-S137.

Figure Captions

Figure 1: Example Price Sequence during Hours of Operation for Different Capacity Utilization Levels f (No release occurs in hours between price bars)
Figure 2: Storage-head curve
Figure 3: Price information year 1999 (Cal-ISO)
Figure 4: Error induced by head effect under filling and drawdown conditions
Figure 5: Effect of MA calculation on estimated revenues without MIF
Figure 6: Price Duration Curve and 2-Block Price Approximation
Figure 7: Comparison between the proposed method and two-block pricing approximation
Figure 8: Ratio of approximated to optimal revenue (Average over MIF values)
Figure 9: Ratio of approximated to optimal revenue (Average over Total Weekly Releases)

Tables

Table 1: Summary of data

Storage Capacity (million m^3)	92.5	
Turbine Flow Capacity (m^3/s)	28.3	
Max. Release (m^3/week)	17.1	
	Week 10	Week 35
Initial Storage (million m^3)	37	61.6
Net Inflow (m^3/s)	5.1	0
Average Price ($/MWh)	17.3	41.6

Table 2: Optimal peak/off-peak price scheme ($/MWH and corresponding % of generation capacity)

Week	f_{PEAK}	P_{PEAK}	P_{OFF}
10	54%	21.02 (23%)	14.78 (73%)
35	16%	98.10 (4%)	34.70 (41%)

Table 3: Summary Results without MIF

Method	**Relative error**	
	Week 10	**Week 35***
Proposed (exact MA)	0.4%	0.7%
Proposed (approx. MA)	0.9%	9.2%
Optimal Peak/Off-peak	2.5%	4.2%
Common Peak/Off-peak	5.8%	6.9%

* Week 35 has more variable prices.