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LEONARDO DA VINCI'S TENSILE STRENGTH TESTS: IMPLICATIONS FOR THE DISCOVERY OF ENGINEERING MECHANICS

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In one of Leonardo Da Vinci's notebooks, an experiment is described where strengths in tension are measured for various lengths of wire. The notebook indicates that the results of these experiments were that longer wires were weaker than shorter wires. This result de[illegible] classical mechanics of materials. This conflict has been explained as a note-taking fai[illegible] by Leonardo. This short note develops an alternative explanation, based on the likely heterogeneity of the mechanical properties of the wire and elementary probability theory. This latter explanation has implications for the difficulty and delays experienced by early investigators into the mechanics of materials.

Keywords: History; Strength of materials; Probability; Imperfection

INTRODUCTION

It took thousands of years for the laws of engineering mechanics to be uncovered and understood and Leonardo Da Vinci's work showed great insight in this field for its time (Duhem, 1906). However, several of his experiments (c. 1500) would have given greater insight into engineering mechanics, but their results were apparently interpreted incorrectly. In at least one case, the apparent incorrect interpretation has another explanation, a physical explanation based on the

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likelihood of heterogeneous material properties in renaissance materials. If such an explanation of Leonardo's apparent error is correct, it raises the possibility that the more complex nature of mechanics for materials with heterogeneous properties could have obscured classical mechanics of materials and the permutations of classical mechanics that we currently use to consider non-homogeneous materials. This short note examines Leonardo Da Vinci's study of the tensile strength of iron wire.

LEONARDO DA VINCI'S TENSILE TESTS

In his notebooks (CA, 82v-b) Leonardo Da Vinci describes an experiment for studying the tensile strength of wire, entitled, "Testing the strength of iron wires of various lengths". In the experiment, a wire of a given thickness and length was used to suspend a basket. The basket was filled slowly with sand, fed from an adjacently suspended hopper (Fig. 1). When the wire suspending the basket breaks, a spring closes the hopper opening, and the basket falls a short distance into a hole, so as not to upset the basket. The sand in the basket was then

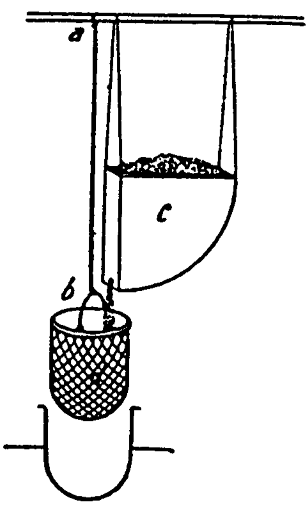

FIGURE 1 Da Vinci's hanging basket.

weighed to establish the tensile strength of the wire (Leonardo Da Vinci, 1972; Parsons, 1939).

Parsons' partial translation of Leonardo Da Vinci's text follows (1939): "The object of this test is to find the load an iron wire can carry. Attach an iron wire 2 braccia long to something which will firmly support it, then attach a basket or similar container to the wire and feed into the basket some fine sand through a small hole placed at the end of the hopper. A spring is fixed so that it will close the hole as soon as the wire breaks. The basket is not upset while falling, since it falls through a very short distance. The weight of sand and the location of the fracture of the wire are to be recorded. The test is repeated several times to check the results. Then a wire of 1/2 the previous length is tested and the additional weight it carries is recorded; then a wire of 1/4 length is tested and so forth, noting the ultimate strength and the location of the fracture".

CONFLICT WITH CLASSICAL MATERIAL MECHANICS

Parsons (1939) notes that the result described, where shorter wires supported a greater weight, conflicts with the classical theory of mechanics of materials. Classical theory holds that the wire's length should be irrelevant, since the stress should be the same along the entire length of such a wire and the weight of the wire should be negligible compared to the weight in the basket. The wire's diameter, however, should be important, with a wire of double the diameter having a tensile strength four times greater.

This apparent conflict with classical engineering mechanics is fundamental. In the case of a wire under tension, classical engineering mechanics holds that the stress in each unit length of the wire will be identical and that, therefore, each unit length of wire should behave identically. Therefore, the entire length of wire should have the same strength as a short length. However, an increase in the diameter of the wire increases the cross-sectional area of the supporting wire and therefore reduces the weight that must be supported by each unit area of the cross-sectional area. *I.e.*, increasing the wire's diameter decreases the stress in the wire, allowing it to support a greater weight before some ultimate failure stress is reached. To solve this problem by elementary classical engineering mechanics, one needs know only the wire's diameter and the ultimate stress the wire's material is capable of supporting. This ultimate stress is a material property that is assumed to be constant throughout the material.

Parsons' explanation of the recorded result is then that Leonardo Da Vinci, in his notes, mistakenly recorded the experiment and its results. Parsons suggests that instead of meaning "length", Leonardo meant "thickness", and instead of meaning "additional" weight, he meant "lesser" weight for the load carried by the smaller wire. With these two changes, the text indicating the result of the experiment is changed to: "Then a wire of 1/2 the previous [thickness] is tested and the [lesser] weight it carries is recorded; then a wire of 1/4 [thickness] is tested and so forth, noting the ultimate strength and the location of the fracture".

While Parsons notes that there are frequent errors in Leonardo's notebooks, the particular errors suggested by Parsons, while plausible, seem unlikely. Two words must be changed to relatively opposite meanings, and the word for length appears in several places in the text, including the section title. It seems unlikely that Leonardo Da Vinci actually studied the effects of wire thickness when the entire test repeatedly discusses wire length.

AN EXPLANATION

An alternative explanation of the apparent error in Leonardo's notes springs from the homogeneity of the wire's material properties and the constancy of the wire's diameter assumed by classical engineering mechanics. These assumptions also were made by Parsons, apparently himself an engineer.

If the material properties and/or the diameter of the wire are assumed to have varying properties, the behavior of the wire can be entirely different, and in agreement with Leonardo's notes. This explanation basically arises from the old saying, "A chain is only as strong as its weakest link", or, in this case, a wire is only as strong as its weakest cross-section. Assume that the ultimate strength of Leonardo's wire is not constant. In terms of classical mechanics of materials, the variation in the ultimate strength of wire per unit length could be due to variation in actual wire diameter or variation in the iron's material properties. Perhaps because of defects or unevenness in the material or in the process of being worked into wire, some unit lengths of wire have different ultimate strengths than other unit lengths. The strength of a length of wire would then be the strength of the weakest unit length. The longer the wire, the greater the likelihood of the wire's length containing a particularly weak unit length. Under these circumstances, longer wire will be weaker, on average.

This can be shown more rigorously with a bit of mathematics. Given a probability distribution of the wire material's ultimate stress, $\Pr(s_u)$, and the probability distribution for the actual diameter of the wire, $\Pr(D)$, the probability of a unit length of wire having an ultimate strength greater than a load W (units of force or weight) is $\Pr(\pi s_u D^2/4 > W)$, assuming that the material's ultimate stress varies only lengthwise and not radially within the wire. A hypothetical distribution for $\Pr(W)$, the probability the ultimate strength of a unit length of wire is greater than some load W is given in Figure 2.

Given the probability distributions for wire diameter and ultimate material failure stress, and assuming that material failure stress is constant across any given cross-section, the following equation gives the probability that a unit length of wire can support some load W.

$$\Pr(W) = \int_0^{s_{u\cdot\max}} \Pr(s_u) \int_{D_l}^{D_{\max}} \Pr(D) dD\, ds_u,$$

where $D_l = \mathrm{SQRT}(4W/(\pi s_u))$ and $s_{u\cdot\max}$ is the upper limit of material failure stress and $D_{\max}$ is the upper limit of wire diameter.

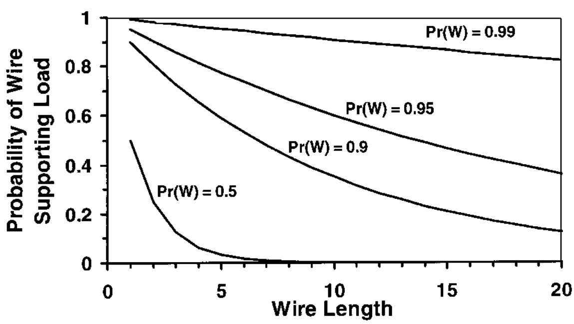

FIGURE 2 Probability distribution for ultimate wire strength per unit length.

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The probability that a wire of length L will be able to support more than some load weighing W, is the same as the probability that the all unit lengths in the wire can support this load. Assuming that the ultimate strength of a unit length of wire is independent of neighboring unit lengths of wire, then the probability of a wire of length L can support a load weighing W is given by:

$$\Pr(W|L) = \Pr(W)^L,$$

where $\Pr(W)$ is the probability that the ultimate strength a unit length of wire exceeds the load W from Figure 2. Since $\Pr(W)$ is less than one for all interesting loads on the wire, the probability that a length of wire L units long can support the load decreases steadily with increasing total wire length. It becomes increasingly likely that weak cross-sections of wire will exist in wires of longer length. This result would agree with Leonardo Da Vinci's apparent findings. It is an especially likely finding since Leonardo recommends repeating the experiment several times. Evidently, the strengths obtained from each wire length were not identical.

This phenomenon was formally studied in modern engineering mechanics beginning in the 1920s and 1930s. As reported by Timoshenko (1956), the importance of material imperfections for the ultimate strength of materials was first published by Griffith in the early 1920s. The effects of increasing size (including diameter and length) on increasing the probability of imperfections and reductions in strength was developed statistically by Weibull (1939). This behavior was demonstrated for high phosphorus steel in experiments by Davidenkov *et al.* (1947), somewhat similar to those by Leonardo Da Vinci and also showing failure stress decreasing for longer samples.

The discussion should now turn to the likelihood that renaissance iron wire had non-constant material properties or diameters.

EVIDENCE OF HETEROGENEITY IN RENAISSANCE METALS

The material properties of wire are somewhat heterogeneous even today. It is common, when performing tensile strength tests on iron or steel to obtain different strengths for the same diameter of wire. Consequently, several replicates are typically made for such testing (Thomas J. McCann, personal communication), something Leonardo also suggests.

The wire-making technique used could have introduced heterogeneities in the wire. The fabrication technique for the wire used in Leonardo's experiments is likely to have been the drawing of wire. Here, iron would be pulled through progressively smaller holes made in steel plates until a desired diameter was reached. This required great force to be placed on the wire. This technology is depicted and discussed by Biringuccio (1540). Leonardo Da Vinci was evidently familiar with this technology. Elsewhere in his notebooks Leonardo suggests metal drawing devices similar to those depicted by Biringuccio for drawing wire (Heydenreich, 1980).

This wire-making technique could have introduced heterogeneous behavior in several ways. Biringuccio's discussion of the manufacture of iron wire indicates the use of annealing periodically throughout the drawing. This would have involved heating the wire, followed by subsequent drawing. Uneven heating in the annealing process followed by inadequate drawing could introduce uneven material properties in the wire.

Biringuccio's text also mentions the need to re-shape the holes in the steel plates used for drawing wire. This indicates that the drawing of wire tended to deform the drawing holes in the steel plates. The wearing of these holes could also introduce heterogeneities in wire cross-section as it was being drawn.

Finally, the drawing of wire as depicted and discussed by Biringuccio often required the use of a heavy clamp to grasp the wire firmly and draw it through the hole in the steel plate. To firmly grasp the wire, it is possible that the clamp bit into the wire, deforming the wire and introducing a weakness. Whenever the clamp was moved, to make the wire longer, a new weak cross-section would be added.

IMPLICATIONS FOR THE HISTORY OF MECHANICS

The discovery of the modern theory of engineering material mechanics required more than mere development of a conceptual framework relating material geometry, properties of material, and force. The development of modern engineering mechanics also required that the important obscuring factor (or noise) of heterogeneous material behavior be realized, controlled for experimentally, and neglected within the basic theoretical framework. This approach allows us to treat heterogeneous material behavior as a variation of the simpler framework of homogeneous material mechanics. Leonardo Da Vinci's experiment might offer an instance where the discovery of an important principle of engineering mechanics was obscured by this failure to overlook an element of real material behavior that obscures our understanding of the material, material heterogeneity.

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