

# Modeling irrigated agricultural production and water use decisions under water supply uncertainty

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[1] Farmers make joint water and land use decisions for economic purposes based in part on water availability and reliability. A two-stage economic production model is developed to examine the effects of hydrologic uncertainty and water prices on agricultural production, cropping patterns, and water and irrigation technology use. The model maximizes net expected farm profit from permanent and annual crop production with probabilistic water availability and a variety of irrigation technologies. Results demonstrate effects of water availability, price, and reliability on economic performance, annual and long-run cropping patterns, and irrigation technology decisions. Variations in water price and availability affect the desirability of different irrigation technologies. Increased water supply reliability can raise the probability of higher economic returns and promote more effective use of water for permanent crops. Such economic benefits can be compared to costs of operational changes and programs to increase water supply reliability for agricultural areas.

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## 1. Introduction

[2] Irrigation water demands depend on farmers' decisions on when and which crops to produce, how much water to apply, and which irrigation technologies to use. Decisions involve short- and long-term commitment of resources. Short-term decisions can respond directly to particular water availability events either to minimize losses in dry years or to take advantage of surplus water supply in wet years.

[3] When water is scarce, farmers seek to optimize water allocation among competing crops and irrigation technologies to maximize production and farm revenue. This problem involves decisions at several timescales: (1) intraseasonal irrigation scheduling decisions, (2) annual decisions on cropping areas, deficit irrigation, and irrigation method, and (3) long-run decisions on "permanent" crops, irrigation equipment purchases, and the area of land to develop for irrigation [Dudley *et al.*, 1971b; Marques, 2004; Cai and Rosegrant, 2004]. For annual crops, decisions on how much to grow are made each year, while decisions on permanent crops are made once, with possible changes every few years, given fluctuations in exogenous factors such as crop prices.

[4] With probabilistic water supply, crop decisions also reflect farmers' flexibility in coping with uncertainty to

maximize yields and profit. High-value permanent crops are usually limited to more reliable water, while annual crop decisions involve annual planning with the possibility of recourse every year depending on water supply. This framework makes the problems of annual and long-run decisions suitable for modeling with multistage, probabilistic optimization methods where decisions are integrated across two timescales, a first stage of "permanent" decisions, and second stage of recourse involving cropping and irrigation decisions based on stochastic water availability and cost of the remaining inputs.

[5] Modeling approaches for simulating agricultural decisions and production include models with detailed physical characterization of climate/soil/plant interaction for water allocation and irrigation scheduling [Dudley and Burt, 1973; Matanga and Marino, 1979; Rao *et al.*, 1990; Verdula and Kumar, 1996], and models focused on policy analysis based on sector behavior and economic relationships [Moore and Negri, 1992; McCarl and Spreen, 1980].

[6] This paper presents a model of agricultural land and water use decisions using two-stage stochastic quadratic programming to simulate decisions on the mix of perennial and annual crops, water use, irrigation technologies and economic performance considering probabilistic water availability. While irrigation scheduling and water use have been extensively modeled with dynamic and multistage programming, the approach presented in this paper contributes to existing literature by integrating the perennial and

long-term elements of cropping and technology agricultural decision framework within a two-stage stochastic programming with recourse decisions. Seasonal scheduling and water use decisions depend on which crops (perennial or annual) are grown and previous acquisitions of irrigation technology, whose combined effects are not yet modeled in the literature. To improve simulation of agricultural decisions under multiple exogenous factors, common crop rotation constraints are replaced by a calibration technique based on positive mathematical programming.

[7] The model's contributions improve the understanding of agricultural decisions under uncertainty and allow evaluation of economic effects of water policies based on farmers' economic responses. The paper begins with a review of agricultural planning issues and stochastic programming, followed by the model concept and formulation, application examples, results discussion, limitations and conclusions. Examples investigate effects of water pricing and water supply reliability on crop production and technology use.

## 2. Stochastic Programming and Agricultural Decisions

[8] Dynamic programming (DP) and stochastic dynamic programming (SDP) has been applied to a variety of real-time, intraseasonal, and interseasonal irrigation and cropping decisions [Tintner, 1955; Dudley et al., 1971a, 1971b; Matanga and Marino, 1979; Rao et al., 1990]. To keep the problem computationally tractable, few crop types are usually considered in SDP approaches. Others apply linear programming (LP) to allocate water within a season, coupled with a DP model to optimize crop areas across seasons and perform interseasonal water allocation [Yaron and Dinar, 1982; Verdula and Kumar, 1996].

[9] Problems involving decision making under uncertainty often can be characterized by multiple scenarios representing combinations of random events with embedded recourse decisions. These problems can be modeled by structuring the process in stages with decisions occurring before the realization of an uncertain events and recourse decisions responding as the future unfolds in different scenarios. The objective is commonly to minimize the expected value cost of all decisions in all stages and their consequences.

[10] Multistage optimization models have been applied to a variety of water resources management problems [Watkins et al., 2000; Huang and Loucks, 2000; Lund, 2002]. Linear two-stage stochastic programming has been applied to long- and short-term water conservation measures for urban water users given probabilistic shortages [Lund, 1995; Wilchfort and Lund, 1997; Garcia, 2002]. Long-term conservation measures are modeled in the first stage, with short-term conservation measures implemented in the second stage responding to particular water shortage events with a given probabilities. Cai and Rosegrant [2004] apply a two-stage stochastic programming to irrigation technology decisions and water allocation among fixed crops based on probability of water availability (second stage) with technology and crop decisions made in the first stage. Other applications simulate farmers' decisions including long- and short-term irrigation technology decisions to evaluate potential water transfers

[Turner and Perry, 1997], and seasonal planting and irrigation scheduling [Ziari et al., 1995]. Maatman et al. [2002] applies multistage stochastic LP to optimize crop production, consumption, storage and marketing decisions during consumption year, based on rainfall uncertainty.

## 3. Model Concept and Formulation

### 3.1. Quadratic Programming of Agricultural Production Decisions

[11] Linear production models are limited in representing real crop diversification by prioritizing production of the most profitable crops based on average conditions. This limitation can be avoided by including linear constraints enforcing observed crop mixes and crop rotation; however, such constraints tend to reduce the model's flexibility in simulating situations outside the range of calibration [Hazell and Norton, 1986; Howitt, 1995].

[12] In practice, crop production equilibrium is determined by marginal conditions [Hatchett, 1997] and is limited by endogenous factors such as crop rotation benefits, heterogeneous land quality, restricted management or machinery capacity (R. E. Howitt, University of California, Davis, Optimization Model Building in Economics, class notes, 2002) as well as exogenous factors such as risk aversion and crop prices. These factors result in diminishing marginal returns to crop production level.

[13] An alternative approach is to use a quadratic objective function that reflects the marginal conditions of a competitive market. Competitive market equilibrium conditions dictate that a price-taking producer will be willing to supply until his marginal revenue (market price  $P_i$ ) equals his marginal cost:

$$P_i = \alpha_i + \gamma_i X_i \quad (1)$$

The right hand side (marginal cost) of (1) is the farmer supply function of a given product  $i$  in the quantity  $X_i$  with intercept  $\alpha_i$  and slope  $\gamma_i$ . To arrive at these marginal conditions we can set the Lagrangean and apply the Kuhn-Tucker first-order conditions to a defined objective function. Starting from the marginal conditions, equation (1) can be integrated in  $X$  to arrive at the desired objective profit function (2).

$$Z = PX - (\alpha + 0.5\gamma X)X \quad (2)$$

The intercept and slope of the supply functions are empirically calibrated with positive mathematical programming (PMP) [Hatchett, 1997; Bauer and Kasnakoglu, 1990; Howitt, 1995]. A similar approach is used by Burke et al. [2004] to calibrate a parameterized economic model and estimate farmers' willingness to sell water. The PMP approach adds calibration constraints to crops in a LP version of the model, and uses the shadow values of these constraints to estimate the slope and intercept parameters of the quadratic profit function (2). The dual values for the binding calibration constraints (3) are defined as the difference between marginal and average products of the inputs for the calibrated crops [Howitt, 1995].

$$\lambda_2 = 0.5\gamma X \quad (3)$$

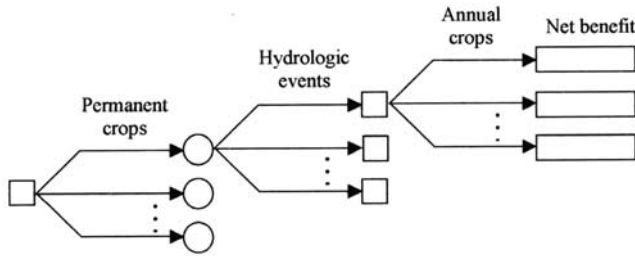


Figure 1. Problem decision tree.

Equation (3) is solved for the supply function slope  $\gamma$  with  $X$  being observed calibration cropping area. The intercept  $\alpha$  is calculated by substituting  $\gamma$  and the observed areas  $X$  in equation (1), since  $P_i$  is equal to the marginal production cost per acre at an optimal allocation.

3.2. Model Formulation

[14] A discrete version of a two-stage stochastic optimization model appears in Figure 1. Permanent crop decisions are simulated in the first stage, and annual crop decisions in the second stage, based on the probability distribution of water available in a given year (however, in the model, permanent and annual crop decisions are continuous). Irrigation technology decisions are made in both stages and are represented as combinations of crop type and technology type. These decisions are omitted from Figure 1 for clarity. Water use per acre is also determined by the model to simulate stress irrigation operations.

[15] Recourse decisions are annual, as it is assumed that local water storage can be used to cover seasonal availability imbalances. Permanent crop and capital irrigation technology purchase decisions are made in the first stage for the entire planning horizon.

[16] The objective function (4) maximizes the net expected economic benefit of crop production and water use decisions with probabilistic water availability, and is subject to constraints on land (5), water (7), stress irrigation (6), (8), (9), and irrigation technology (10). It includes permanent crops ( $X_{1ik}$ ) establishment cost and irrigation equipment investment ( $IR_k$ ) in the first stage, and annual net benefits of annual crops and permanent crops in the second stage. Second stage net benefits are calculated by multiplying marginal production costs ( $\alpha + 0.5\gamma X_2$ ) from equation (2) by crop production  $X_2$  and subtracting from gross benefits  $RE \cdot X_2$  (in the second stage,  $X_2$  is used for annual crops and  $Y_1$  for permanent crops). Marginal production costs include irrigation technology  $k$  operation and maintenance. The last cost term in the second stage penalizes production with  $CA_{1i}$  per unit area of permanent crops lost  $K_{1jik}$  due to excessive stress irrigation.

$$\begin{aligned}
 MaxZ = & - \sum_{i=1}^m \sum_{k=1}^h (INI_i X_{1ik}) - \sum_{p=1}^u IR_k + \sum_{j=1}^g P_j \left( \sum_{l=1}^n \sum_{k=1}^h \right. \\
 & \cdot (RE_{2l} X_{2jlk} - (\alpha_{2jlk} + 0.5\gamma_{2jlk} X_{2jlk}) X_{2jlk}) \\
 & + \sum_{i=1}^m \sum_{k=1}^h (RE_{1i} Y_{1jik} - (\alpha_{1ik} + 0.5\gamma_{1ik} Y_{1jik}) Y_{1jik}) \\
 & \left. - \sum_{i=1}^m \sum_{k=1}^h CA_{1i} K_{1jik} \right) \tag{4}
 \end{aligned}$$

subject to  
Land constraint

$$\sum_{i=1}^m \sum_{k=1}^h X_{1ik} + \sum_{l=1}^n \sum_{k=1}^h X_{2ljk} \leq L \dots \forall j \tag{5}$$

Second stage permanent crops

$$Y_{1jik} \leq X_{1jik} \dots \forall j, \forall i, \forall k \tag{6}$$

Water constraint

$$\sum_{i=1}^m \sum_{k=1}^h TAW_{1jik} + \sum_{l=1}^n \sum_{k=1}^h X_{2jlk} AW_{2jlk} \leq a_j \dots \forall j \tag{7}$$

Permanent crop water allocation

$$Y_{1jik} = \frac{1}{AW_{1jik}} TAW_{1jik} \dots \forall j, \forall i, \forall k \tag{8}$$

Stress irrigation threshold

$$K_{1jik} \geq X_{1ik} - \xi_i TAW_{1jik} \dots \forall j, \forall i, \forall k \tag{9}$$

Irrigation technology constraint

$$\sum_{i=1}^m X_{1ik} IC_{ik} + \sum_{l=1}^n X_{2jlk} IC_{lk} \leq IR_k \dots \forall j, \forall k \tag{10}$$

where model parameters are

- $\alpha_{1ik}$  supply function slope for permanent crop  $i$  and irrigation technology  $k$  (\$/acre\*acre);
- $\gamma_{1ik}$  supply function intercept for permanent crop  $i$  and irrigation technology  $k$  (\$/acre);
- $\alpha_{2jlk}$  supply function slope for annual crop  $l$  in year type  $j$  with irrigation technology  $k$  (\$/acre\*acre);
- $\gamma_{2jlk}$  supply function intercept for annual crop  $l$  in year type  $j$  and irrigation technology  $k$  (\$/acre);
- $\xi_i$  stress irrigation threshold for permanent crop  $i$  (acre/acre-foot);
- $a_j$  water available in year type  $j$  (acre-foot/year);
- $CA_{1i}$  annualized re-establishment cost for permanent crop  $i$  (\$/acre);
- $IC_{ik}, IC_{lk}$  irrigation capital value to supply an acre of permanent crop  $i$  or annual crop  $l$  with technology  $k$  (\$/acre);
- $INI_i$  annualized establishment costs for permanent crop  $i$  (\$/acre);
- $L$  land available (acre);
- $p_j$  probability of hydrologic event (year type)  $j$ ;
- $RE_{1j}$  annualized gross revenue of permanent crop  $i$  in year type  $j$  (\$/acre);
- $RE_{2l}$  annualized gross revenue of annual crop  $l$  (\$/acre);

and the model variables are

- $AW_{1jik}, AW_{2jlk}$  water supply to annual crop  $l$  with technology  $k$  in year type  $j$  (acre-foot/acre);
- $IR_k$  annualized first stage investment in irrigation technology  $k$  (\$);

$K_{1jik}$	area of permanent crop $i$ lost in year type $j$ due to water scarcity (acre);
$TAW_{1jik}$	water supply to permanent crop $i$ with technology $k$ in year type $j$ (acre-foot);
$X_{1ik}$	area of permanent crop $i$ established with technology $k$ (acre);
$X_{2jik}$	area of annual crop $l$ irrigated with $k$ in year type $j$ (acre);
$Y_{1jik}$	area of permanent crop $i$ irrigated with technology $k$ in year type $j$ (acre).

### 3.3. Stress Irrigation

[17] If agricultural production were modeled as if it depended only on crop areas, these would be constrained solely by water availability, with permanent crops limited to the lowest water availability (driest year). To avoid this limitation and to represent agricultural production decisions more realistically, stress irrigation water use decisions are included as decision variables. This allows the model to reduce water use for permanent crops in drier years (up to a limit) while still maintaining (reduced) production.

[18] Permanent crop establishment costs appear in the first stage of (4) that includes planting costs plus operating costs during the several years until the crops start producing. These costs are included in the  $INI_i$  variable. Equation (8) limits the area of permanent crop  $i$  irrigated in a given year  $j$   $Y_{1jik}$  to a given amount of water  $TAW_{1jik}$ . The ratio  $1/AW_{1jik}$  (acres per acre-feet of water) indicates how many acres of  $Y_{1jik}$  can be grown for a given quantity of water  $TAW_{1jik}$ . If stress irrigation is applied ( $TAW_{1jik}$  less than the full evapotranspiration demand),  $Y_{1jik}$  will be less than the planted area of permanent crops  $X_{1ik}$ . Since stress irrigation is likely to be applied over the whole area,  $Y_{1jik}$  is used as an area-equivalent supply term. The whole  $X_{1ik}$  area will receive water and produce crops, but the water supply per acre will be reduced to  $TAW_{1jik}/X_{1ik}$  and crop production will be reduced by a factor of  $Y_{1jik}/X_{1ik}$ . Constraint (6) limits the second stage irrigation of permanent crops to the area established in the first stage.

[19] Constraint (9) sets a limit for stress irrigation based on a stress threshold  $\xi_i$ , representing the area of permanent crop  $i$  that can be maintained per unit of water. Multiplying  $\xi_i$  by the water allocated to a given permanent crop  $TAW_{1jik}$  (af) results in the entire crops area being maintained. The difference from the planted area in the first stage  $X_{1ik}$  represents permanent crop area lost in the second stage  $K_{1jik}$  due to water stress. For water allocation  $TAW_{1jik}$  above the threshold, the second term of the right hand side of equation (9) equals the permanent crop area grown in the first stage, resulting in zero crops lost. If  $TAW_{1jik}$  is enough to avoid crop losses, but insufficient to supply all of  $X_{1ik}$  with full evapotranspiration demand, stress irrigation is applied reducing production by  $Y_{1jik}/X_{1ik}$ . Any area of crops lost  $K_{1jik}$  is multiplied by a replanting penalty ( $CA_{1ji}$ ) in the objective function (4).

[20] This method adds a simplified, linear penalty to production resulting from stress irrigation, and it is not intended to accurately simulate real impacts of stress irrigation in agricultural yields. One issue not considered is the potential additional yield reduction if stress irrigation is applied for several consecutive years. A calibration parameter could be added to adjust the factor  $Y_{1jik}/X_{1ik}$  to

more realistic yield impacts. However, such analysis is beyond the scope of this study.

### 3.4. Irrigation Technology

[21] The adoption of higher irrigation technology increases the percentage of water applied being used to meet the agronomic objectives, but also implies higher capital investment, energy and labor costs. Crops differ in irrigation requirements and the adoption of a given irrigation technology may be desirable or not depending on water demand, water supply, crop value, climate and soil conditions. Variations in water availability and reliability affect farmer's decisions on water use and consequently on the technology adopted. The model includes irrigation technology decisions for different crops to maintain yield while varying water application (and cost) per area. The water saved by more efficient irrigation will be available to irrigate other crops, and the optimal decision is a balance between irrigation costs, water use and increased production. *Cai and Rosegrant* [2004] present a two-stage stochastic model to incorporate hydrologic uncertainties on irrigation technology decisions, with irrigation technology decisions and a fixed cropping pattern in the first stage, and crop water allocation in the second stage. The model presented in this paper includes variable cropping and irrigation technology decisions in both first and second stages, plus water allocation and stress irrigation in the second stage, depending on crops' growth cycle.

[22] Beneficial uses of applied irrigation water include crop evapotranspiration, salt leaching and climate control. To meet these objectives, efficient and uniform application of water is necessary. *Burt et al.* [1997] define multiple performance indicators commonly used and describe irrigation efficiency as the ratio between the volume beneficially used and the applied water.

[23] In this paper, the term irrigation efficiency (IE) indicates performance in meeting the beneficial use of evapotranspiration. The crop applied water target is defined through the evapotranspiration of applied water (ETAW), which is the portion of irrigation water consumptively used by the plants. This discards consumptive demands met by rainfall or water previously stored in the soil. Defining irrigation applied water as AW we have:

$$IE_{ETAW} = \frac{ETAW}{AW} \quad (11)$$

Irrigation technology is modeled with decision variables for investment in technology types  $IR_k$ , and permanent and annual crop area irrigated with technology type  $k$ ,  $X_{1ik}$  and  $X_{2jik}$  respectively in the first and second stage. Irrigation technologies in  $k$  require previous investment in equipment (drip irrigation, sprinkler and Low Energy Precise Application – LEPA), and furrow irrigation, which is available in any year without previous investment in dedicated equipment. The parameters  $IC_{ik}$  and  $IC_{lk}$  (\$/acre\*year) are the irrigation capital requirements needed to supply permanent crop  $i$  and annual crop  $l$  using technology type  $k$ .

[24] Irrigation costs are estimated based on irrigation technology functions developed by *Hatchett* [1997], which used irrigation performance and cost characteristics for 8 crop types and 15 irrigation systems developed by *CH2M HILL* [1994]. In the work by *Hatchett* [1997],

feasible technology management combinations for each crop and region were plotted and fitted with a constant elasticity of substitution isoquant, with the form:

$$a \left( b \left( \frac{AW}{ETAW} \right)^\rho + (1-b)IC^\rho \right)^{\frac{1}{\rho}} = 1 \quad (12)$$

where  $a$ ,  $b$ , and  $\rho$  are estimated parameters and  $IC$  is the annualized irrigation cost in \$/acre\*year. This curve allows trade-offs between irrigation technologies and cost, while maintaining the same yield. Irrigation technology is represented by the ratio  $AW/ETAW$ .

[25] The two-stage model uses equation (12) to estimate the irrigation cost (\$/acre\*year) for a decision on a given irrigation technology for a given crop. The irrigation technology choice will affect the applied water  $AW$  (af/acre) based on equation (11). More technology (higher  $IE$ ) results in a lower  $AW$  and consequently higher  $IC$  (equation (12)). A set of  $AW$  values is precalculated for each combination of crop and irrigation technology. The irrigation cost parameters  $IC_{ik}$  and  $IC_{lk}$  are used in the calculation of total production costs and are reflected in the supply function parameters for permanent and annual crops  $\alpha_{1ik}$ ,  $\gamma_{1ik}$ ,  $\alpha_{2jlk}$ ,  $\gamma_{2jlk}$ . Constraint (10) limits the use of each irrigation technology in the second stage to the investment made in the first stage  $IR_k$ .

### 3.5. Calibration Approach

[26] The model is calibrated by calculating the slope and intercept of supply functions with equations (1), (2) and (3) [Howitt, 1995]. The observed acreage  $X$  for each crop type must be split among irrigation technologies. This approach generates a diversity of technology use based on the calibration values, which may temper the purely cost-based desirability of a given technology. For example, if water is abundant and available at a very low price one would expect to see most crops irrigated with furrow, given its low irrigation cost. However, the quadratic revenue functions present diminishing returns, so as the furrow irrigated area approaches the calibration value, other technologies will present higher marginal gains and enter the solution.

[27] This approach allows the model to represent irrigation technology diversification in a fashion similar to that for crop diversification, without using artificial constraints. However, this approach may limit the model's response to variations in extreme situations (i.e., very low water prices). Because of the lack of detailed data on irrigation technology use, the observed cropping areas are split equally among the available irrigation technologies. Current values of technology diversification can be used in future model developments. Annual crop observed acreages also depend on the water availability in the respective year, requiring the supply function parameters to be calibrated for different year types.

### 3.6. Model Runs and Data

[28] Model runs are intended to demonstrate the model concept and capabilities, rather than to provide an accurate simulation of regional crop production and water use. Production and hydrologic data are used from irrigation districts in California's Central Valley. The model is implemented with the optimization package GAMS (General Algebraic Modeling System) [Brooke et al., 1998] and it simulates the decisions of a single irrigation district with

access to a major surface water supply source. Data on crop prices, technical coefficients and input costs are obtained from the Statewide Agricultural Production Model (SWAP) [Howitt et al., 2001] and University of California Cooperative Extension, Department of Agricultural and Resource Economics. The U.S. Bureau of Reclamation (USBR) operates surface reservoirs in the region and delivers water to irrigation districts under contract using the Friant-Kern canal. Water contracts have a price structure based on water reliability; the most reliable supply is priced at \$44/acre-foot [Marques, 2004]. Crop areas in the region are from the California Department of Water Resources (DWR) 1999 land survey. Three permanent crops (grapes, citrus and nuts), and five annual crops are included (cotton, field crops, truck crops, alfalfa and miscellaneous grain crops).

[29] The model runs for a group of possible hydrologic years, each one with a probability of occurrence based on water availability. Rather than a time series, this group tries to capture the range of water availability outcomes. The results present the combination of short-/long-term decisions to be made on each year that produces the greatest expected benefit.

[30] The sets of events (year types) representing probabilistic water availability are developed based on observed water deliveries. Initially, ten equally probable water deliveries are used for exploration of water pricing impacts on agricultural production and technology use. This simplification makes the results and model concept easier to interpret. Two larger sets (25 year types lognormally distributed) are used to study the effects of water supply reliability on agricultural production, water and technology use and the economic value of different probability distributions of irrigation water availability.

[31] The group of year types can be obtained by generating a histogram of a long time series of water deliveries to the irrigation district such as might be derived from historical data or water resource system model outputs. In the present study, a time series of surface water deliveries from Marques et al. [2003] provided the moments to generate a longer, synthetic time series based on log normally distributed random numbers.

### 3.7. Water Pricing, Technology Use, and Agricultural Production

[32] Technology use results show the expected diversification based on the cropping areas used for calibration. Cropping areas are reasonably evenly distributed among the irrigation technologies (27.8% for sprinkler, 29.7% for LEPA, 30.9% for drip, and 11.6% for furrow). Factors affecting irrigation technology choices in this model include water availability, water price and crop consumptive demand (other factors such as soil type and climate are not considered). Results from initial runs with the water price at \$44/af appear in Tables 1, 2, and 3. The technologies are organized from the least efficient to the most efficient as furrow, sprinkler, LEPA and drip. These runs use the 10 equally probable hydrologic events.

[33] Given the low water availability in most hydrologic events, water use is concentrated in the most profitable permanent crops (grapes and nuts) and tends to use more efficient irrigation technologies. The limited amount of water allocated to citrus crops is mostly through high-efficiency drip irrigation (62% of the total citrus area in Table 1).

**Table 1.** Technology Decisions for Permanent Crops

	Furrow Irrigation		Sprinkler Irrigation		LEPA Irrigation		Drip Irrigation	
	Acres	Percent From Total of Crop	Acres	Percent From Total of Crop	Acres	Percent From Total of Crop	Acres	Percent From Total of Crop
Citrus	0	0.0	0	0.0	7	37.6	12	62.4
Grapes	515	8.2	1781	28.5	1928	30.8	2028	32.4
Nuts	729	16.5	1188	26.9	1243	28.1	1258	28.5

[34] Water availability constraints bind for hydrologic events 1 through 8, which motivates higher investment in technologies that conserve more water (Table 2). Both permanent and annual crops share the initial investments in irrigation technology. Initial investment is based on expected future value of irrigation equipment, and not all equipment acquired is used in all hydrologic events.

[35] Drip irrigation is almost half of all investment and twice the investment in sprinkler irrigation. In the first stage, drip irrigation receives 43% of all investment in high-value permanent crops (Table 2). The remaining equipment purchased is either used to irrigate annual crops when there is enough water available, or remains idle if water is too scarce for annual crops. In hydrologic events 1 through 6 about 97% of irrigation investment is used entirely in permanent crops. When more water is available in events 7 to 10 the annual crop acreage is expanded using the remaining 3% of irrigation equipment investment (Table 3).

[36] Water availability also affects decisions on technology use on a year basis, however the flexibility of such changes depends on the technology used as frequent removal of equipment already installed (e.g., drip and sprinkler systems) is not common given operation costs. The model results follow this behavior with small changes verified in acreages of drip, LEPA and sprinkler systems (Table 3) regardless of year type, while furrow irrigation varies significantly. Most of the technology mix change when water becomes very scarce or abundant is due to fluctuations in the acreage of furrow irrigated crops, given its lower setup and operation costs comparing to drip and sprinkler systems.

[37] Furrow irrigation shares about the same percentage of the annual crop acreage as drip irrigation in the event with 47 thousand acre-feet (kaf) ( $1 \text{ kaf} = 1.23 \cdot 10^6 \text{ m}^3$ ) of water available, but when water is abundant (e.g., 149 kaf available) furrow irrigation takes up 1063 acres out the total 1099 acres of annual crop increase, while the acreages of drip and LEPA remain practically unchanged accounting for about 11% of the total annual crops.

[38] The profit function for all crops is quadratic, thus presenting bigger gains with more acreage (i.e., steeper) in the beginning. Annual crops are severely constrained by water availability in the driest events, so as more water is available, up to 141 af/year, larger portions of land are brought into production. As the acreage increases the profit function gets flatter and reaches the maximum (diminishing marginal returns) and no significant increases are verified in acreages. As pointed out, these changes are more significant for furrow irrigation, given its low cost and high water consumption. This aspect explains the shifts in the results presented in Table 3.

[39] Water price is an important factor in these results. To further investigate the effect of water price on technology

choice, multiple runs were made varying water price from \$10/af to \$190/af. Results in Figure 2 compare acreages of annual crops irrigated with different technologies for different water prices and water availability. When water is cheap and abundant, acreage of annual crops is high and furrow irrigation predominates over higher efficiency technologies. Annual crop area is significantly reduced and furrow irrigation is abandoned when water is very expensive. Increasing the water price from \$10/af to \$190/af reduces annual crop acreage by 89% (for the wettest hydrologic event) leaving only higher-value truck crops. No annual crops are produced in very dry years (water availability less than 41 kaf/year), regardless of irrigation technology used or water price. Permanent crop decisions also are subject to changes in water price. Total permanent crop acreage (10,796 acres at \$10/af) is reduced by 11% (to 9605 acres at \$190/af) mostly from eliminating 1,390 acres irrigated with furrow.

[40] Total investment in irrigation equipment in the first stage (mostly for permanent crops) is slightly reduced as water becomes more expensive (due mostly to acreage reductions), but is concentrated in more efficient technologies (Table 4) resulting in higher investment per acre. High-efficiency technology remains widely applied even when water is inexpensive. Compared to water availability variation, water price has less effect on technology decisions for the case investigated here.

[41] Other agronomic variables are also important in production and may enhance or diminish the effectiveness of water pricing policies. *Green and Sunding* [1997] modeled adoption of low-pressure (higher efficiency) irrigation as a function of water price and field characteristics; and found that agronomic factors such as soil permeability and field gradient trigger different technology decisions leading to some crops being less sensitive to changes in irrigation technology with water price change than others. This issue highlights the importance of calibrating the model to current

**Table 2.** First Stage Irrigation Technology Investment and Permanent Crop Decisions

	Initial Investment		Permanent Crops	
	10 <sup>3</sup> Dollars	Percent From Total Invested	Acres	Percent Use From Total Invested Year Types HYD1 to HYD6
Furrow	0	-	1,245	-
Sprinkler	302	23.9	2,969	22.7
LEPA	412	32.5	3,178	31.5
Drip	552	43.6	3,298	42.8
Total	1267	100	10,690	97

**Table 3.** Annual Crop and Irrigation Technology Decision in the Second Stage<sup>a</sup>

Year Type Water Availability, kaf/year	Furrow		Sprinkler		LEPA		Drip	
	Acres	Percent From Total Annual Crops	Acres	Percent From Total Annual Crops	Acres	Percent From Total Annual Crops	Acres	Percent From Total Annual crops
46	0	0.0	0	0.0	0	0.0	1	100.00
47	54	14.8	151	40.9	111	30.1	52	14.2
50	373	45.2	189	22.8	110	13.3	52	6.3
141	1117	76.1	188	12.8	110	7.5	52	3.6
149	1117	76.1	188	12.8	110	7.5	52	3.6

<sup>a</sup>Water price is \$44/af.

land and technology use when analyzing the effects of different water pricing policies.

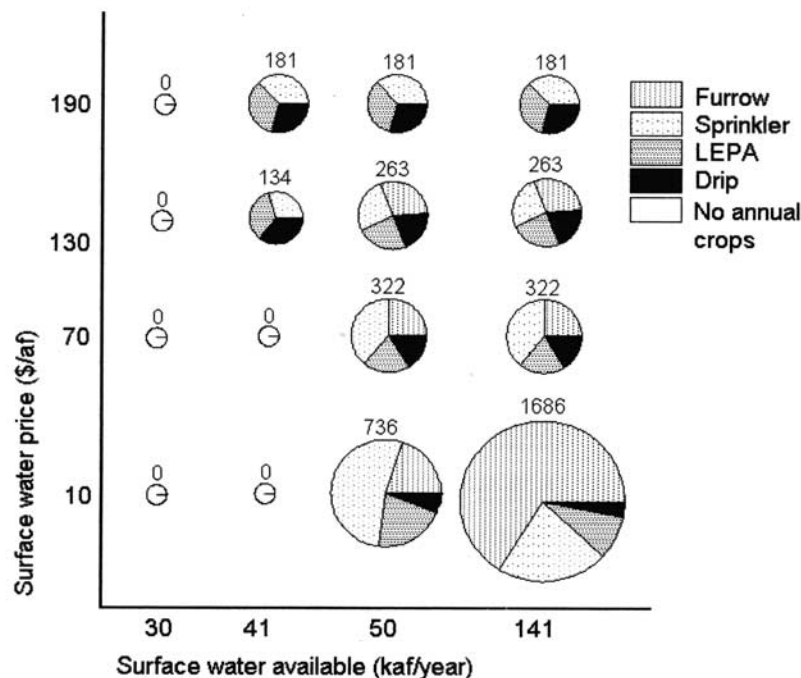
**3.8. Water Supply Reliability**

[42] Water supply variability and reliability may be affected by local water infrastructure design and operation or factors that modify the probability distribution of runoff and inflows, such as climate variability and land use. The two-stage model can be used to investigate the effects on agricultural production when the probability distribution of water availability is modified by such factors. To evaluate supply reliability benefits and the effects on crop production and irrigation technology use, two model runs are executed with different variances for a lognormal probability distribution of water availability, but the same average, as depicted in Figure 3 [Marques, 2004]. The original run has a 93,000 af/year average and 15,800 af/year standard deviation. The second run has the same average and but a lesser standard deviation of 8000 af/year. The range of hydrologic events is represented by 25 hydrologic year types.

[43] Reducing the variance of water availability raises the total net expected value benefit from \$47.8 million to \$49.2

million per year (3% increase). Figure 3 presents water marginal expected values (right y axis) for different water availability scenarios (year types) on the x axis. Marginal expected values are the water marginal value in a given year type (gain in expected net revenue for one additional unit of water on that year), divided by the probability of occurrence of that year. In Figure 3, curves for water marginal expected values are plotted for both original data (continuous line) and the less variance data (dashed line). With less variance, the chances of having a year with less water available than 62 kaf, and more than 125 kaf/year are virtually zero (and thus the less variance data water marginal expected value curve is not defined in this range). This reduces the chances of severe droughts. In drier years (from 63 to 87 kaf/year), for the run with less variance, marginal water values are slightly higher, reflecting higher willingness to pay for water when it is more reliable.

[44] The reduced supply variance allowed a 3.8% expansion in the area of permanent crops (at the expense of a small reduction in annual crop acreage). The larger permanent crop area takes advantage of the higher probability of average supply conditions, (around 90 taf/year) increasing the expected benefit. The trade-off is some increase in stress



**Figure 2.** Annual crops production and irrigation technology use for different water availability and water price.

**Table 4.** Variation in Irrigation Technology Investment for Different Water Prices

Water Price, \$/af	Total Investment, 10 <sup>3</sup> \$	Total Irrigated Area, acres	Percent of Investment Applied to Irrigation Technology			Irrigation Technology Investment, \$/acre
			Sprinkler	LEPA	Drip	
10	1286	10,796	25.1	32.3	42.6	119
40	1267	10,722	23.9	32.5	43.6	118
100	1255	10,351	23.2	32.5	44.4	121
160	1254	9,764	22.8	32.5	44.7	128
250	1189	8,930	21.9	32.6	45.5	133

irrigation in drier years (water supplies between 63 and 78 taf/year), but since the probability of these years occurring is smaller (Figure 3) they have little effect on the overall expected benefit. Increase in stress irrigation is also reflected in the slightly higher water marginal expected value (lower variance model run) in Figure 3.

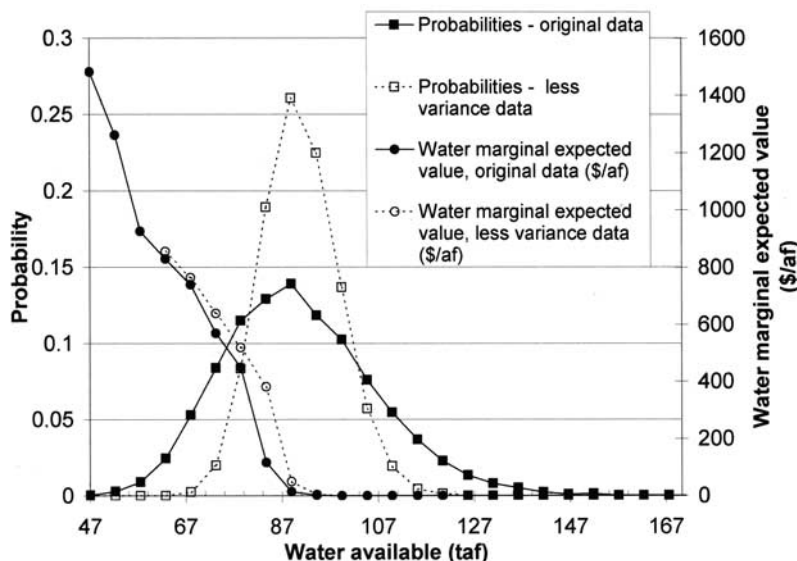
[45] To accommodate the expansion in permanent crops, investment in irrigation technology in the first stage is increased by 1.8% with lower water supply variance. The increase in investment in the highest-efficiency technology (drip) is slightly greater than for other technologies (3% increase against 1.6% in LEPA). Permanent crops irrigated with drip increase from 4670 acres to 4812 acres (3% increase), while the acreage of furrow irrigated permanent crops increases from 3860 to 4100 (6% increase), given the lower cost of furrow irrigation. However, for years of average supply conditions (which have higher probability in the less variance model run) annual crop acreages are reduced to increase water supply to permanent crops and most of the reduction is made in the crops irrigated with low efficiency technologies (Table 5) and crops with highest consumptive water demand. This indicates that with more reliable water supply, more efficient irrigation technologies are preferred.

[46] Annual crop decisions are more flexible to changes in the variance of water availability. With less variance in water availability, annual crop acreages decrease in every

hydrologic event, by up to 72% in some drier years and by almost 10% in some wetter years with water supply slightly above average (99 to 125 taf/year) (Figure 4). One would expect annual crop acreage to be maintained for wet years where the water marginal value is zero (99 taf/year and above); however, some water application is influenced by irrigation technology investments made in the first stage, which has more permanent crops in the lower variance run. The curves in Figure 4 are not defined for events with less than 62 kaf/year and more than 125 kaf/year since the probabilities of these events are virtually zero.

[47] Acreages of crops grown with technologies not requiring an initial investment (furrow irrigation) do not decrease in wetter years. If the desirability of furrow irrigation was only based on costs, it could expand in wetter years to use the available water. However, the profit function has diminishing returns and the current acreage of furrow irrigated crops in the original run is already close to the maximum economic return point, so further expansion results in little benefit. This behavior can be adjusted to better match real situations by calibrating the model to current technology diversification and water availability conditions.

[48] Further benefits of more reliable water supply are less variability in farmers' income. The minimum return increases from \$19.8 to \$35.8 million/year in the less variance run (Figure 5). Also, the probability of returns



**Figure 3.** Water availability and marginal values for runs A5a and A5b.



**Table 5.** Reduction in Annual Crops Acreage From Run With Original Water Availability Data to Run With Less Variance Water Availability

Year Type Water Availability, kaf/year	Reduction in Annual Crops Area			
	Furrow, acres	Sprinkler, acres	LEPA, acres	Drip, acres
78	0	9	10	9
83	46	42	58	42
89	313	182	60	0
94	227	139	61	1
99	0	139	61	1
104	0	139	61	1

between \$35.8 million/year and \$48 million/year increased significantly (e.g., the probability of returns exceeding \$44.6 million/year increases from 82% to 95%). The desirability of this solution depends on user’s risk aversion. More risk averse users may trade higher average water supply against a smaller, more predictable return. The model could help evaluate this trade-off between expected returns and return reliability by performing different runs with less water available (smaller average supply) but higher reliability (smaller deviation). Depending on user’s risk aversion, conditions can be improved with the use of less water, but with more demand for operational changes (i.e., more reservoir carry-over storage use to reduce supply variability).

**4. Limitations**

[49] The model has several limitations. These limitations identify areas for future model improvement. Crop prices are a major factor affecting cropping decisions. Permanent crop decisions are not subject to recourse in the model and crop prices are fixed. Fluctuations in crop prices can result in permanent crop acreage changes in the long run. This issue could be addressed in the model by representing crop prices as a second random variable if probabilistic estimates

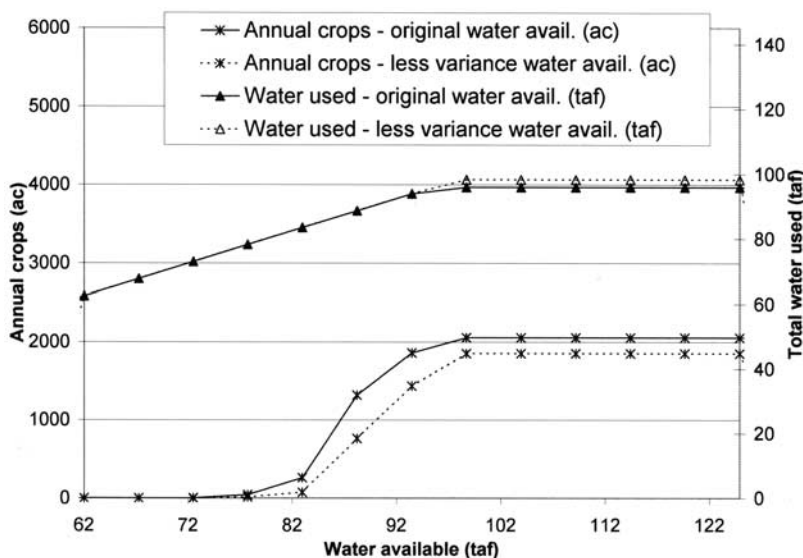
on future crop prices (and perhaps their covariance with water availability) are available.

[50] Crop yields are primarily fixed and do not vary directly with water application. In stress irrigation conditions, yields are reduced by the factor  $Y_{1ji}/X_{1ji}$  to represent the penalty of reducing supply. This factor could be adjusted based on production functions developed with detailed agronomic relationships of plant/soil/water/climate. Farmers also use crop rotation to increase productivity. The model currently simulates decisions in random, independent hydrologic events and does not consider benefits from alternating crops from one year to the other. This issue also limits representation of stress irrigation long-term negative effects. If stress irrigation is applied in multiple, consecutive dry years, yields of permanent crops may be more adversely affected.

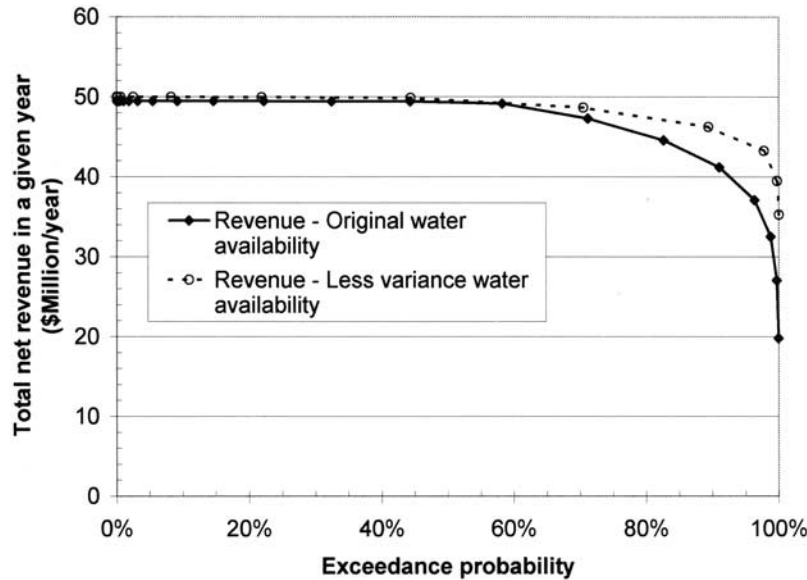
[51] Water requirements here do not vary for a given crop. Crop water requirements are determined by many factors, including climate and climate variability affecting evapotranspiration and effective precipitation. Inclusion of climate uncertainty would improve simulation of real water demands, while adding more complexity and requirement for new data. However, such variability could be included by varying unit crop water use coefficients within a set of second-stage crop production functions.

[52] The model assumes that water deliveries are known in advance. This assumption is reasonable for systems relying on snowmelt and having ample seasonal water storage, as is common in the western United States.

[53] The model does not incorporate water reuse and water quality effects. Agricultural water use often includes more complex operations with use and reuse of return flows, which vary in quality from initial supply. Use of return flows reduces the overall demand for applied water but also may reduce yields if salinity problems are present. Thus a given total amount of water delivered to an irrigation district may supply different crop acreages depending on return flow use, water salinity and crop tolerance to salts. The model could be improved to represent water with



**Figure 4.** Water consumption and annual crops production for runs with original and less variance water availability.



**Figure 5.** Probabilities of return for runs with original and less variance water availability.

varying quality and crops with varying tolerance to salts. This improvement would enable modeling of decisions on water reuse. Reuse might increase in very dry periods to grow more salt tolerant annual crops instead of fallowing land.

[54] Soil, climate, and other conditions also affect decisions on irrigation technology use and are not considered. Irrigation efficiency is considered for meeting ETAW only, but beneficial uses for salt leaching and climate control are neglected. The combination of these objectives with specific soil or climate conditions can affect the desirability for a given irrigation technology regardless of its efficiency or cost.

[55] Groundwater is not available in the model. Groundwater is a common water source for agricultural use given its vast, often convenient storage capacity. However, groundwater use should be properly managed to avoid negative overdraft impacts. Conjunctive use operations of groundwater and surface water can improve supply reliability and flexibility without compromising groundwater resources in the long run [Marques, 2004].

[56] The model is calibrated to real, observed crop acreages in Delano Earlimart irrigation district, California central valley. However, model results were consistently below observed acreages as the current model formulation does not include groundwater, resulting in strong water constraint and not all the available land being brought to production. Further calibration and groundwater operation improvements would be needed for field application.

[57] Other second stage decisions, such as irrigation scheduling, are not included. Although irrigation scheduling is an important second-stage decision for farmers to use water more efficiently and avoid crop losses, it represents a higher level of detail on farmers' decisions and it is beyond the scope of the model as a policy analysis tool.

## 5. Conclusions

[58] The model presented provides an explicit economic engineering representation of agricultural production deci-

sions for permanent and annual crops, irrigation technology and stress irrigation with probabilistic water availability. Agricultural water demands are consequences of these decisions, oriented by marginal conditions in market economies, and their understanding provides a basis for developing water management solutions across conflicting water uses.

[59] Model results provide causal insight into farmers' decisions and valuation of water and other aspects of production, such as irrigation technology use and water demand management through stress irrigation. Model results indicate that variations in water price, availability and reliability significantly affect agricultural decisions; with variation in one factor affecting other factors. For example, water availability affects use of irrigation technologies more with lower water prices. When water is very expensive, low efficiency irrigation technologies are not used regardless of water availability. Irrigation technology use decisions may change little from variations in water availability depending on setup and operating costs. The use of more expensive technologies for annual crops practically do not vary from dry to wet years, while less expensive technologies (i.e., furrow irrigation) are more flexible and may vary significantly from one year to another according to water availability.

[60] There are clear benefits from reducing the variance of a given average supply, and the model presented can help evaluate these benefits and potential changes in water demands due to variations in crop and technology choices. The model provides a method to derive the economic value of operation of local surface and groundwater storage, and water transfer programs for improved probability distribution of water deliveries.

[61] Farmer's preferences are important for identifying the desirability of water management solutions, trading off expected returns for less return variability. Development of multiple model runs with varying expected water deliveries and water delivery variance may provide a straightforward approach to present these trade-offs to decision makers.

[62] Further conclusions from the improved water supply reliability model runs are as follows.

[63] 1. Higher reliability increases benefits under average conditions. Permanent crop acreages increase slightly to take advantage of more reliable water under average conditions at the expense of some stress irrigation in drier (and less probable) years.

[64] 2. Capital investment in irrigation technology in the first stage increases to support additional permanent crops. Concentration of irrigation equipment on permanent crops reduces the annual crop area.

[65] 3. As water is reallocated to high value permanent crops, more efficient technologies are prioritized for annual crops to improve conservation of limited water supply.

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