

## SPATIAL COMPLEXITY AND RESERVOIR OPTIMIZATION RESULTS

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*(Received 17 June 2002; Revised 27 July 2003)*

The complexity of a model can significantly affect its costs of development, ease of use, and the reliability of its output. However, it is difficult to find the levels of complexity appropriate for a particular model application because few quantitative studies of the effects of complexity on model results exist. This paper attempts to classify various types of model complexity. Using three indicators of spatial complexity, network flow models of Northern California's water system are formulated and compared at six levels of spatial aggregation. The results show that less complex spatial formulations result in fewer modeled shortages and lesser economic costs. However, the model continues to respond realistically to changes in hydrology. Additional research is needed to more completely understand the effects of model complexity on results, but these results provide an indication of the magnitude of these effects for a large water resource system.

*Keywords:* Complexity; Aggregation; Optimization; Network flow model

### 1 INTRODUCTION

Recent advances in computer technology and data management and acquisition have made possible the development of increasingly complex models. The high level of complexity incorporated in many recent models has renewed debate over the merits of simple versus complex models. Advocates of complex models (Nihoul, 1994) argue that they are more reliable, represent the system more comprehensively, and are less likely to be used inappropriately. However, simpler models are said to be less time-consuming and expensive to develop, require less data, and produce results that are easier to understand and interpret (Ward, 1989). The most common sentiment (Jackson, 1975; Palmer and Cohan, 1986; BDMF, 2000) seems to be that modelers should attempt to develop models that contain just enough complexity to accomplish accurately the project objectives, but no more.

These arguments underscore the importance of selecting an appropriate level of complexity for each model application. The level of complexity incorporated in a model has important implications for the costs and availability of input and calibration data, for model run time, and for interpreting model output. The recent trend for reservoir systems has been to

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develop larger and more complex models. In California, recent examples include CALSIM, a network flow simulation model of California's Central Valley water system (DWR, 2001), and CALVIN, an economically based network flow optimization model of California's inter-tied water system (Jenkins *et al.*, 2001; Draper *et al.*, 2003). Models of this complexity can only be developed with significant time and money. While increased model complexity

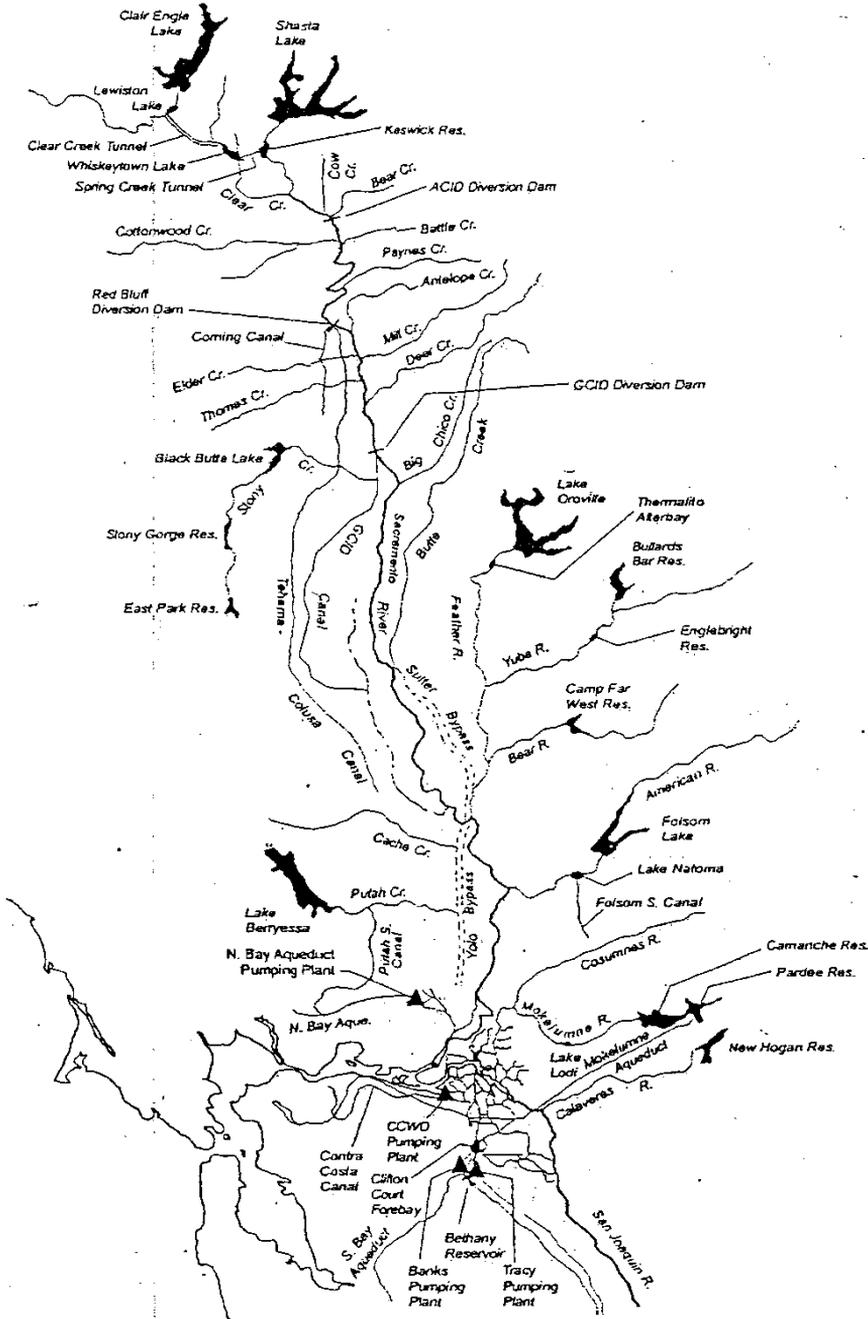


FIGURE 1 Sacramento valley water system.

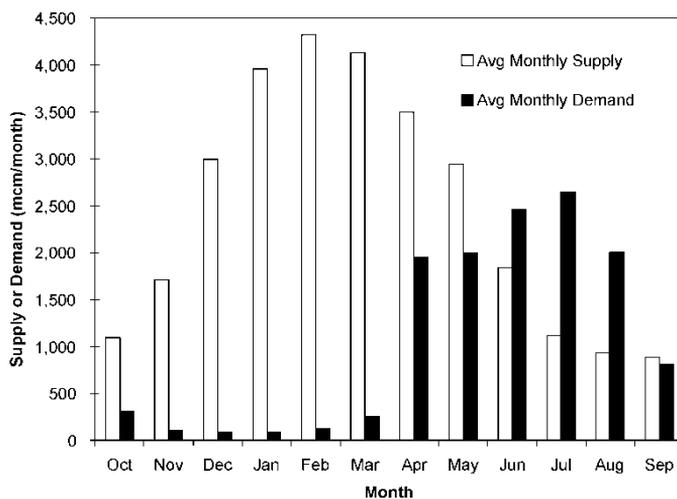


FIGURE 2 Average monthly demand and supply.

will presumably improve the reliability and accuracy of results (given sufficient data to describe the system), little data exist to evaluate the expected benefits of incorporating additional complexity into a model. The purpose of this study is to provide some insight into the magnitude of possible benefits of greater spatial detail in a reservoir systems model.

This paper presents the results of numerical experiments on a large scale network flow model of California's Sacramento valley water system. A map of the water system can be seen in Figure 1. Figure 2 shows the monthly hydrologic inflows and the total monthly demands in an average year during the model period. While the majority of the natural supply is available during the winter months, most of the demand occurs during the summer months. Therefore, the system depends on storage to supply the system's demands. Water that cannot be stored in reservoirs flows out through the Sacramento-San Joaquin delta and is lost to the system.

While the results cannot be generalized with great rigor, they offer some practical anecdotal evidence of the relative importance of spatial complexity in the modeling of reservoir systems. The study was performed by modeling the water system using multi-period optimization at different levels of spatial aggregation. The paper begins with a definitional discussion of model complexity, followed by a review of previous studies of the effects of complexity on model results. Next, the formulation of network flow optimization models of the Sacramento valley at six different levels of spatial complexity is described and the results of these models are presented and discussed. Finally, some conclusions are made on the impact of model complexity on the results and their interpretation.

## 2 WHAT IS MODEL COMPLEXITY?

There is no single accepted definition of model complexity. Most authors on the subject do not even attempt a definition. Applying the dictionary definition of complexity, Brooks and Tobias (1996) define model complexity as "a measure of the number of constituent parts and relationships in the model." This is the definition used here.

Model complexity can be divided into several distinct types and numerical indicators can be developed to quantify each type of complexity (Palmer and Cohan, 1986). Some types of complexity, with example indicators, appear in Table I. Among these types a distinction can

TABLE I Types of Model Complexity.

<i>Complexity type</i>	<i>Example indicator</i>
Spatial	Number of spatial variables
Temporal	Number of time steps in the model
Uncertainty	Number of stochastic variables in the model
Programming	Lines of programming code
Interface	Complexity of user interaction with the model
Input	Amount of input required to run the model
Calibration	Amount of data needed to run the model
Run-time	Time required to run the model
Interpretation	Time required to interpret model results

be made between those types of complexity that can be determined during model selection (spatial, temporal, uncertainty, programming, and interface) and those that are products of the overall complexity of the model (input, run-time, interpretation, and calibration).

In this study, the CALVIN model of California (Jenkins *et al.*, 2001; Draper *et al.*, 2003) is used to model the water system at six different levels of spatial complexity. Run-time and interpretation are used as additional measures of complexity for each case. Because the same model is used in each case, the indicators for the uncertainty, programming, and interface classes of complexity are the same for each case. Temporal complexity is identical for all cases since the same monthly 72-year period of record is used for all six test cases. Input complexity is neglected because the test cases are simplifications of an existing model and so it is difficult to evaluate how much effort would have been required to assemble the data for each individual alternative. In some cases, data were already available in disaggregated form, so the input data cost may actually increase slightly for the less complex models. Finally, calibration complexity is neglected because this study is not concerned with the accuracy of any of the individual model runs, but only with the differences between them. The indicators used to measure spatial, run-time, and interpretation complexity are discussed below.

## 2.1 Spatial Complexity

In this study, the spatial complexity is measured as the sum of the number of inflow links, reservoirs, and demand nodes contained in the system. A disadvantage of this measure is that certain aspects of spatial complexity, such as the representation of conveyance facilities, are neglected. However, in general the overall schematic complexity of each test case approximately corresponds to the relative values of spatial complexity.

## 2.2 Model Run-Time

Model run-time complexity is measured in terms of the number of decisions required of the optimization model and the number of iterations needed to find a solution. These are considered more reliable measures of complexity than the actual run-time because they are not influenced by the computer's processing speed. With recent advances in computer technology, model run-time complexity is becoming less important as a practical consideration in model selection.

## 2.3 Interpretation Time

The interpretation time is the amount of time required by the modeler to analyze the results produced by the model and to interpret their practical meaning. An important factor to

consider when making such a comparison is what output should be generated from each model. A more complex model usually generates more detailed results than a simpler model. The time needed to generate and evaluate all of the potential results from a complex model can be much greater than is required to interpret more aggregated results similar to those produced by a simpler model. However, it is often necessary to look at a model's detailed results to ensure that the model is behaving reasonably. In this study, it would be difficult to gauge accurately the amount of time required to evaluate each case individually because all six test cases involve the same water system. Interpretation time is estimated simply by the number of output time series reviewed in the results analysis for each case.

### 3 STUDIES OF MODEL COMPLEXITY

Despite the perceived importance of model complexity, relatively few studies compare the results of models at different levels of complexity, both generally and for reservoir systems models. This section provides a summary of available water-related studies and attempts to classify each study according to which type of complexity from Table I that each study is designed to evaluate. All of the studies analyzed either spatial or programming (*i.e.*, solution scheme) complexity, or both. Two studies looked at temporal complexity and one group of studies looked at uncertainty complexity as well.

Studies of spatial complexity include Palmer and Cohan (1986), who modeled the hydro-power system of the Columbia river using both a single reservoir and a multi-reservoir model and found a net annual difference of only about 1%. This is the only study found that directly compared the results of simpler and more complex reservoir models. Spatial complexity has received more attention in the field of hydrologic modeling. Wood *et al.* (1988) and others (Wood, 1995; Woods *et al.*, 1995) have attempted to find an intermediate scale at which the average hydrologic response is invariant or varies only slowly with increasing catchment area. Warwick and Cale (1987) proposed a method for water quality models that achieves a desired reliability by balancing errors caused by choosing a model of inappropriate spatial complexity and errors caused by uncertainties in parameter characterization to minimize overall modeling error. Warwick (1989) found that reducing one type of error often increases the other kind of error, often with an overall reduction in model reliability.

Loague and Freeze (1985), Jakeman and Hornberger (1993), and Boyle *et al.* (2001) studied the effects of spatial and programming complexity on rainfall-runoff models and found that simpler model formulations provided as good or better results than more complex formulations. However, Fontaine (1995) showed how a rainfall-runoff model with greater programming complexity produces better results for extreme flood situations. Ponce *et al.* (1978) and Keskin and Agiralioglu (1997) compared the accuracy and computational effort of numerical solution schemes of the Saint Venant equations and found that a simpler model formulation usually gave good results.

Studies of temporal scale include Sinha *et al.* (1995), who studied the effects of time and space scales flood routing results model, finding that the order of accuracy with time discretization is more important than with space discretization. Costanza and Sklar (1984) studied 87 models of freshwater wetlands with varying solution schemes and time and space scales, finding that simpler models tended to produce a better match with historical data.

A common technique for modeling multi-reservoir hydropower systems is to use spatial aggregation methods to simplify the solution of a stochastic dynamic program (Turgeon, 1981; Saad *et al.*, 1996; Archibald *et al.*, 1997; Turgeon and Charbonneau, 1998). Turgeon (1981) and Archibald *et al.* (1997) compared the solutions obtained with their stochastic methods with those obtained by solutions of the same system using deterministic

optimization models. Turgeon found a difference in the average annual value of less than 1% for a system of reservoirs in series, while Archibald *et al.* found a difference in the average annual value of 3.1% for a system of 17 reservoirs.

Other studies of model complexity exist in a variety of fields. These include comparison of two point snowmelt models under different weather and snowpack conditions (Bloschl and Kirnbauer, 1991), a study of models describing the decrease of galactic cosmic rays applied at both one and two dimensions (Le Roux and Potgieter, 1991), Stockle's (1992) study of the performance of plant canopy models at different levels of complexity, Palsson and Lee's (1993) study of red blood cell metabolism models, and a study of the effects of model complexity on the performance of automated vehicle steering controllers (Smith and Starkey, 1995). With the exception of Stockle (1992), all of these studies concluded that simpler models yield inadequate results in some situations. Thus, these models differ from reservoir, hydrologic, and water quality models in that the accuracy of their results is greatly influenced by model complexity. Perhaps more complete data sets are typically available in these fields to characterize more complex formulations.

#### 4 THE CALVIN MODEL

CALVIN is an optimization model developed to better understand California's inter-tied water system (Jenkins *et al.*, 2001; Draper *et al.*, 2003). CALVIN uses the network flow reservoir optimization model HEC-PRM (USACE, 1994) to maximize economic benefits by allocating water over a 72-year period of historical inflows. Economic benefits are represented by piece-wise linear economic value functions at each demand location and time. The entire CALVIN model contains 51 surface water reservoirs, 28 groundwater reservoirs, 19 urban demand regions, and 24 agricultural demand regions. The model results contain monthly time series of flow and storage for every element in the system. The alternative used as a base case for this study is an unconstrained water market, with water allocations limited only by physical and environmental constraints. This study was performed using a preliminary version of CALVIN, before calibration flows were introduced to normalize the available water supply. Because preliminary CALVIN runs showed few shortages under this scenario, water availability was artificially reduced for this study by reducing the external inflows by 20% and increasing the losses on return flows by 30%. While these assumptions are acceptable for studying model complexity, the optimization results presented here are not intended to represent accurately the system's current or potential operation.

For this study, only the northern portion of the CALVIN model (the Sacramento valley and Sacramento-San Joaquin delta) is used. The portions of the state outside of the modeled area are assumed to be operated identically for each alternative and are not modeled. The Sacramento valley portion of CALVIN contains 17 surface water reservoirs, 9 groundwater reservoirs, 7 urban demand regions, and 9 agricultural demand regions. Groundwater storage in each agricultural region is represented by a single reservoir. This detailed representation is considered the base case (test case A) for the present study and is the most complex model tested. Simpler model formulations are aggregated versions of this base case.

#### 5 TEST CASES

Six test cases have been developed at different levels of aggregation. Table II shows the spatial complexity of each case, which is defined as the sum of the number of inflow, reservoir,

TABLE II Complexity Measures for Each Case.

Case	Spatial complexity	# Iterations	# Decision variables	Interpretation complexity
A	80	1,523,663	442,603	129
B	65	1,383,246	382,551	103
C	44	816,591	203,695	69
D	26	373,372	118,660	37
E	8	105,396	44,498	15
F	5	53,037	32,114	14

and demand locations in each model formulation. Figure 3 depicts the spatial complexity of each case, with brief descriptions of each case given below. Complete descriptions can be found in Van Lienden (2000).

**Test case A: Full CALVIN Representation** – Case A is the full CALVIN representation of the northern portion of the California water system as described above.

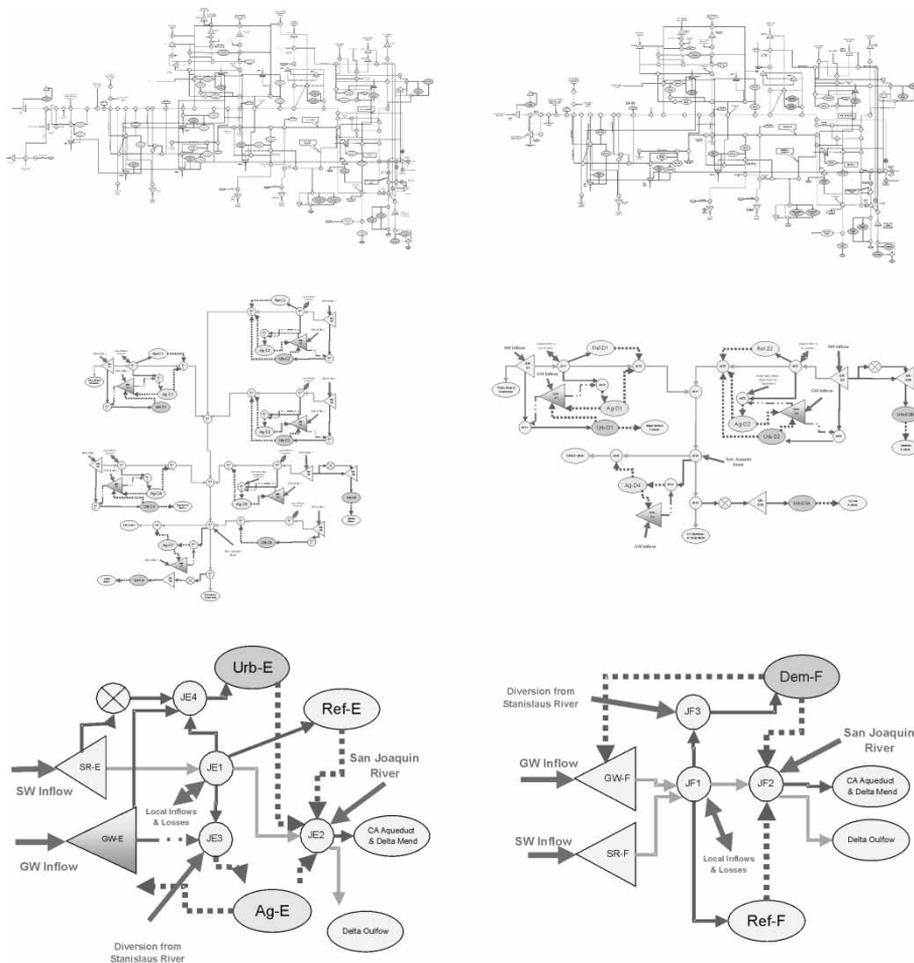


FIGURE 3 Schematics of cases A–F.

**Test case B: Local Aggregation** – Some regional storage and demand elements are aggregated.

**Test case C: Aggregation by River System** – The system is divided into nine regions. Within each region, the surface water storage nodes, groundwater storage nodes, agricultural demand nodes, and urban demand nodes are combined into single aggregate nodes of each type.

**Test case D: Aggregation of Eastern and Western Sacramento Valley** – Case C is expanded so river regions emanating from the eastern portion of the Sacramento valley and those emanating from the northern and western portions are combined.

**Test case E: Aggregation by Group Types** – Surface water storage, groundwater storage, agricultural demand, urban demand, and the environmental refuges are each aggregated into individual nodes representing the entire contents of the modeled system.

**Test case F: Full Aggregation** – All agricultural and urban demands are combined into a single demand node.

## 6 ISSUES AFFECTING CASE COMPARISONS

Some aspects of spatial aggregation make it difficult to compare the results of particular runs. These limitations can be divided into two classes – those caused by the aggregation of spatial elements and data and those caused by limitations of the network flow optimization model.

### 6.1 Aggregation Limitations

When developing cases B–F, the data in case A was duplicated as thoroughly as was possible given the simplified spatial formulations. In doing so, however, assumptions had to be made to combine data from several different links and storage nodes in case A onto individual elements in the aggregated systems. Of these, the most problematic involved aggregating demand regions and the constraints and costs of conveyance facilities.

Aggregation of demand regions was done such that the total demand of each aggregated region would equal the sum of demands of the component demand regions. While this assumption allows for one-to-one comparisons of deliveries between different cases, it does not account for the reduction in demand caused by reuse between the demand regions. To account for reuse, reuse links were added to each aggregated demand region that allowed return flows to be re-routed back to the region's delivery node. The amount of return flow available for reuse was conservatively limited to the amount available to the downstream demand if the upstream demands were fully supplied. While this exaggerates the amount available for reuse in virtually all months for most of the aggregated agricultural regions, the actual implementation of reuse was very low. For example, while case F has a reuse capacity of 986 million cubic meters (mcm) per year, the addition of this reuse link increased the average annual deliveries by only about 12 mcm/year.

The aggregation of conveyance facilities affected the representation of canal capacities, minimum flow requirements, and pumping costs in the simpler cases. Because aggregated elements essentially have infinite and costless capacity between them, many canal capacity constraints are neglected altogether, and many of the canals that do appear are limited by the demand of a particular region within an aggregated region rather than by the size of the canal. While most minimum flow constraints and pumping costs are represented in all cases, they are aggregated in the simpler cases and therefore do not reflect local conditions. In a few cases it was necessary to eliminate certain data altogether. For example, in case F the

pumping costs on deliveries to San Francisco Bay Area demand nodes are neglected because the urban and agricultural demands are combined. This limitation may affect the economic results of case F, and could explain why the shadow values for case F are consistently higher than those shown for case E.

## 6.2 Limitations of Network Flow Optimization

For many water resource problems, numerous near optima may arise. Thus, small variations in formulation (such as might arise in aggregation) can result in different solutions (Rogers and Fiering, 1986). While these solutions will have similar objective function values, and most likely similar overall results, the values in any given year or local region can be very different. It can therefore be difficult to interpret differences in results between different cases for particular time periods because small changes in the input data can cause the model to arrive at a different optimum and yield different results. For example, although average annual outflows from the Sacramento-San Joaquin delta for case E are less than those for cases A–D, for the wet 1982–83 water year case E has much larger delta outflows. To test the effect of such deviations, case E was re-run with an arbitrarily imposed cost of \$0.08 per thousand cubic meter (tcm) added for every tcm of delta outflow above 13,600 mcm in any given month. The overall results of case E with this cost were very similar to those without the cost included, with the same average annual shortages and delta outflows, but with delta outflow in March 1983 reduced from 30,800 to 13,600 mcm.

## 7 MODEL RESULTS

The primary indicators used to evaluate the performance of the six cases are the average shortages and groundwater mining resulting from each model formulation. These results can be found in Table III. In CALVIN, shortage is defined as the difference between the maximum economic demand and the actual delivery. Each case has 2176 mcm/year of urban demand and 11,175 mcm/year of agricultural demand. Cases with higher complexity show larger urban and agricultural shortages and more groundwater mining. Because urban deliveries are valued much more highly than agricultural deliveries, all cases show much fewer urban shortages. Only cases A–C show significant urban shortages. While all cases show significant agricultural shortages (due to the artificial reduction of available water), there is a gradual increase in shortage quantity as the complexity increases. The groundwater mining in cases A–D is a function of the water balance in individual groundwater basins.

To evaluate the accuracy of each case, it is assumed that the case A results are 100% accurate and that deviations from case A result from spatial aggregation. While in actuality case A

TABLE III Comparison of Results for Cases A–F.

Case	Total shortages (mcm/year)	Urban shortages (mcm/year)	Groundwater mining (mcm/year)	Net delivery (mcm/year)	% Error in net delivery	% Error in total shortage
A	2814	10	52	10,485	0.0	0.0
B	2642	9	42	10,667	1.7	6.1
C	2623	9	42	10,686	1.9	6.8
D	2365	0	42	10,944	4.4	16.0
E	2120	0	0	11,231	7.1	24.7
F	2017	n/a	0	11,334	8.1	28.3

(as an optimization model) represents a major deviation from actual system operation, it is assumed that, given sufficient data, the most complex formulation will be the most accurate and therefore can be used as a benchmark for evaluating other formulations. Two measures used for comparison are the total shortages and the average annual net delivery, which is calculated as follows:

$$\text{Net delivery} = \text{Total demand} - (\text{total shortages} + \text{groundwater mining}).$$

The percent errors are calculated for each case by taking the percent difference between the net delivery or total shortage for the case in question and that for case A. The percent errors in net delivery and shortage are shown for each case in Table III. Table II contains the complexity measures for each case. Figure 4 shows the percent error in net delivery versus percent complexity for each complexity measure. These results show good correlation between the complexity measures. As complexity decreases, the percent error increases. However, the percent net delivery error does not increase linearly with complexity. The error increases the most between cases E and F but very little between cases B and C. When measured in terms of shortage rather than net delivery, the percent errors of cases B–F are much larger. Figure 5 shows the percent errors in shortage and net delivery for each case. The percent shortage errors are less than 7% for cases A–C but greater than 16% for cases D–F, which indicates that some aggregation is possible with minimal error but that greater aggregation may produce unacceptable errors.

The remainder of this analysis focuses on more specific aspects of the results. First, the annual time series results will be analyzed for the entire system. Then, monthly time series analysis will be performed on the entire system for specific time periods. Finally, the differences in results for surface water reservoirs at different levels of aggregation will be analyzed.

### 7.1 System-Wide Analysis of Annual Time Series

Figure 6 shows the reliability of total annual deliveries for each case. All six cases show the same basic pattern of reliability. With a few exceptions, the curves are ordered as might be expected, with the more complex cases suggesting less reliability of delivery. The greatest

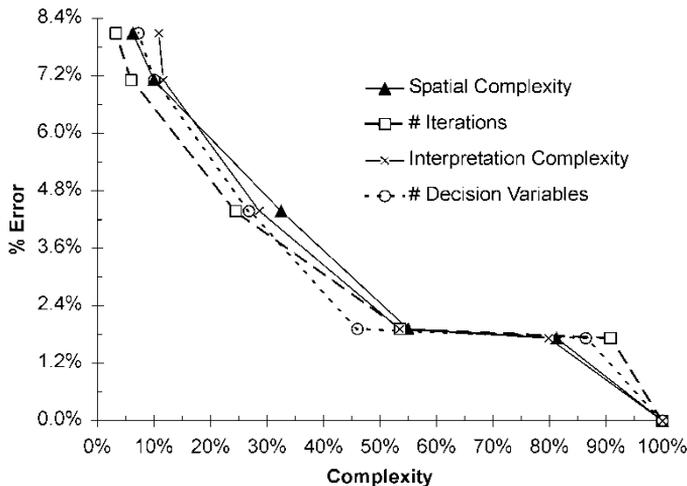


FIGURE 4 Net delivery error versus complexity.

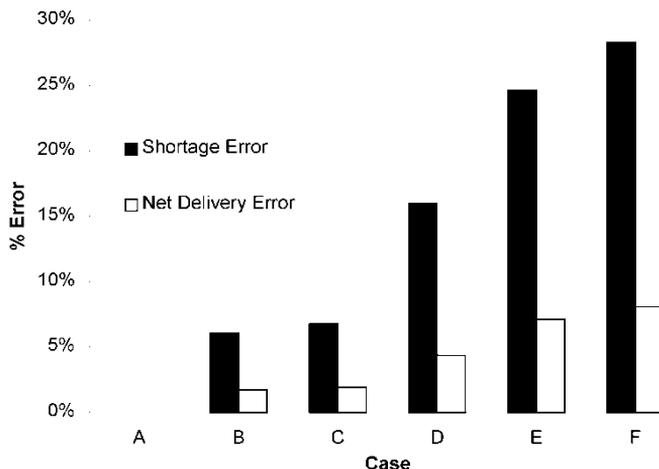


FIGURE 5 Percent error for each case.

exception is case B, which is less reliable than case C for deliveries that are exceeded more than 60% of the time. While spatial aggregation tends to allow greater delivery reliability during the years with more water available, these gains can be made at the cost of greater shortages during drier years. Aggregation of storage allows more efficient water use during wetter years but is less of a factor during drought years. It is unclear why the model does not use the greater flexibility of aggregated formulations to mitigate the more severe droughts, in which the marginal cost of shortage would be higher. This may be caused by the problem of flat objective function surfaces in the CALVIN model, by which several different possible solutions give similar objective function values. The spatial aggregation may cause minor deviations in individual years that skew the shape of the reliability curves. This explanation is supported by the annual time series of shortages which show frequent fluctuations in magnitude order between cases from year to year. For example, Figure 7 shows shortages for each case from 1973 to 1993. In 1974, case B has slightly more shortage than case A, which is followed by cases C, D, F, and finally E. By 1976 case B has fewer shortages than case

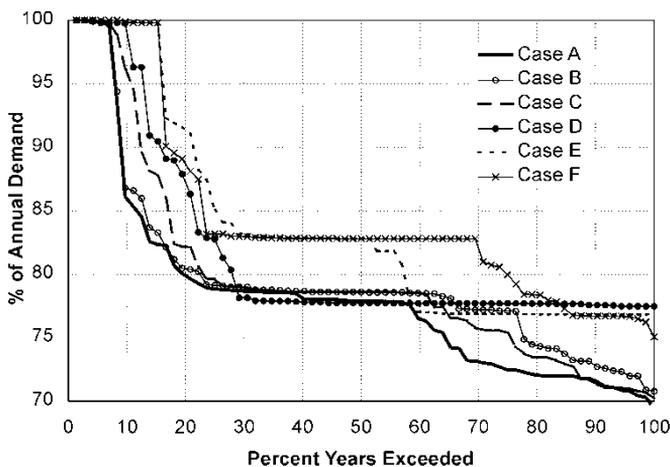


FIGURE 6 Reliability of total deliveries for each case.

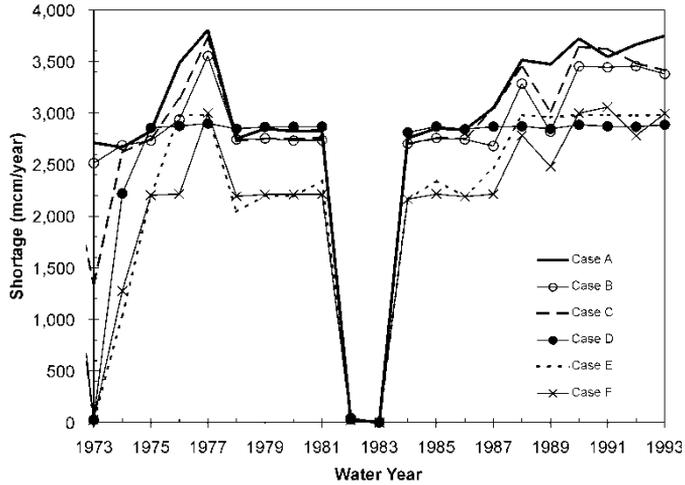


FIGURE 7 Total annual shortage.

A, C, or E. Between 1988 and 1989, both cases B and C show sharp drops in shortage while the other four cases stay flat. Case D shows a unique response to both droughts in that its level of shortage rises the fastest of all six cases in both 1975 and 1984, to about 2700 mcm, but then remains flat so that case D has the lowest shortage in 1977 and from 1990 to 1993. Such fluctuations in curve order make it difficult to draw definite conclusions about the differences between the models in individual years. However, the results in Figure 7 indicate that all model formulations are reacting realistically to changes in hydrology, with shortages during the drought years of 1976–77 and 1987–92 and virtually no shortages during the extremely wet years of 1982–83.

Urban shortages for this period appear in Figure 8. All cases with urban shortages (A–D) experience them during the 1987–93 drought and case A, with the largest total urban shortages, has additional shortages from 1976 to 1978. Although the differences in urban

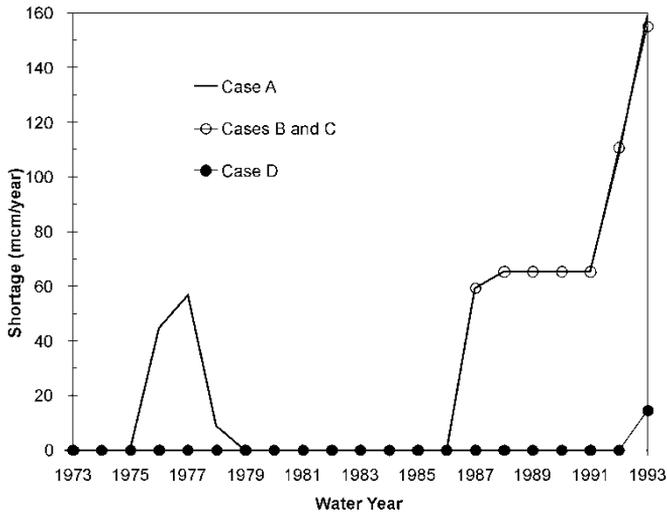


FIGURE 8 Total annual urban shortage.

shortage are small, they are significant because of the high economic value of urban demands. All urban shortages occurred in the demand node for the East Bay Municipal Utilities District (EBMUD), which is isolated from the rest of the system and can only receive water from the Pardee reservoir on the Mokelumne river via the Mokelumne aqueduct. In cases B and C, the Pardee reservoir is combined with the Camanche reservoir, providing additional storage space for EBMUD. With this change, the 1976–78 shortage is eliminated. The further aggregation in case D of the Mokelumne river with the rest of the Eastern Sacramento valley eliminates almost all of EBMUD's shortage, while additional aggregation in cases E and F eliminates the shortage altogether. Thus, there is a gradual reduction in EBMUD's shortage as the amounts of storage and external inflows available to supply the region increase.

## 7.2 System-Wide Analysis of Monthly Time Series

On an average monthly basis, there is very good correlation between the model complexity and the level of deliveries, as shown in Figure 9. All cases have little shortage from October through to March, a time of few agricultural water demands. During the summer, all cases have higher shortages and, with a few exceptions, more complex cases have higher shortages than simpler cases for each month. The remainder of this section is focused on the 1976–77 drought.

As shown in Figure 7, each case responded differently to the 1976–77 drought. While cases A, B, C, and F experienced shortage peaks during 1976 and 1977, shortages in cases D and E tended to plateau at smaller levels, but maintained that level longer. In addition, although case B had larger overall shortages, case C tended to have larger annual shortages during the drought years. These trends also can be seen in the plot of monthly agricultural shortages shown for each case from January 1976 to January 1978 in Figure 10. While all six cases had the largest shortages during June and July, cases D and E had the smallest peaks in both years with the exception of case B in 1976. In addition, case C had the largest peak in 1976 and virtually the same shortages as case A in 1977. The magnitude of particular shortages for each case in 1976–77 may help to explain the economic differences between the cases in those years.

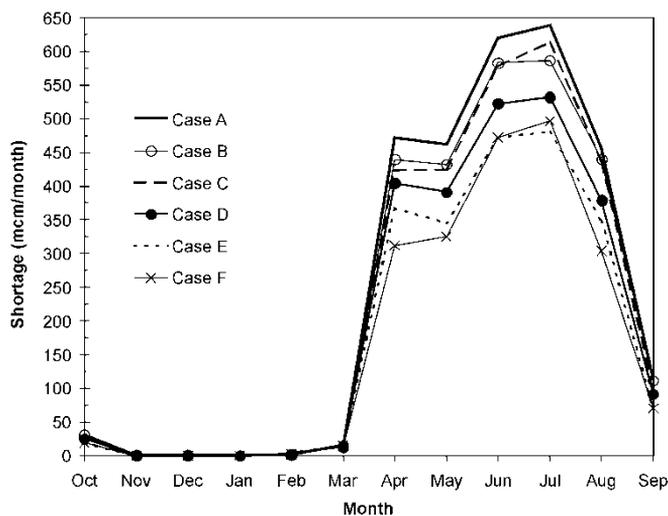


FIGURE 9 Average total monthly shortages.

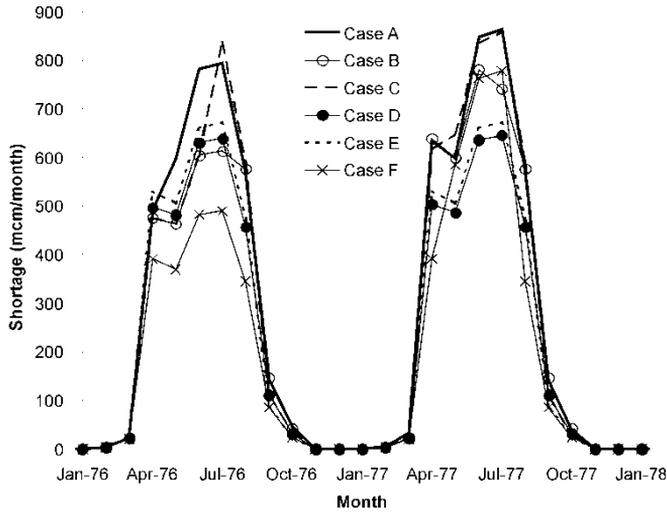


FIGURE 10 Total agricultural shortages.

Because HEC-PRM is a deterministic optimization model, it can anticipate a drought and fill the reservoirs to capacity to increase later deliveries. The model also can anticipate the drought's end and completely drain the reservoirs during the last dry year. Thus, all cases reach a peak in total surface water storage in March 1974, reach another peak in May 1975, and then are depleted until a minimum in November 1977. During March 1974, some reservoirs in all six cases have a very high shadow value on reservoir capacity. This shadow value reflects the cost of shortage during the coming drought. The reservoir containing the Shasta lake is at capacity in every case, and the shadow values in March 1974 for this reservoir for each case appear in Table IV. Table IV also shows the shadow values for required delta outflow and required deliveries to Sacramento valley wildlife refuges in April 1974. Each of the economic values are highest in case C, followed by cases A, B, D, F, and E. The low values shown for case E are caused by the unusually low shortages of case E in 1974 (see Fig. 7). While the other cases have agricultural marginal willingness-to-pay values in 1974 comparable to those in 1976–77, the maximum agricultural marginal willingness-to-pay for case E is only \$35.0/tcm in 1974. With the exception of the Sacramento valley refuges, all values are similar. The increased economic values for cases C and F may be due to their unusually high shortages during 1976–77. The economic values for case F may be higher than those for case E because of the absence of pumping costs to urban demand regions. The Sacramento valley refuges shadow value is \$6–16/tcm less than the

TABLE IV Economic Output (in \$/tcm) for Each Case.

Case	Lake Shasta storage shadow value (March 1974)	Required delta outflow shadow value (April 1974)	Sac valley refuges shadow value (April 1974)	Agricultural marginal willingness-to-pay (1976–77)
A	108.3	108.6	94.1	112.3
B	107.5	107.9	94.1	92.1
C	112.3	112.7	95.5	119.0
D	97.5	97.9	81.9	93.1
E	40.2	40.3	34.1	84.3
F	84.5	84.8	71.6	84.3

storage shadow value in every case. This difference occurs because the refuge is typically in parallel with the agricultural demand region while the reservoir and the delta outflow are in series. Thus, if the agricultural delivery increases 1 tcm because of reduced refuge demand, all return flow from that 1 tcm would remain needed to supply delta outflow. However, if the delta outflow requirement was reduced by 1 tcm, all of return flow from that 1 tcm would be available for reuse, increasing the value of that unit of water.

Marginal and shadow values are related to marginal willingness-to-pay values for the agricultural regions. The marginal willingness-to-pay is defined as the amount that the demand regions would be willing to pay to receive one thousand additional cubic meters of water in a given time step. Table IV shows the largest marginal willingness-to-pay value for any agricultural region in the Western Sacramento valley for every case during 1976 or 1977. These marginal willingness-to-pay values are of the same magnitude as the marginal values and shadow values. It is difficult to draw conclusions from a comparison of the marginal willingness-to-pay across different cases because for the more complex cases they represent the maximum value of many different demand regions, each of which has a different marginal willingness-to-pay.

### 7.3 Reservoir Shadow Values

When two or more reservoirs are combined, the probability that the aggregated reservoir will have a storage equal to its maximum or minimum storage is less than the probabilities of any of the reservoirs represented individually. The reduced pressure on reservoir storage is reflected in the average storage shadow values. Table V shows the reservoir in each case with the largest average shadow value on its capacity constraint. In case A, lake Englebright has the highest average shadow value, \$7.39/tcm. In case B, lake Englebright is aggregated with New Bullards Bar, reducing the aggregated reservoir's average shadow value to only \$2.85/tcm, much less than Black Butte lake's \$4.61/tcm. In case C, Black Butte lake is aggregated with other reservoirs, leaving the Pardee/Camanche aggregated reservoir with the highest average shadow value. In case D the Pardee/Camanche reservoir is aggregated with additional reservoirs and the EBMUD local storage has the highest shadow value. The EBMUD local storage is one of two disaggregated reservoirs in case D – the two aggregated reservoirs have average shadow values of \$0.79 and \$1.22/tcm, which are very similar to the average shadow values for the aggregated surface water reservoirs of cases E and F. All of the reservoirs with the highest shadow values in cases A–D are relatively small reservoirs. Englebright lake has a capacity of 81 mcm, Black Butte lake has a capacity of 185 mcm, Pardee/Camanche reservoir has a capacity of 790 mcm, and the local EBMUD storage has a capacity of 189 mcm. Because aggregated reservoirs tend to have relatively large capacities, they are likely to have smaller average shadow values. Thus, reservoir aggregation can significantly affect the valuation of surface storage. For capacity expansion planning and valuation purposes, more aggregate representations are likely to

TABLE V Storage Shadow Values.

<i>Case</i>	<i>Reservoir</i>	<i>Average shadow value (\$/tcm)</i>
A	Englebright lake	7.39
B	Black Butte lake	4.61
C	Pardee/Camanche reservoir	3.25
D	EBMUD local storage	2.77
E	Total aggregated reservoir	0.80
F	Total aggregated reservoir	0.91

reduce the estimated values of new capacity and change the locations of preferred expansion locations.

## 8 CONCLUSIONS

The results in this paper provide insight into the effects of spatial complexity on the results of a network flow optimization model. Given sufficient quantity and quality of data, more complex model formulations are presumed to give more accurate results. However, time, budget, or data constraints may require simpler formulations. Based on the results of this study, the following differences can be expected when using spatially simpler formulations.

1. Less detailed models tend to generate higher deliveries and consequently fewer shortages. Here, the simplest model had 8% higher average deliveries and 28% fewer shortages than the most detailed model. (Because deliveries tend to exceed shortages, a percent error in shortage will usually exceed the percent error in deliveries.)
2. Because its shortage magnitudes are less, the simpler model will generate lower shadow and marginal willingness-to-pay values, and so may underestimate the economic values of facility expansion and increased deliveries.
3. While shortage magnitudes and economic values will be reduced in a more aggregated model, these values will continue to respond realistically to annual changes in hydrology. All cases experienced the largest shortages and economic costs during historical drought periods.
4. A simpler model will not produce results for localized regions that are not represented explicitly. Greater levels of aggregation diminish the value of results for localized questions.

These conclusions make general and common sense, and would be applicable to most water resource systems. Even with greater spatial aggregation it may still be possible to develop a reasonable and useful model, depending on the particular concern. Because both agricultural and urban shortages were reduced with aggregation, modelers might compensate when developing a simpler model by making parameter and input adjustments that increase the demand, reduce aggregated reservoir storage capacities, or reduce quantities of water available. Perhaps water reuse within aggregated demand areas can be neglected. In many cases, the magnitude of error may not be a major concern because the study involves questions of differences in results between two or more alternatives, each of which may be similarly affected by errors from spatial aggregation.

Choosing an appropriate level of model complexity might be easier if more studies were conducted to broaden understanding of the effects of different types of model complexity on results. This study is just an example of what could be studied concerning the complexity of reservoir network flow models. Such studies could be expanded to explore the effects of proposed policy alternatives at each level of aggregation. In addition, any of the types of model complexity listed in Table I could be examined for their relative effects, and these studies could be expanded to test the effects on many different systems. The next logical step may be to explore the effects of temporal complexity by running the model at different time steps.

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