Derived Willingness-to-Pay for Household Water Use with Price and Probabilistic Supply

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Abstract: Stochastic optimization is used to estimate the willingness-to-pay (WTP) of individual households and groups of households for changes in a combination of probabilistic water supply reliability and retail price of water. By modeling the financial and "perceived" costs of implementing long- and short-term conservation options and assuming rational (expected value cost minimizing) behavior, economic demand curves for water and expected water use can be estimated for a household. Monte Carlo-simulation techniques are used to represent variability in the household model parameters and derive estimates of aggregate household WTP for water supply reliability, and demand curves for water and conservation measures. Examples are provided to illustrate the approach.

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Introduction

A goal of water managers is to deliver reliable water supply at a "reasonable" cost. Indeed, any increase in reliability involves a cost that should be balanced against the benefits associated with resulting reductions in frequency of water scarcity. Economically optimal water supply reliability occurs where the marginal cost of increased reliability equals the marginal cost of increased shortage (Howe and Smith 1994; Hoagland 1998). Estimating the value of increased capacity has long been a practical question for water resources planning (Dupuit 1844).

Valuing willingness-to-pay (WTP) for a probabilistic supply is useful for reliability planning. If a reliability enhancement project's cost (for water recycling, extra capacity, water transfers, etc.) is less than consumers' WTP, the project is economically attractive (Abrahams et al. 2000). Conversely, in highly reliable systems consumers might willingly accept a greater frequency of shortages in exchange for reduced water bills (Howe and Smith 1994).

Relatively little effort has been devoted to valuing urban water supply reliability. Most studies have been empirical, using either price elasticity (Howe and Linaweaver 1967; Howe 1982; Renwick et al. 1998; Jenkins et al. 2003) or contingent valuation (CV) techniques (Carson and Mitchell 1987; CUWA 1994; Howe and Smith 1994; Griffin and Mjelde 2000). These empirical studies typically ignore much of the interaction between long-term and short-term conservation actions and look only at a single shortage event, defined by a given level of shortage with a given frequency (i.e., a 10% shortage once every five years). Yet there is a need for estimating shortage losses over the entire range of possible shortages (Howe and Smith 1994). Indeed, investments in water supply reliability enhancement can alter the frequency of all shortage levels so estimating the economic value of an entire probability distribution of shortages is desirable.

Several studies (Griffin and Mjelde 2000; CUWA 1994) show that consumers have difficulty interpreting probabilistic information, leading to inconsistent results. The CUWA (1994) study concluded that these limitations make it difficult to apply CV data to "a real world hydrology that produces a mix of shortages." Thus, traditional empirical methods used for valuing the benefits of urban water supply reliability are in some ways ill-suited for probabilistic settings.

Lund (1995) and Wilchfort and Lund (1997) propose two-stage optimization models to estimate WTP to avoid shortages considering users' responses to an entire shortage probability distribution. This study extends this approach to include retail water price and household variability. This extension allows derivation of individual and aggregate demand curves for water and for conservation options as well as probabilistic estimates of WTP for water supply reliability. Including retail water prices in the formulation also allows examination of the water conservation and financial effects of this water utility policy on both water utilities and households.

This paper begins with a simple analytical treatment of long- and short-term conservation options in the context of a probability distribution of water rationing levels and retail prices for water. Some general theoretical optimality conditions are discussed. A two-stage linear program is then formulated to numerically estimate the WTP of a single household to avoid an entire probability distribution of shortages (in favor of 100% water supply reliability) under different retail water prices. The optimization approach is applied to derive household water demand curves without rationing (100% water supply reliability). Using Monte Carlo methods, this approach is extended to develop aggregate demand curves for a group of households. Individual and aggregate household demand curves are then produced with probabilistic water supply reliability (probabilistic water rationing). Demand curves for conservation options also are presented. The overall intent of

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the paper is to establish a theoretical framework for a derived approach to urban water demand that will provide conceptual insights and perhaps serve later empirical and practical study. The focus of this paper is on households' economic and water conservation responses to a set of rations, prices, and conservation policies set by a water utility. Better knowledge and understanding of consumer responses should be useful for utilities in setting rationing, pricing, and water conservation policies.

General Formulation for Individual Households

The formulation of Lund (1995) and Wilchfort and Lund (1997) can be extended to include the given price of water for each shortage event k (p_{Qk}). The household's objective is to minimize the expected value of total annualized costs needed to meet each level of probabilistic shortage (f_k) subject to household water rationing (r_k) and management constraints. Expected cost minimization is a rational objective where water costs are a small part of total household expenditures (Arrow and Lindh 1970). This is formulated as a mathematical program

$$\text{Minimize } Z = v(\mathbf{X}_1) + \sum_{k=1}^{n} f_k(g_k(\mathbf{X}_1,\mathbf{X}_{2k}) + p_{Qk}Q_k) \tag{1}$$

Subject to

$$h_k(\mathbf{X}_1,\mathbf{X}_{2k}) = Q_k, \quad \forall\ k \tag{2}$$

$$Q_k \leqslant r_k, \quad \forall\ k \tag{3}$$

where Z=expected value of total household water costs to accommodate (or WTP to avoid) the entire shortage probability distribution (in favor of 100% reliability), with component costs for long- and short-term water conservation efforts and the purchase cost of water for each rationing (and nonrationed) event; the latter two terms are weighted by the probability of each rationing event to account for variability in water supply availability; X_1=vector of long-term conservation decision variables (e.g., xeriscaping, plumbing retrofits, and appliance purchases), with a total annualized cost of $v(\mathrm{X}_1)$; and X_{2k}=vector of short-term conservation options available for each shortage event k (e.g., installing toilet displacement devices and temporarily reducing lawn watering and shower use), with total annualized costs of $g_k(\mathrm{X}_1,\mathrm{X}_{2k})$.

The predetermined function $h_k(\mathrm{X}_1,\mathrm{X}_{2k})$ calculates water use in event k resulting from implementing short- and long-term conservation options, and should be specified to only allow total water use Q_k to be nonnegative. (In principle, by purchasing enough bottled water, a household could have zero municipal water use, though this would be costly.) Eqs. (2) and (3) state that water use is a function of the permanent and short-term conservation efforts $h_k(\ )$ and require that water use Q_k not exceed each event ration r_k.

This problem can be examined analytically using the method of Lagrange multipliers. Substituting Eq. (2) into Eqs. (1) and (3), the Lagrangian function is

$$L = v(\mathbf{X}_1) + \sum_{k=1}^{n} f_k[g_k(\mathbf{X}_1,\mathbf{X}_{2k}) + p_{Qk}h_k(\mathbf{X}_1,\mathbf{X}_{2k})] - \sum_{k=1}^{n} \lambda_k[h_k(\mathbf{X}_1,\mathbf{X}_{2k}) - r_k] \tag{4}$$

The resulting first-order conditions are

$$\frac{\partial L}{\partial X_{1i}} = 0 = \frac{\partial v(\mathbf{X}_1)}{\partial X_{1i}} + \sum_{k=1}^{n} f_k\left[\frac{\partial g_k(\mathbf{X}_i,\mathbf{X}_{2k})}{\partial X_{1i}} + \left(p_{Qk} - \frac{\lambda_k}{f_k}\right)\frac{\partial h_k(\mathbf{X}_1,\mathbf{X}_{2k})}{\partial X_{1i}}\right], \quad \forall\ i \tag{5}$$

$$\frac{\partial L}{\partial X_{2jk}} = 0 = f_k\left[\frac{\partial g_k(\mathbf{X}_1,\mathbf{X}_{2k})}{\partial X_{2jk}} + \left(p_{Qk} - \frac{\lambda_k}{f_k}\right)\frac{\partial h_k(\mathbf{X}_1,\mathbf{X}_{2k})}{\partial X_{2jk}}\right], \quad \forall\ j,k \tag{6}$$

where i=long-term conservation activity; and j=short-term conservation activity. Rearranging Eq. (5) results in Eq. (7)

$$\frac{\partial v(\mathbf{X}_1)}{\partial X_{1i}} = \sum_{k=1}^{n} f_k\left[\left(\frac{\lambda_k}{f_k} - p_{Qk}\right)\frac{\partial h_k(\mathbf{X}_1,\mathbf{X}_{2k})}{\partial X_{1i}} - \frac{\partial g_k(\mathbf{X}_1,\mathbf{X}_{2k})}{\partial X_{1i}}\right], \quad \forall\ i \tag{7}$$

This result shows the effect of price on the optimal marginal cost of implementing any permanent conservation option i. The implementation of permanent conservation options is encouraged to the extent of decreased household water expenditures over the entire shortage probability distribution. Thus, price effects on conservation implementation are proportional to the effectiveness of the conservation option. If price does not vary between shortage events (p_{Qk}=p_Q), then the price effect is directly proportional to the expected value of water savings from a given conservation option.

The condition in Eq. (6) is rearranged to

$$\frac{\dfrac{\partial g_k(\mathbf{X}_1,\mathbf{X}_{2k})}{\partial X_{2jk}}}{\dfrac{\partial h_k(\mathbf{X}_1,\mathbf{X}_{2k})}{\partial X_{2jk}}} = \left(\frac{\lambda_k}{f_k} - p_{Qk}\right), \quad \forall\ j,k \tag{8}$$

This condition holds that the marginal implementation cost-effectiveness should be equal across all short-term conservation actions j for each event k. Higher water prices make additional (less cost-effective) conservation options j desirable. Similarly, events k with large scarcity values for water (high absolute values of λ_k) also encourage greater short-term conservation for those events, although this tendency is reduced by higher event probabilities, which encourage longer-term water conservation options instead.

Putting Eq. (8) into Eq. (5) yields a modified first-order condition for permanent conservation options. This first-order condition no longer explicitly includes the retail water price; price is implicitly included through Eq. (8) on the optimal magnitude of short-term implementation cost-effectiveness.

$$0=\frac{\partial v(\mathbf{X}_1)}{\partial X_{1i}}+\sum_{k=1}^{n} f_k\left[\frac{\partial g_k(\mathbf{X}_1,\mathbf{X}_{2k})}{\partial X_{1i}} - \left(\frac{\frac{\partial g_k(\mathbf{X}_1,\mathbf{X}_{2k})}{\partial X_{2jk}}}{\frac{\partial h_k(\mathbf{X}_1,\mathbf{X}_{2k})}{\partial X_{2jk}}}\right)\frac{\partial h_k(\mathbf{X}_1,\mathbf{X}_{2k})}{\partial X_{1i}}\right], \quad \forall\, i,j \tag{9}$$

If implementation of long-term (permanent) conservation options does not affect event-specific (short-term) implementation costs, $\partial g_k(\,)/\partial X_{1i}=0$. Further, if the implementation of permanent options has the same water-conservation effectiveness for each rationing event k, then $\partial h_k(\,)/\partial X_{1i}=$constant (not varying with k). In this case, Eq. (9) becomes

$$\frac{\frac{\partial v(\mathbf{X}_1)}{\partial X_{1i}}}{\frac{\partial h_k(\mathbf{X}_1,\mathbf{X}_{2k})}{\partial X_{1i}}}=\sum_{k=1}^{n} f_k\left(\frac{\frac{\partial g_k(\mathbf{X}_1,\mathbf{X}_{2k})}{\partial X_{2jk}}}{\frac{\partial h_k(\mathbf{X}_1,\mathbf{X}_{2k})}{\partial X_{2jk}}}\right), \quad \forall\, i,j \tag{10}$$

Where these conditions hold, marginal implementation cost-effectiveness should be equal across all implemented permanent conservation options i and also should equal the expected value of marginal implementation cost-effectiveness for each implemented short-term conservation option j.

While these analytical solutions provide some theoretical insight, many water conservation actions are discrete at a household scale, with discontinuous effects on costs and effectiveness. Moreover, implementing long-term conservation options often reduces the effectiveness of short-term options (demand hardening, such as permanent xeriscaping reducing the effectiveness of short-term banning of lawn watering) and sometimes increases the effectiveness of short-term options (complementarity, such as when meter installation allows unit price to vary with drought conditions). This reflects the complex forms of substitution that can sometimes occur among various water conservation options. In practice, specific numerical formulations are more useful. Fortunately, this problem can be nicely formulated into linear programs for several cases.

Linear Program Formulation for a Single Household

The household's objective is to minimize the expected value of total annualized costs necessary to meet each level of probabilistic shortage (f_k) subject to household water rationing (r_k) and management constraints. Economic motivations for water conservation are the retail price of water and limits placed on water availability due to shortage events k (rationing levels or outages). The overall effect of water shortages depends on a shortage probability distribution such as those commonly generated from water supply models (Jenkins and Lund 2000). This stochastic optimization problem can be represented as a two-stage linear program shown in Eqs. (11)–(16).

First-stage decisions concern long-term conservation options, which must be implemented before the shortage and have a long life span and fixed annualized costs. Once a long-term conservation option (X_{1i}) is implemented the household bears its full annualized cost (c_{1i}). Second-stage actions respond to particular shortage events, representing short-term water conservation options (X_{2jk}) and their costs (c_{2jk}), weighted by the probability of each shortage event f_k. Water bills also are included ($p_{Qk}Q_k$), allowing price p_{Qk} to vary with shortage event k

$$\mathrm{Min}Z=\sum_{i=1}^{n_1} c_{1i}X_{1i}+\sum_{k=1}^{m} f_k\left(\sum_{j=1}^{n_2} c_{2jk}X_{2jk}+p_{Qk}Q_k\right) \tag{11}$$

Subject to

$$X_{1i}\leqslant u_{1i}, \quad \forall\, i \tag{12}$$

$$X_{2jk}\leqslant u_{2jk}, \quad \forall\, j,k \tag{13}$$

$$d_k-\left(\sum_{i=1}^{n_1} q_{1i}X_{1i}+\sum_{j=1}^{n_2} q_{2jk}X_{2jk}\right)=Q_k, \quad \forall\, k \tag{14}$$

$$Q_k\leqslant r_k, \quad \forall\, k \tag{15}$$

$$X_{1i},X_{2jk}\geqslant 0, \quad \forall\, i,j,k \tag{16}$$

where X_{1i}=level of implementation of long-term option i; X_{2jk}=level of implementation of short-term option j in shortage event k; c_{1i}=cost of long-term measure i (annualized); c_{2jk}=annual cost of short-term option j in event k; f_k=probability (frequency) of occurrence of shortage event k; q_{1i}=unit annual water saved by long-term option i; q_{2jk}=unit annual water saved by short-term option j during shortage k; u_{1i}=upper limit of long-term conservation option i; u_{2jk}=upper limit of short-term conservation option j under event k; p_{Qk}=retail price of water for each shortage event k; Q_k=water use for each shortage event k; r_k=ration amount for event k; and d_k=full service water use for event k.

Each long- and short-term water conservation option has a maximum level of implementation [Eqs. (12) and (13)] and a lower limit of application [Eq. (16)]. If more detailed water end use data is available, these limits could be a function of water use affected by each conservation option (i.e., 10% of water used for toilet flushing). More elaborate equations representing the upper bounds of conservation options exist where long-term conservation options (such as xeriscaping) affect the effectiveness of short-term conservation (such as restricting lawn watering) and vice versa. Eq. (14) calculates the quantity of water purchased for each event k (full service water use d_k minus the effectiveness of water conservation efforts).

Eq. (15) defines the water availability or ration r_k for event k. This ration amount is determined by utility policy, and can be either a fixed number (rationed water volume/household), as assumed here, or a proportion of some base water use [such as 75% of some unrationed water use, where Eq. (15) would not exist for an unrationed condition]. This might allow use of this approach to examine the household water use effects of different water rationing schemes.

Water price (p_{Qk}) should include all variable utility charges based on water use (sewer, environmental surcharges, etc.). Specific rate structures can be accommodated in the model. If consumers are operating under an increasing (convex) block rate structure, this can be accommodated by separating Q_k into pieces for each block. A decreasing (concave) block rate structure can similarly be accommodated, with the addition of binary integer variables as an integer-linear program. A convex rate structure can also be adapted to represent enforcement penalties to households for exceeding rationed amounts of water use [allowing a later constraint, Eq. (15), to be eliminated].

Table 1. Illustrative Parameter Values

	Unit implementation cost ($/year)	Conservation effectiveness (gpd)	Cost-effectiveness ($/1,000 gallons)	Limits on conservation options (u_{1i}, u_{2jk})
Long-term options				
Toilet retrofit	150 (30)	30 (6)	13.70	2
Xeriscape I	500 (100)	100 (20)	13.70	1
Xeriscape II	1,000 (200)	150 (30)	18.26	1
Short-term options				
Toilet dam	5 (1)	2 (0.4)	6.85	2
Dry lawn	400 (80)	100 (20)	10.96	1
Dry shrubs	1,200 (240)	70 (14)	46.97	1

Note: Parenthetical numbers are standard deviations used for Monte Carlo simulations.

If we are concerned about the effect of water conservation on water utility revenues, a constraint can be set on net utility revenues by making the price parameter p_{Qk} into a decision variable and adding a constraint on revenue, $\Sigma_{k=1}^{m} f_k p_{Qk} Q_k$=desired revenue. A constraint limiting revenue variability also could be added. In both these cases, the formulation above becomes nonlinear. A more difficult aspect to integrate in the model is to know exactly what price variable consumers respond to (if any). An abundant (and controversial) body of literature has considered how consumers respond to the price of water (Howe and Linaweaver 1967; Nordin 1976; Billings and Agthe 1980; Howe 1982; Nieswiadomy and Molina 1991). Yet, consumers' demand for water is also affected by household characteristics, season, location, income, and other factors. One might want to capture the effects of these variables by restating the problem using different coefficients for different locations and classes of household, a possibility discussed later in the paper.

In any case, the cost coefficients here are annualized and include both financial and perceived costs. Financial costs are the annualized monetary costs of implementation, including materials and labor. Perceived costs would include any additional time and effort required of the household to implement a conservation option, such as time and discussions needed to select a particular low-flush toilet, change out the toilet, or oversee the work conducted by others. Perceived costs also could include the inconvenience of water function with the conservation option compared with current function (such as inconveniences with low-flow showerheads or low-flush toilets, disruptions to current water use behaviors, or reduced aesthetic value from xeriscaping). Implementation of conservation options typically entails both financial and inconvenience (perceived) costs, where the perceived costs could be the largest component. The model's cost coefficients are estimates of consumers' overall WTP to avoid implementing specific water conservation measures (from financial and option-specific CV or hedonic valuation studies). Where perceived costs increase with the level of implementation of a water conservation action, these costs can be represented in piecewise linear fashion or by moving to a quadratic programming formulation.

The approach and method is illustrated for a series of examples. For simplicity the retail price of water and the cost and effectiveness of conservation measures will be fixed over the range of shortage events as shown in Table 1. These values are representative of the literature, and are chosen for purposes of illustrating the method (Berk 1993; Schulman and Berk 1994; Berk et al. 1995). Significant additional research is needed to estimate household WTP to avoid implementing particular water conservation actions, particularly regarding household variability in WTP to avoid such actions.

Formulation for a Single Household without Rationing

We first examine the effects of price on a single household's water use without rationing. For this case, $m=1$ in the problem formulation (only one event) and Eq. (15) is eliminated (the only event has no rationing). The linear program is solved for different price levels to generate a water demand curve.

As price increases, the household substitutes water use for conservation actions. At some point increasing price makes a given conservation option cost-effective and it is implemented; then the marginal reduction in the water bill from conservation exceeds the costs (financial and perceived) of conservation efforts. Conservation options will be implemented in order of their cost-effectiveness. A household's demand for conservation options can be derived indirectly from the solution to this linear program (LP) problem.

If the model's cost coefficients are estimates of a household's WTP to avoid implementing specific water conservation options, then the objective function minimizes the loss of consumer surplus (CS) due to an increase in the retail price of water (where free water serves as the baseline). A price increase reduces consumer surplus by increasing household expenditures for water and increasing expenses to reduce water use (conservation measures). The optimization minimizes the sum of those losses, and hence minimizes the loss of consumer surplus. Graphically, the CS is given by the area between the demand curve and a horizontal line drawn at a given price level. The equivalence of this theoretical and optimization formulation is demonstrated by Garcia Alcubilla (2002).

Formulation for a Class of Consumers without Rationing

Variability in water demand is widely recognized even for households with similar characteristics. Household occupancy rates and plumbing can vary, but more importantly, lifestyle characteristics and water use patterns vary between households and individuals in ways not well understood (Vickers 2001). Such factors affect a household's total water use and enhance or reduce the effectiveness and perceived costs of conservation options.

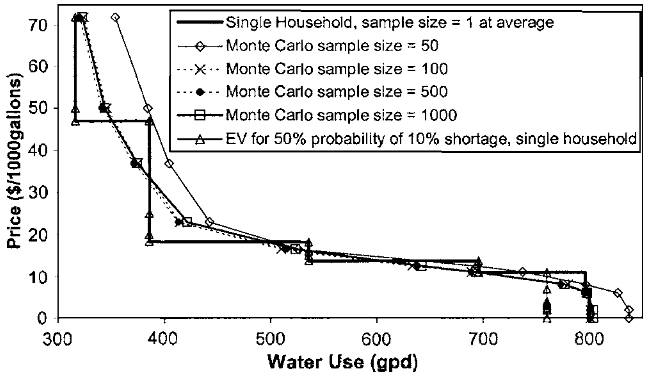

Fig. 1. Derived household water demand without restricted supply (except for last curve)

Within a class or group of similar (but not identical) customers responding to the same prices, variability among individual users can be represented by varying perceived conservation implementation costs (c), effectiveness of water conservation options (q), and normal levels of water use (d). The availability of conservation options among households also may vary.

A given conservation action's cost can be decomposed into a financial expense for the cost of materials and installation of conservation devices, and an inconvenience cost including other aspects of consumer's preference for conservation measures, such as the value of the time spent implementing a conservation option, resistance to bothering to conserve water, reductions in utility from changes in water use (i.e., taking shorter showers or flushing the toilet less often), or (in the opposite direction) positive values from activities felt to be ethically or publicly desirable.

Financial cost may seem easier to estimate; yet choices of conservation devices are rapidly increasing and costs vary widely. Vickers (2001) reports ranges of $75 to $650 for low-flush toilets. The inconvenience costs associated with the implementation of a specific conservation measure cannot be easily ascertained. The CUWA (1994) contingent valuation study reported that the major conservation cost was the time and effort to monitor water use. The variability of perceived costs might exceed financial costs. Contingent valuation studies of WTP to avoid implementation of a specific conservation option would be required to gain insight on this matter (Lund 1995). A lower bound for cost coefficients would consider only financial costs.

Water conservation effectiveness can range widely for the same type of conservation option (Dziegielewski et al. 1993; Vickers 2001; CUWCC 2000; Maddaus 1984; Walski et al. 1985). Effectiveness depends on the appliances being replaced, on the new ones installed, on technical considerations such as local water pressure, and on behavioral aspects of water use patterns. New developments in measuring and modeling end uses of water provide empirical evidence of the effectiveness and interactions of specific conservation measures at the local level quickly and at relatively low expense (Weber 1993; Mayer et al. 1999).

Monte Carlo simulations can be used to examine the importance of variability in the model parameters within a class of customers. By solving the LP model for a set of random parameters, confidence intervals can be calculated for the loss of consumer surplus, water use, and adoption of conservation options. The Monte Carlo approach has the advantage of explicitly accounting for variability through the model's parameters.

For this work we assumed model parameters are normally distributed with known means and standard deviations (Table 1). Fig. 1 shows results without rationing for different numbers of Monte Carlo iterations. By accounting for variability in parameters, the points at which different conservation options become cost-effective shift to smooth the average aggregate demand curve. The water use curve stabilizes after a hundred iterations. This aggregate curve can be used to estimate price elasticities of water demand for different prices. These elasticity values should not be taken literally because this model's parameters are not empirically based, yet their relative values and variation along the demand curve are consistent with the theory of residential water use. The almost perfectly inelastic response observed at low water prices can be attributed to the limited range of low-cost conservation options considered for this model. Also, when the price of water is low, the effects of a price increase on household income may be small and lead to little change in water use. For such low price levels, bother about water use monitoring exceeds foreseeable benefits. As the price of water increases ($6–$23 per 1,000 gal.), water use becomes more responsive to changes in price until water use becomes essential (drinking, cooking), so that water use barely responds to price increases ($p >$ \$50 per 1,000 gal.).

Table 2. Calculation of Loss of Consumer Surplus and WTP

	No rationing	Rationing	
I	II	III	IV
Price ($/1,000 gal.)	Objective function value ($/year)	Objective function value ($/year)	WTP ($/year)
0.00	0	157	157
2.00	584	712	128
4.00	1,168	1,267	99
6.80	1,997	2,054	57
11.00	3,191	3,192	0

Formulation for a Single Household with Rationing

Households expecting frequent reductions in water availability (water rationing) are likely to change their water-related investments in landscaping and plumbing to reduce normal water use and ease further reductions during periods of shortage. The response of a single household to water rationing can be studied by solving the LP problem for several shortage probability distributions (f_k). The solution output provides the mix of conservation measures that minimizes total cost to the consumer while ensuring that water use in each of the shortage events (Q_k) is within the ration for that event (r_k).

To illustrate some aspects of rationing, let us focus on a simple case with a 50% probability of no shortage (event 1) and a 50% probability of a 10% shortage (event 2). The relevant results for this case appear in Table 2. For low price levels (Fig. 1, last curve), rationing forces use of conservation options at price levels where these would not otherwise be cost-effective. With low prices, no conservation would be implemented without rationing. For this range of prices (0–$6.8 per 1,000 gal.), rationing raises total costs to the household (column III on Table 2). However, price increases dilute the ration's effects as conservation actions resulting from rationing become cost-effective (and would therefore be implemented even without rationing). At higher water prices conserving water is economically appealing for the house-

Table 3. Example Shortage Probability Distributions and Results

Shortage level (reduction in full service demand)	Probability distributions											
	A	B	C	D	E	F	G	H	I	J	K	L
Event 1 (0%)	0.9	0.95	0.97	0.6	0.8	0.7	0.6	0.5	0.75	0.5	0.25	0.5
Event 2 (10%)	0.1	0	0	0.1	0.2	0.3	0.4	0.5	—	—	—	0.2
Event 3 (20%)	0	0.05	0	0.1	—	—	—	—	—	—	—	0.1
Event 4 (30%)	0	0	0.03	0.1	—	—	—	—	—	—	—	0.1
Event 5 (40%)	0	0	0	0.1	—	—	—	—	0.25	0.5	0.75	0.1
EV shortage (gpd)	8	8	8	80	16	24	32	40	80	160	240	88
	(1%)	(1%)	(1%)	(10%)	(2%)	(3%)	(4%)	(5%)	(10%)	(20%)	(30%)	(11%)
WTP ($/year) at $p_i=0$	31	68	383	933	63	94	126	157	1,133	1,378	1,481	933
WTP ($/year) at $p=\$2/1000$ gal.	26	63	326	806	51	77	102	128	994	1,183	1,266	806

hold; the effect of rationing is no longer felt and the average water use curves coincide with the "single household" (without rationing) water use curve (Fig. 1).

Increased objective function cost with rationing represents loss of consumer surplus compared with the "no ration" case. This "extra loss" provides an estimate of the consumer's maximum WTP to avoid a particular shortage distribution, establishing the value of water supply reliability for the household. The household's WTP to avoid a specific shortage probability distribution decreases as the retail price of water increases (column IV in Table 2) because higher price levels provide economic incentive to implement conservation options and reduce use voluntarily. These results are consistent with contingent valuation findings on water supply reliability (Griffin and Mjelde 2000).

In practice some level of unreliability is associated with every water supply system, and therefore in most cases, the relevant analysis would be for the incremental value of moving from one distribution of unreliability to another. This can be simulated in this model by comparing results for different shortage probability distributions. For policy and utility planning purposes, the interaction between utility decisions (with resulting prices and ration quantities) and household decisions becomes important.

For the analysis of WTP to avoid shortage, two aspects of the reliability of a water supply system are relevant. The reliability of a system is characterized by (1) the probability of having no shortages, and (2) the levels and probabilities of shortage associated with given levels of unreliability. This is illustrated by comparing the consumer's WTP to avoid shortage probability distributions A, B, and C described in Table 3. Though distributions B and C might seem more reliable than A, when we consider not only the probability of experiencing no shortage but the entire probability distribution, they all have an expected shortage value of 8 gal per day (gpd). However, these shortage probability distributions impose widely different costs on the consumer and different WTP to avoid them (Table 3). To cope with the shortages in Distribution C, long-term conservation options must be implemented, boosting the consumer's WTP to avoid this situation. The cost of long-term options is somewhat fixed as opposed to short-term costs that occur only during shortages. Therefore, the optimal solution would often be to implement short-term options first and turn to long-term options only if needed. This simple example supports the common notion that frequent small shortages should be preferred to big infrequent ones (CUWA 1994; Koss and Khawaja 2001). As discussed before, WTP to avoid rationing decreases as the price of water increases and the expected value (EV) demand curves converge with the no rationing case.

This example illustrates the importance of considering the probability distribution of the entire range of shortages. The distribution of unreliability between different shortage levels is an aspect of the consumer's WTP that contingent valuation studies of reliability cannot grasp because they consider only one shortage level.

To further illustrate the interaction between the shortage probability distribution and user's WTP to avoid those shortages, let us analyze the examples in Table 3. Distributions A, E, F, G, and H (Table 3) show a series of shortage probability distributions where the frequency of small shortages gradually increases, making each distribution less reliable than the previous. For small shortages, the WTP to avoid shortages increases linearly with the EV of shortage and decreases (also linearly) with price, where small shortages can be handled by implementing only short-term conservation measures, so increasingly frequent shortages will impose costs directly proportional to the frequency of occurrence.

In contrast, for severe shortages (distributions I, J, and K in Table 3) WTP does not respond linearly to increases in the expected value of shortage. For this example problem, the cost (and the consumer's WTP) of moving from no shortage to 10% shortage exceeds the cost of going from 20 to 30% shortage. This nonlinear response is triggered by the need to implement more expensive long-term conservation and incur fixed costs to accommodate occasional large shortages. Moving from situations with small or nonexistent shortages to a distribution including large shortages (such as I) can force implementation of long-term conservation actions, which incur costs independent of shortage frequency. After implementing long-term conservation, if that same large shortage becomes more frequent (distributions J and K), long-term options will already be in place and conservation costs might be unaffected. As water price increases long-term conservation becomes more cost-effective relative to short-term conservation regardless of the level of rationing. Fig. 2 shows the very significant reduction in average water use imposed by Distribution I (with respect to the nonration case) compared to the milder curtailment of water use needed to absorb increased frequency of shortage. The average demand corresponding to Distribution G is not a perfect step function as one would expect in a linear optimization model. This results from a shift from short-term conservation to long-term conservation as price rises to extreme levels. When price becomes high enough, those short-term options that

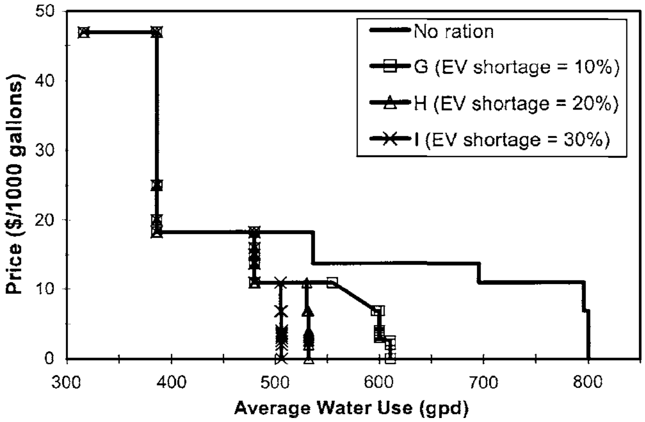

Fig. 2. Average single household water demand for different shortage distributions

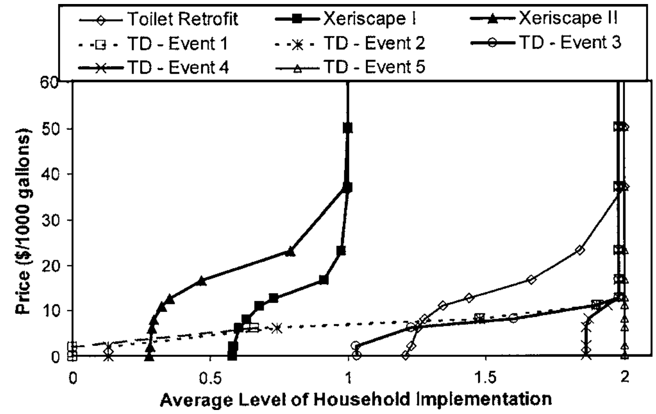

Fig. 4. Derived average demand for long-term conservation actions and toilet dams from Monte Carlo run (500 samples) for each given shortage in Distribution D

were being used because of rationing but were not otherwise cost-effective (dry shrubs) are replaced by permanent actions (toilet retrofitting or xeriscape II). The benefits of paying the high cost of the dry shrubs option only during shortages are less than the benefits of extra water conservation made possible by long-term options (reducing both the water bill and the need for the dry shrubs option).

To emphasize the complex relationship between the WTP and the shortage probability distribution, the comparison between distributions D and L illustrates the particular case that for a particular consumer there is no WTP to go from Distribution L to the more benign D, which represents a 1% reduction in the EV of shortage! This is because the possibility of having a very severe event (40% shortage) makes long-term conservation necessary. For this particular set of parameters these long-term conservation actions alone are more than enough to cope with small shortages of 10% and no extra short-term options are required. Therefore, changing the probability of a small 10% shortage has no effect on costs and therefore on WTP.

Formulation for a Class of Consumers with Rationing

The analysis presented above can be extended to a class of consumers. Fig. 3 presents the average demand curves for a class of customers confronted by shortage probability Distribution D. The demand curve for event 1 (no shortage) illustrates the permanent effects of long-term conservation actions (reducing water use for all events). The curves for the no ration case with both deterministic and probabilistic cost and effectiveness parameters are provided for comparison. Required conservation through rationing reduces consumers' responsiveness to price for an intermediate range of prices.

Statistical analysis of the Monte Carlo simulation results of household response to rationing can provide useful information about the structure of demands for water and conservation options within a particular class of customers. Fig. 4 provides the derived average demand for long-term conservation options and shows how higher water prices increase use of conservation. Fig. 4 also shows how average implementation of a specific conservation option [installing a toilet dam (TD), up to two per household] in the Monte Carlo run responds to price for each shortage event.

For nonimplemented conservation options their "reduced cost" sensitivity analysis results indicate the reduction in implementation cost necessary for an option to be employed and the effect of that option's cost on its market penetration among households. Fig. 5 shows the cumulative percentage of people using the xeriscape II option. All households would use this option if its cost were reduced by $774.

Perhaps the most relevant information concerns the variability in willingness-to-pay to avoid shortage within a class of customers. Fig. 6 shows that for a given retail price of water ($6 per 1,000 gallons in this case), some customers are willing to pay more than $947 per year to avoid shortage probability Distri-

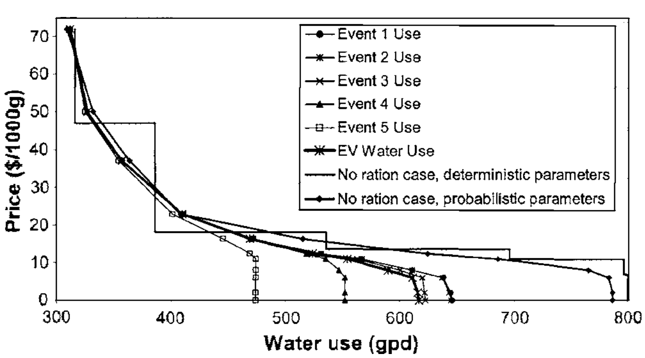

Fig. 3. Customer class demand curve for Distribution D {0.6,0.1,0.1,0.1,0.1}, sample size=500

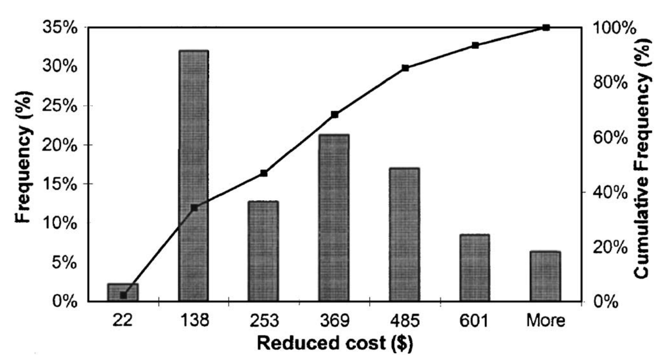

Fig. 5. Reduced cost of xeriscape II and effect on implementation for shortage probability Distribution D

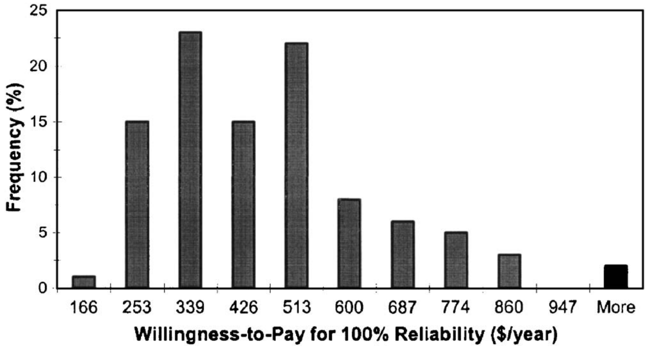

Fig. 6. Histogram of Monte Carlo results (price=$6/1,000 gal.), WTP to avoid shortage probability Distribution D

bution D while others are willing to pay only $167. Such differences in WTP are derived from the structure and variability of preferences, options, and costs of each customer and have practical and ethical implications when customers are asked to fund reliability improvements.

Some customers would pay a premium for a preferred level of reliability. The funds generated by a premium could finance long-term conservation or reliable supply options (desalination plants, dry year water contracts with farmers, etc.). A system of contracts could exist to offer higher than standard reliability for customers willing to pay for it and discount rates for lower reliability (i.e., interruptible service during drought). Such reliability pricing already operates in the electric industry (Flory and Panella 1994). In the water industry such a pricing might allow industrial users or emergency service facilities higher reliability or priority during drought, but at a higher price.

This modeling approach can be extended to a whole water service area by restating the problem using different coefficients and conservation options for different locations and classes of consumers. The service area can be discretized into relatively homogeneous groups and the partial results from each group aggregated to study the overall effect of different parameters and shortage distributions on consumers' WTP. Alternatively, we could account for differences in the water service area by varying the variability in model parameters.

Discussion

The probabilistic optimization approach presented here enables the derivation of water demand curves consistent with our current understanding of residential water use and management. Demand curves for different shortage events can be derived easily and used to study the effects of rationing, technology, and price on customers' water use. This approach could provide information to understand and perhaps predict the effects of retail water price and the interaction of long-term and short-term conservation actions on water demands. These interactions are important for water utilities because long-term conservation can significantly affect water utility revenues and operation. Implementing long-term conservation options entails significant (and permanent) reductions in water use that reduce utility revenue, which lead to price increases likely to make water conservation more attractive to customers. However, if appropriately planned, long-term conservation can be integrated with drought water pricing, other drought management actions, agreements to sell surplus conserved water, and expansion to serve new customers in a way coordinated to provide sufficient revenue for the utility at the least cost to consumers.

The main contribution of this model is to derive estimates of consumers' WTP for changes in reliability in ways consistent with economic theory, avoiding inconsistent results sometimes obtained from contingent valuation studies (CUWA 1994). Another advantage over contingent valuation studies is the model's capability to examine a complete shortage probability distribution and the ability to account for price effects.

The WTP interpretation of the results rests on the assumption that the model's costs coefficients are estimates of the WTP of customers to avoid implementing specific water conservation options and that the household optimization process is costless. This is likely to be more reasonable in areas experienced with drought shortages or where public conservation education has been particularly effective. The Monte Carlo probabilistic optimization approach presented has the advantage of providing information about consumer variability in WTP as well as information for exploring such innovative management options as "priority pricing" or urban water markets.

This approach could also contribute to the design of cost-effective conservation programs by using the information provided by the model (market penetration estimates, implementations for different events and prices, reduced costs for each option, sensitivity analysis) to adapt conservation programs to the characteristics of each group of customers. The model can be used, for example, to estimate the effects of a utility's water conservation campaign. This can be done by fixing to the extent possible the financial costs of a conservation measure and letting the variation involve only the "perceived or inconvenience costs" (time and effort spent implementing a conservation option). The approach should also provide a way of integrating retail water price into studies of the economic impact of shortage or water resources planning models that explicitly consider urban shortage management (Hoagland 1998; Jenkins and Lund 2000).

The model's parameters may vary for different events. Indeed the effectiveness of some short-term conservation actions may increase with the severity of the shortage. Similarly, cost coefficients might be reduced in extreme events if customer concern about drought reduces perceived costs. Appropriate long-term monitoring of conservation effectiveness and specific contingent valuation studies might provide useful estimates.

There is long-standing literature regarding problems with the expected cost-minimizing behavioral assumptions (Allais 1953; Kahneman et al. 1982). Perhaps the greatest problems are the assumption of perfect rationality and the absence of risk aversion in the expected value formulation. Risk aversion would tend to increase use of permanent conservation options over short-term options beyond those suggested by an expected value model. Subrational decision making (within the constraints) would raise the level of conservation (reduce water use) associated with any given level of price. Also, the small number of options in this model probably reduces responsiveness of water use to price and increases the cost of response to rationing and prices, perhaps mimicking imperfect information.

Finally the formulation presented constrains the household to meet the ration level for all events independently of their probability of occurrence. This constraint results in extremely risk-averse behavior by the household; a very small probability of occurrence (i.e., 0.1%) of a shortage event can force the household to reduce water use and incur long-term conservation costs. Further it assumes that households can perfectly monitor their water use and that the water agency can cut water supply to the household when it has consumed its ration. An alternative approach would be to introduce price penalties for exceeding the ration level.

The approach taken is one of microscale modeling of consumer demand decisions. This requires a great deal of model calibration and computational effort for demand studies of classes of consumers and service area studies. An alternative approach would be to use a more semiempirical approach such as positive mathematical programming (Howitt 1995). Here, a quadratic matrix in the objective function of a quadratic program or two-stage quadratic program might be calibrated based on aggregate consumer decisions and water uses, either by customer class or by water service area.

Conclusions

The two-stage linear programming approach presented by Lund (1995) is extended to include the retail price of water and to allow for variability in the model's parameters for an urban population. This allows derivation of demand curves for water (with or without rationing) consistent with current knowledge of residential water demands. Derived demands for conservation options also can be obtained using this approach. The model provides information about the interaction between long- and short-term conservation, the retail price of water and water use in the residential sector (which should prove some understanding and perhaps greater predictability of complex household water conservation decisions), and for water agencies designing water conservation programs. Under the assumptions that the model's cost coefficients represent the consumer's WTP to avoid implementation of specific conservation options and that the customer behaves rationally (expected cost minimization behavior), probabilistic estimates of WTP to avoid specific shortage probability distributions are obtained.

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