Priority Preserving Unit Penalties in Network Flow Modeling

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Abstract: A general algorithm is presented for determining values for unit cost coefficients that reflect water use priorities for network flow programming models of water resource systems. The overarching principle for setting unit penalties for priority-based operations is that senior unit penalties must exceed the combined junior unit penalties for any feasible competing space-time path through the system for any unit of water potentially available at the senior location. The algorithm accommodates both storage and flow related water uses over multiple periods and accounts for the effects of return flow on flow allocation, which can introduce a complexity that inhibits the use of intuitive or trial-and-error methods for determining cost coefficient values. The approach is formulated initially as a linear program that can be used as a preprocessor to the network flow modeling and is applied to a water-rights model of the Truckee-Carson system. The formulation is generalized for a location connectivity matrix and vector of use priorities.

INTRODUCTION

As competition for water resources increases, so does the need to include a representation of the institutional framework governing regional water allocation in modeling efforts. This ''institutional framework'' often refers to water use priorities as specified by the existing water-rights structure. However, water rights are not necessarily the only ''prioritized'' water uses in a system. Often, because of historical precedent or judicial or legislative action, non-water-righted purposes, such as environmental or recreational uses, gain stature and are given a priority in system operation plans. Incorporating such institutional constraints in technical analysis is fundamental for reaching solutions in conflict situations. This is particularly so in over or fully appropriated systems in which institutional criteria may be more influential than physical or economic factors in determining flow allocation among uses.

The prior appropriation doctrine used extensively in the western United States is founded on the principle of ''first in time, first in right.'' Water-right seniority and use priority are established by the date a user first appropriated water. This priority system dictates that in periods of reduced water supplies, users are shorted according to their seniority, with the most junior users shorted first. However, strict enforcement of the prior appropriation doctrine is rare. More often, diversions to senior water-rights holders are reduced and the shortage is allocated among all water users, rather than having junior rights holders bear the full brunt of the shortage.

Prioritization of water use also can be mandated by judicial or legislative action. For instance, under the federal endangered species act, the water supply necessary to aid the recovery or delisting of a threatened or endangered species can be accorded the highest priority in a system. Young (1995) makes a similar case in reference to the operation of the Colorado River: ''Although the Law of the River [Colorado River] is not technically a priority system, either expressed or implied priorities are created among those legally entitled to use water by the compacts, court decisions, statutes, and operating regulations that comprise the Law. . . [and] these priorities would presumably govern allocations in a severe drought situation.''

Network flow programming (NFP) models have been used extensively to model water rights and water use priorities (Sigvaldason 1976; Shafer et al. 1981; Graham et al. 1986; Kuczera and Diment 1988; Brendecke et al. 1989; Chung et al. 1989; *Kaministiquia* 1990; Andrews et al. 1992; Martin 1992; *Winnipeg* 1994; Labadie 1995; *ARSP* 1998). However, as dependent as these methods are on their unit cost coefficients, no systematic approach is available for determining these coefficients such that the preservation of water rights or use priority is guaranteed.

In simple systems with few water users, assigning unit cost coefficients that preserve priority rank may appear to be a relatively straightforward task. However, even in these simple systems, the unit cost coefficient c_k for water use k must satisfy several conditions if true system priorities are to be guaranteed under a variety of system configurations and flow conditions. Intuitive rules such as $c_1 << c_2 << c_3$ may not always work. Selecting arbitrarily large values, potentially resulting in a large range in the unit cost coefficients, also may induce scaling problems in some optimization algorithms. Other rules such as the total penalty incurred by a water use with senior priority that must be greater than the sum of penalties incurred by all water uses with lower priorities also may not always provide the desired result, as discussed below. Complications and difficulties with the determination of cost coefficients typically arise if return flows are incorporated in the NFP formulation. When return flows are modeled, the relative location of prioritized water uses is significant because it matters if the diversion and/or return flow points are upstream or downstream of senior or junior priorities. The interaction between flow and storage penalties also is difficult to capture with intuitive rules.

In this paper, an algorithm for determining priority-preserving unit cost coefficients in an NFP framework is proposed. HEC-PRM, a network flow optimization model developed by the U.S. Army Corps of Engineers Hydrologic Engineering Center (HEC 1994) was used to develop and test the algorithm. However, the algorithm is valid for any network flow or linear programming based model. The algorithm is first presented conceptually in the form of rules and then as a linear program (LP) that serves as a preprocessor to an NFP model. Following an example application to the Truckee Carson system, the formulation is then generalized based on a system connectivity matrix and vector of use priorities.

COST COEFFICIENTS FOR NFP

NFP can be used to model situations in which a resource, such as water, must be moved through a system from one or

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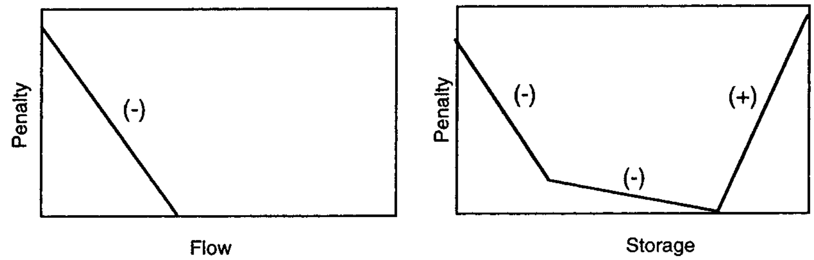

FIG. 1. Typical Flow and Storage Penalties with Sign of Unit Cost Coefficient

more sources to one or more points of use (Jensen and Barnes 1980). A water resource system is represented as a series of nodes and arcs, in which the nodes can represent reservoirs, diversion points, inflow or return flow points, stream junctions, and other control points. Arcs connect nodes and can represent river reaches, channels, pipelines, or reservoir storage (flow in time). The arcs can have upper and lower capacity constraints, as well as a flow multiplier coefficient for incorporating linear gains and losses. The unit cost coefficient c_k is used to assign penalties for flow through the arcs. Arc flow q_k representing reservoir storage, reservoir releases, and flows at selected points in the system is the decision variable. The network flow model can be represented by the following set of equations, in which the objective function [(1)] is to minimize total costs of flow through the system.

minimize:

$$Z = \sum_{k=1}^{K} c_k q_k \tag{1}$$

subject to:

mass balance at each node

$$\sum_{k \in K_{in}} a_k q_k - \sum_{k \in K_{on}} q_k = 0 \quad \text{for all nodes } n = 1, 2, \ldots, N \tag{2}$$

upper and lower capacity constraints for each arc

$$0 \le l_k \le q_k \le u_k \quad \text{for all arcs } k = 1, 2, \ldots, K \tag{3}$$

where Z = total system cost (penalty); N = number of nodes (e.g., demand points, reservoirs); K = number of arcs (e.g., river reaches, canals); q_k = flow entering arc k; c_k = cost or penalty per unit flow in arc k; a_k = flow multiplier for arc k, $0 \le a_k \le 1$ (e.g., canal losses); K_{in} = subset of arcs flowing into node n; K_{on} = subset of arcs flowing out of node n; l_k = lower bound flow for arc k; and u_k = upper bound flow for arc k.

Because NFP is a minimal cost algorithm, the selection of proper cost coefficients for storages and flows is an essential part of the analysis. Assigning increasing unit costs for shorting water uses as their priority increases can be used to assure that the highest priority demands are satisfied first in the cost minimization problem. Moreover, depending on how the cost coefficient c_k is defined, the network flow model of (1)–(3) lends itself to both optimization and simulation analyses of water resource systems (Andrews et al. 1992). If realistic economic values are used to characterize the penalty functions, the network flow model can be used to optimize system operation to minimize costs. However, if cost coefficients are set to reflect flow priorities (i.e., as pseudocosts), then the network flow solution would reflect flow allocation under the predetermined operating policies. Different management policies can be represented simply by changing priorities on the various water uses.

Analyses based on pseudocosts to reflect water use priorities are motivated largely by the common need to represent aspects of the institutional system. Pseudocost methods also are used to avoid the difficulties associated with developing realistic economically based penalty functions for all water uses of interest, particularly the noneconomic water uses, such as instream flow, recreation, or preservation of fish and wildlife. Research efforts to derive economic value functions for these nonmarket uses have not always provided widely accepted metrics. Often, representative and well-behaved penalty functions are used as surrogates for realistic cost information (Karamouz et al. 1992). Yet, the accuracy of these hypothetical functions is always suspect. Indeed, developing economically based penalty functions, whether real or hypothetical, can often be a time-consuming and laborious part of an analysis and also the greatest source of controversy and disagreements over the results.

Whereas economic penalty functions may be complex convex functions and require piecewise linearization, priority-based penalty functions have a simple structure. In most instances, one or two linear segments are sufficient to adequately represent a priority-based penalty, either flow or storage (Fig. 1).

Regardless of the basis for the penalty functions, the network flow algorithm requires greater simplifications in representing the water system than would be required by linear or nonlinear programming models. This can limit the accuracy with which system penalties and priorities are represented. Although specialized algorithms have been developed that permit storage-release and some nonlinear relationships [e.g., hydropower (HEC 1994) and some reservoir evaporation losses (Sun et al. 1995)], typically, system constraints in NFP models must be represented as a function of a single network variable (e.g., arc flow), and no interaction among system components is possible. Thus, contingent constraints cannot be modeled directly. A final point about NFP is that water uses with very high priority may be represented by upper- or lower-bound constraints on flow in the appropriate arc. Minimum instream flow requirements, for instance, can be incorporated as a fixed lower-bound constrain on flow in this arc, recognizing, of course, that such hardwiring of constraints reduces model feasibility under extreme hydrologic conditions.

APPROACH FOR DEVELOPING PRIORITY-BASED PENALTIES

The overarching principle for setting unit penalties for priority-based operations is that senior unit penalties must exceed the combined junior unit penalties for any feasible competing space-time path through the system for any unit of water potentially available at the senior location. Moreover, the set of priority-preserving unit penalties is nonunique. Many sets of unit penalty values will ensure that an optimization model allocates water in accordance with given priorities. The problem here is to ensure that one such set is chosen. It also is desirable that the set chosen not span too wide a range of values (to avoid potential numerical difficulties for the solution algorithm) and that identification of the priority-preserving set of values not require too much analysis time in itself. Thus, a great deal of approximation will often be possible, satisfactory, and desirable, in identifying a priority-preserving set of unit penalties for the optimization model.

The explicit identification of all possible competing flow-paths would be needed to ensure finding the minimum range of unit penalties. This identification exercise would be too time-consuming. Thus, taking advantage of the nonuniqueness of penalty-preserving unit penalty sets, the rules chosen here make it easier to implement simplifications of this general principle, which ensure strict priority preservation.

Flow Related Penalties

Some guidelines, presented as rules, for determining a priority-preserving set of unit cost coefficients for flow related penalties are presented below. The guidelines embody the basic notion that until diverted from the stream, a unit of water released from storage can satisfy multiple nonconsumptive instream demands, neglecting evaporation and seepage. However, once water is diverted from the stream, it is lost unless there is a return flow component. The interest in establishing the proper unit cost coefficients lies with the penalty incurred by the next unit of water to be allocated (i.e., the marginal penalty). Thus, the actual water-right entitlement (or demand quantity) is not important in the formulation of the rules. Priorities can be correctly represented as long as the unit cost coefficients, or slopes of the penalty functions, satisfy the conditions specified below.

The cost coefficient for the most junior rights holder (i.e., the lowest priority on the system) is used as the baseline measure, and unit penalties are established for other users by applying the following rules in order of increasing priority. The rules are described in terms of the penalty avoided for satisfying a unit of demand. The algorithm yields the absolute value of the unit cost coefficient. In implementing the algorithm, care must be taken to provide the proper sign for the unit cost coefficients, which represent the actual slope of the penalty function (Fig. 1). In the following, the terms ''unit penalty'' and ''unit cost coefficients'' are used interchangeably. In the example accompanying each of the following rules, it is assumed for now that there are no storage related penalties and that each demand is equal to 10 units, which is also the entering flow rate for the single time period considered.

Rule 1: Upstream Senior without Return Flows and Downstream Juniors

> The unit penalty, P_s, on a diversion with senior priority and no return flow must be greater than the maximum of the sum of unit penalties, P_j, on all junior priorities (N_i) for all possible stream paths i between the senior water use and the flow sink

$$P_s > \max_i \left\{ \sum_{j=1}^{N_j} P_j \right\}_i \quad (4)$$

A simple example is illustrated in Fig. 2. The highest priority water use, P_s, is also the most upstream diverter, and several junior rights holders are situated downstream. P_1, P_2, and P_4 are instream uses, and P_3 is an off-stream diversion; the two possible stream paths are $\{P_1, P_2, P_3\}$ and $\{P_1, P_2, P_4\}$. If water were allocated according to the unit penalty coefficients shown in Fig. 2, in which $P_s > P_1 > P_2 > P_3 > P_4$, the most senior rights holder would be shorted even though it has the largest unit penalty coefficient. By not diverting to P_s, a penalty of 900 is incurred (80 times 10 units demand for P_s plus 10 times 10 unit demand for P_4; water would be diverted to use P_3), but a penalty of 1,000 for shorting all junior priorities is avoided. Therefore, to ensure that demands with the highest priority are satisfied first, the total penalty incurred for

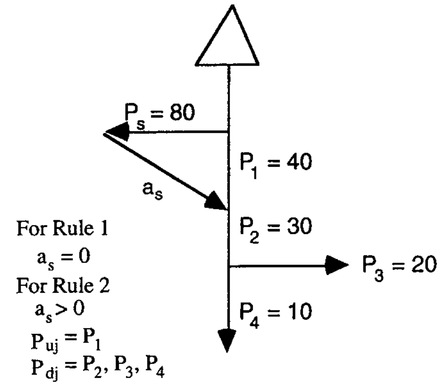

FIG. 2. Schematic for Rules 1 and 2

diverting must be less than the total penalty incurred for not diverting. Thus, in the example, P_s must be >90.

Rule 2: Upstream Senior with Return Flows and Downstream Juniors

> The unit penalty, P_s, on a diversion with senior priority and return flow fraction a_s ($0 < a_s < 1$) must be greater than $(1 - a_s)$ times the sum of unit penalties, P_{dj}, on all junior priorities (N_i) downstream of the return flow point plus the sum of unit penalties, P_{uj}, for all junior priorities (M_i) situated between the senior diversion point and the return flow point, for all flow-paths i

$$P_s > \max_i \left\{ (1 - a_s) \sum_{j=1}^{N_i} P_{dj} + \sum_{j=1}^{M_i} P_{uj} \right\}_i \quad (5)$$

Again referring to Fig. 2, but now considering a return flow component, a_s, from the diversion of the most senior priority, the condition $a_s = 1$, in which all diverted flow is returned to the river, is equivalent to an instream use by the senior-rights holder, and only those uses, P_{uj}, located between the senior diversion point and the return flow point are of concern. For $a_s = 0$, (5) reduces to Rule 1. Using the unit penalty coefficients of the example in Rule 1 (Fig. 2) and a return flow of 30% ($a_s = 0.3$), P_s would have to be >75. The condition with $a_s > 0$ is less stringent than that of Rule 1 because the return flow component reduces shortage and penalties to junior rights holders. In fact, this rule could give rise to a situation in which P_s is less than some of the P_j and still be priority preserving.

Similarly, other flow related rules can be derived. These are presented below.

Rule 3. Downstream Senior with Upstream Junior Return Flows

> The unit penalty, P_s, on a senior use located downstream of a diversion with junior priority and return flow fraction a_j must be greater than the unit penalty, P_j, of the junior priority, divided by $(1 - a_j)$

$$P_s > \frac{P_j}{1 - a_j} \quad \text{for all upstream } j \text{ with } a_j < 1 \quad (6)$$

It should be noted that as a_j approaches 1, but $a_j \neq 1$, P_s must become very large relative to P_j to prevent water from going to junior user j before the senior user's right is satisfied. Where the upstream junior flow use is entirely nonconsumptive, $a_j = 1$ and the junior user becomes unimportant.

Rule 4: General Flow-Based Seniority Penalty

> If the point of diversion for the senior rights holder is such that junior rights holders are both upstream and downstream, the unit penalty for the senior priority, P_s, must be

greater than the larger of the upstream (Rules 1 and 2) and the downstream (Rule 3) values

$$P_s > \max\left\{\max_i\left\{(1-a_s)\sum_{j=1}^{N_i} P_{dj} + \sum_{j=1}^{M_i} P_{uj}\right\}_i, \frac{P_j}{1-a_j}\right\} \quad (7)$$

A generalized flow penalty including storage is developed later following the discussion of storage penalties [(12) and (13) in Rule 9].

Storage Related Penalties

Reservoir storage is by far the most dynamic and complex arc in the network flow formulation. Unlike diversion, return flow, or flow-through arcs, which are independent in time, reservoir storage arcs are linked in time by carryover storage. Moreover, in addition to upper- and lower-bound constraints on storage capacity, storage arcs are bounded for the period of analysis by initial conditions and end-of-period target specifications. Although not considered herein, reservoir storage is subject to evaporative losses, which are nonlinear functions of storage.

Rule 5: Storage versus Storage Priorities

The following is for the unusual case where only storage priorities exist in a system:

> The unit penalty for a storage use of higher priority P_{ss} must be greater than the unit penalty for a storage use with the next highest priority P_{sj}

$$P_{ss} > P_{sj} \quad (8)$$

This rule would consider, for example, competing recreation uses at different reservoirs or different uses within the same reservoir. For instance, to balance percentage storage in various reservoirs, Martin (1992) used storage penalty functions having approximately equal unit penalty coefficients at the same percentage of total capacity in each reservoir. To prioritize the order in which different reservoirs fill and draw down, the associated unit costs would be established according to the desired ranking, with the reservoir to be emptied last (highest priority storage) having the largest penalty. Storage penalties within the same reservoir are represented by a piecewise linear penalty function (Kirby 1994).

The following rules apply to systems with mixed storage and release priorities.

Rule 6. Senior Storage with Downstream Junior Flow Priorities

> The unit penalty, P_{ss}, incurred at a storage arc with senior priority should be greater than the penalty incurred by all downstream flow demand points (N) with lower priority

$$P_{ss} > \sum_{j=1}^{N} P_j \quad \text{for all downstream } j \quad (9)$$

This rule could be elaborated to include downstream branching and diversion considerations discussed for Rule 1 [(4)]. However, this simpler version of the rule will always work.

Rule 7: Senior Storage with Upstream Junior Flow Priorities

> The unit penalty, P_{ss}, incurred at a storage arc with senior priority must be greater than the penalty incurred by upstream flow demands with lower priority divided by $(1 - a_j)$ if a return flow is associated with the junior water use

$$P_{ss} > \frac{P_j}{1-a_j} \quad \text{for each upstream } j \text{ where } a_j \neq 1 \quad (10)$$

Rule 8: Mixture of Storage and Flow Priorities

> The unit penalty, P_{ss}, incurred at a storage arc with senior priority must exceed the maximum of the upstream (Rule 7), downstream (Rule 6), or in-reservoir storage (Rule 5) condition values

$$P_{ss} > \max\left(\sum_{j=1}^{N} P_j, \frac{P_j}{1-a_j}, P_{sj}\right) \quad (11)$$

The total penalty incurred is important in establishing unit penalty coefficients to represent relative priorities between storage and flow allocations. In principle, the senior penalties in (9)–(11) could be reduced to allow for the repeated storage penalties incurred by allocating a marginal unit of water to a nonstorage use. A unit of water that is released from storage to satisfy downstream demands is a one-time use of water (although it may serve several purposes as it passes through the network). However, storing that same unit of water in the reservoir yields storage benefits (or avoided costs) each time step until it is released, whereas downstream flow related penalties suffer shortages every time period. Strictly speaking, a value less than or equal to the number of modeled time steps might be used as a divisor for the storage penalties from (9)–(11). For systems with annual refill cycles, the appropriate divisor would be the number of time steps until refill. However, for senior storage uses, this divisor is not needed to preserve priority-based operations and would add complications. However, for senior flow uses in a system with storage, this model duration factor may be important.

Rule 9: Senior Flow versus Junior Mix of Storage and Flow Priorities

For this case, the senior flow must have a unit penalty greater than penalties on any alternate upstream or downstream set of feasible competing uses. These competing junior uses include upstream consumptive diversions, upstream storage, and downstream use and potential storage. Thus

$$P_s > \max\left[\max_j\left\{\frac{P_j}{1-a_j}\right\}, T\cdot\max(P_{sj}), \max_i\left\{(1-a_s)\sum_{j=1}^{N_i} P_{dj} + \sum_{j=1}^{M_i} P_{uj} + T\cdot\max(P_{dsj})\right\}_i\right] \quad (12)$$

where the first term is the greatest competing upstream consumptive use; the second term is the greatest competing storage use (multiplied by the number of time steps in the analysis or until refill, T); and the third term is the greatest competing combined downstream flow and storage use. In the third term, a storage penalty appears if the water can serve one or more downstream junior uses and then be stored in a junior downstream reservoir. If the NFP is used to optimally allocate flows within each time step of a simulation run, then $T = 1$.

Because only the strict inequalities must hold, this last set of flow conditions can be simplified to

$$P_s > \max\left[\max_j\left\{\frac{P_j}{1-a_j}\right\}, T\cdot\max(P_{sj}), \max_i\left\{(1-a_s)\sum_{j=1}^{N} P_j + T\cdot\max(P_{dsj})\right\}_i\right] \quad (13)$$

A curious variant on this rule applies for multiperiod optimizations with upstream storage, where it is desirable under

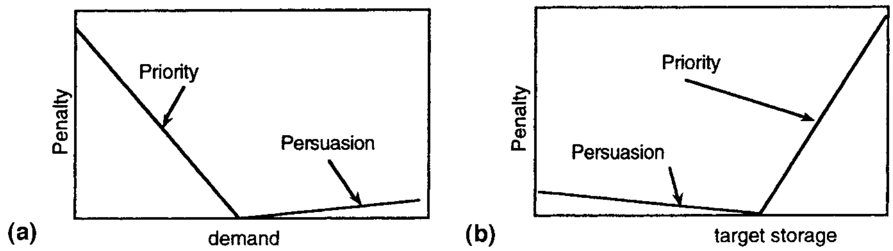

FIG. 3. Penalty Functions: (a) Typical Flow; (b) Storage

water-short conditions to make immediate releases to senior flow uses, rather than storing water to satisfy the same senior release uses at a later time. Here, to prevent the reservoir from storing water in the near-term (reducing junior storage penalties) and providing water to the senior use in later time steps, the senior release penalty must decrease with time. For these conditions, the senior release penalty is also subject to the following condition for each time step *t*.

$$P_{s,t} > (T - t)P_{sj} + P_{s,T} \tag{14}$$

where P_{sj} = junior upstream storage unit penalty; and $P_{s,T}$ = senior release unit penalty in the last time step.

Impure Priority Rules

In shortage situations, rarely will the prior appropriation doctrine or other rank-specifying mechanisms be strictly enforced to the extent that some users receive no water. Water conservation or some type of water rationing or water reallocation program would be implemented to avoid 100% of the shortages for many users. Economic-based penalties can reflect the decreasing marginal benefit of water as supplies approach the demand level. Similarly, priority-based penalty functions can be used to represent the reduced priority to the user for these last units of water.

For example, if water conservation were an option in dry years, this could be represented by dividing the entitlement into two parts. The second component representing water conservation would have a lower priority than the first and thus a lower unit penalty coefficient. The unit penalty on the second segment is set such that some or all junior water-right holders receive water before the senior receives the full entitlement. In wet years, all demands would be satisfied and this additional priority would be inconsequential.

Persuasion Penalties

Operators of complex water resource systems must contend not only with primary project purposes (e.g., prioritized water uses and uses with high economic value) but also with a myriad of secondary operating criteria. These comprise the subtle rules that experienced operators have developed to answer questions such as, ''If there is excess water, where would we rather have it, in storage (if so, in which reservoir), in the stream, or at a specified diversion point?'' Flow persuasion penalties can be used to dissuade releases or diversions in excess of demand, and storage persuasion penalties can be used to keep water in storage if not needed to satisfy downstream demands (Fig. 3).

In a network flow formulation, these criteria can be represented with ''persuasion penalties,'' so called because of their secondary, lower priority nature. These penalties must have the lowest overall priority. In the event that persuasion penalties of varying priorities are needed, the ranking rules presented above can be used to properly set the penalty coefficients and ensure that persuasion penalties are low enough to not interfere with primary operating purposes.

Persuasion penalties are commonly used in reservoir optimization studies, although not always explicitly. For example, Martin (1992) used a penalty function with a unit cost of 1 to keep water in the desired reservoir. Similarly, HEC (1993) used ''. . . very minor non-economic penalties which encourages releases within the physical limits of the project.'' Insight on existing flexibility in system operation can be gained by systematically changing the magnitude of cost coefficients on the persuasion penalties.

Persuasion penalties also can be used to overcome some of the inherent quirks of the network flow algorithm. For instance, if the unit cost coefficient is the same for two or more arcs, or if several arcs have no associated penalties, the NFP algorithm will internally allocate flow quantities through these arcs. The final allocation will depend to a large extent on how the network is defined in the input file (i.e., the order in which the arcs and nodes are specified). In such cases, inclusion of persuasion penalties on selected arcs can reduce or altogether eliminate this internal flexibility. Thus, persuasion penalties can be used to ''tighten up'' the system, and let the user, not the algorithm, guide allocation decisions.

LP FOR ASSIGNING UNIT PENALTIES

The rules presented above for determining unit penalty coefficients for priority-based penalty functions are linear. Therefore, the problem of assigning such coefficients can be formulated as a LP, as long as the objective function is also linear. This section presents such an LP. One of the issues mentioned earlier with regard to the trial-and-error selection of unit penalty coefficients is the potential for scaling problems in the NFP solution algorithm, which may arise if the final coefficients vary too greatly in magnitude. To avoid this problem, the objective function for the proposed LP is to minimize the range of unit penalty coefficients. The mathematical model is formulated as follows

minimize:

$$Z = P_1 - P_N \tag{15}$$

subject to:

$$P_p \geq P_{p+1} + \varepsilon, \quad \forall p = 1, \ldots, N - 1 \tag{16}$$

$$P_p \geq (1 - a_p) \sum_{j>p}^{K} P'_j + \sum_{j>p}^{L} P''_j + TP_{ds,p+1} + \varepsilon, \quad \forall p = 1, \ldots, N \tag{17}$$

$$P_p \geq \left(\frac{1}{1 - a_j}\right) P_j + \varepsilon, \quad \text{for all upstream juniors } j; \quad p = 1, \ldots, N \tag{18}$$

$$P_p \geq T \cdot P_{s,j} + \varepsilon, \quad \text{for all junior reservoirs } j; \quad p = 1, \ldots, N \tag{19}$$

$$P_{sp} \geq P_{s,j} + \varepsilon, \quad \text{for all junior reservoirs } j; \quad p = 1, \ldots, N \tag{20}$$

$$P_{sp} \geq \sum_{j=1}^{N} P_j + \varepsilon, \quad \forall j = p + 1, \ldots, N; \quad p = 1, \ldots, N \tag{21}$$

$$P_{sp} \geq \left(\frac{1}{1 - a_j}\right)P_j + \varepsilon, \quad \text{for all upstream juniors } j; \quad p = 1, \ldots, N \tag{22}$$

$$P_N = \text{Base} \tag{23}$$

where P_p = penalty coefficient on water use with priority p, for $p = 1, \ldots, N$, where P_1 has highest priority and P_N the lowest; a_p = return flow fraction for water use with priority p; P'_j = penalty coefficient for water use with priority j located downstream of return flow confluence, for $j = 1, \ldots, K$; P''_j = penalty coefficient for water use with priority j located between point of diversion and return flow confluence, for $j = 1, \ldots, L$; P_{p-1} = penalty coefficient for upstream water use having the next lowest priority to water use with priority p; P_{sp} = penalty coefficient on storage with priority p; T = number of time steps in analysis, or maximum time until refill; ε = arbitrarily small positive number, $\varepsilon > 0$; and Base = penalty for lowest priority water use.

A limitation of the algorithm, and priority-based penalty functions in general, is that it does not guarantee convex composite penalty functions, as required by network flow optimization programs. It is possible to generate a nonconvex penalty function in some cases where several priority-based penalties coincide on one arc. Also, reservoir evaporation is neglected in this formulation.

The LP is solved as a stand-alone component, and the resulting unit penalty coefficients are used to generate the necessary penalty functions for the NFP. For large complex systems, the formulation of the LP and then entering its results into the NFP, with checking for convexity, can be a tedious process. This LP approach to assigning unit penalties is illustrated in the next section of the paper. Later in the paper, the LP formulation is generalized from a location connectivity matrix for the system and a vector of arc priorities, suggesting that LPs for deriving unit penalties could be generated automatically (although this was beyond the scope here).

EXAMPLE APPLICATION

In this section, the priority-based water allocation in the Truckee-Carson River system in Nevada and California (Fig. 4) is used to demonstrate the proposed algorithm for determining unit penalty coefficients. Water allocation in the Truckee-Carson system has been contentious for most of this century. The current system operating policy has evolved over the years in response to numerous conflicts and complicated agreements, compacts, and legislated requirements and mandates (Israel 1996).

The operation of the Truckee-Carson system is highly regulated by a myriad of legal and legislative agreements. The most important of these are the 1935 Truckee River Agreement that establishes satisfying Floriston rates as the primary operating criterion of the Truckee-Carson system, and the 1944 Orr Ditch Decree that prioritizes water use on the Truckee River. The so-called Floriston rates specify seasonal flow requirements for the Truckee River near the California-Nevada stateline. It is assumed that if Floriston rates are satisfied, then the majority of downstream demands also are satisfied. If Floriston rates cannot be met, shortages are allocated according to water-right seniority as established by the 1944 Orr Ditch Decree.

A hierarchy of decisions has been established to guide operation of the upper Truckee River reservoirs. These criteria, which are based on storage and flow conditions throughout the system, establish the order in which reservoirs fill and release to best satisfy Floriston rates and other downstream demands, including water for spawning requirements of the endangered Cui-ui fish in Pyramid Lake. U.S. Army Corps of Engineers flood control requirements are incorporated in the operating criteria. Many of the management criteria stipulated in the legal documents, such as storage credit and storage exchange arrangements, cannot be adequately represented in a network flow formulation and neither can many of the conditional operating criteria. However, the gist of reservoir operations can be represented by the use of persuasion penalties. These penalties can be used to capture, for example, the order in which reservoirs accumulate storage.

A focus of the analysis was to determine if the many institutional requirements may be inadvertently reducing overall system performance. Thus, the modeling proceeded by stripping away much of the complicated operating criteria and focusing primarily on the essentials of total water availability and water use priority. Results of the analysis point to promising avenues for further system evaluation and provide some motivation for revisiting the intricate legal requirements currently governing system operation [see Israel (1996)].

Prioritized Penalties

The water-righted uses considered for the Truckee-Carson system are identified below in order of decreasing seniority (Fig. 4):

1. Pyramid Lake Paiute Indian Tribe irrigation water
2. Sierra Pacific Power Company's initial claim (for Reno M&I)
3. Truckee-Meadows irrigation
4. Sierra Pacific Power Company's second claim (for Reno M&I)
5. Irrigation in the Truckee and Carson Divisions of the Newlands Project

Satisfying Floriston rates is not a water right per se, but it is considered the highest operational priority in the system. Floriston rates are an instream demand, so that the application of Rule 2 reduces to $P_s > 0$, and the unit penalty coefficient is determined by $P_s > P_{s-1}$. Because Floriston rates are not considered in all management alternatives analyzed, they are not incorporated in the algorithm. When required, the unit penalty coefficient for Floriston rates is determined by $P_s > P_{s-1}$.

Although five prioritized uses are identified above, six are considered in the LP model because diversions for the Truckee and Carson Divisions of the Newlands Project occur at different points in the system and both must be taken into account. Diversions for the Truckee-Meadows irrigation (Priority 3) and Reno M&I are made from the same node, and the Reno M&I demand is composed of two parts (Priorities 2 and 4).

Persuasion Penalties

As discussed above, persuasion penalties often are used to represent secondary operating criteria. In the Truckee-Carson system, several such criteria exist. Persuasion penalties are used for the following purposes:

- To keep water in storage in the upper Truckee reservoirs rather than make releases in excess of downstream demands
- To minimize diversions through the Truckee Canal, thereby directing excess Truckee River flows to Pyramid Lake
- To minimize over-diversion at all withdrawal points

The unit penalty coefficients for the persuasion penalties must be sufficiently small so that they can be easily distinguished from the priority penalties and not interfere with the

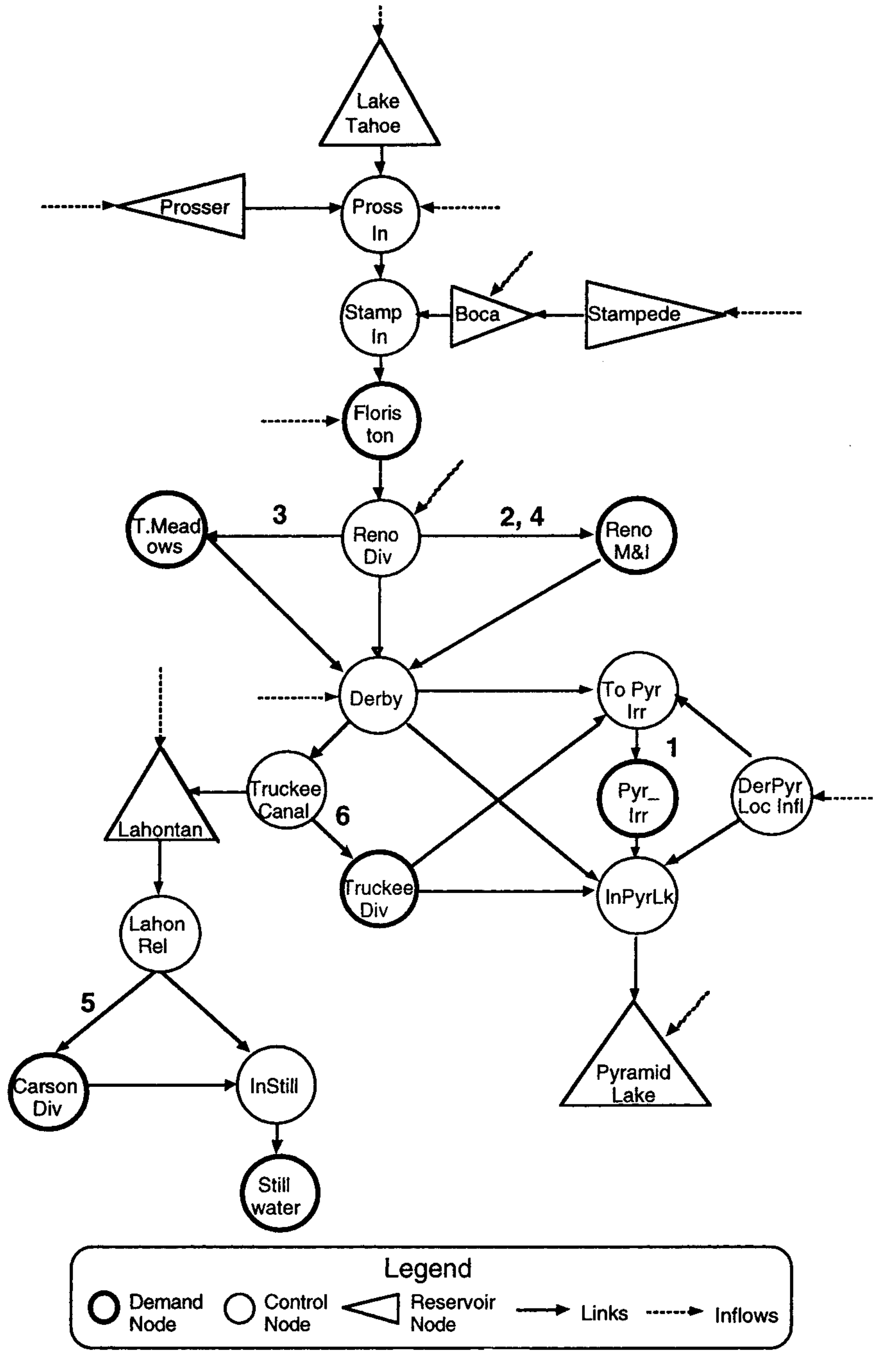

FIG. 4. Schematic of Truckee-Carson System Used in HEC-PRM

system's major operating priorities. In the following formulation, for ease of interpretation, unit penalty coefficients for persuasion penalties are three orders of magnitude lower than the value of the lowest prioritized penalty.

LP for Unit Penalty Coefficients

The LP represented by (15)–(23) yields the system of equations shown in Fig. 5 for the prioritized uses and persuasion penalties. Several of the $P_s > P_j$ are redundant constraints in the LP and not strictly required by the algorithm. They are included to maintain order between prioritized water uses and the unit penalty coefficients. Priority P_4 is an example of a situation in which the algorithm would have determined a lower unit penalty coefficient for a higher priority use were it not for the constraint $P_4 > P_5$. The unit penalty coefficients for the prioritized uses that result from this simple LP, solved in MS-Excel, are (P_1, P_2, P_3, P_4, P_5, P_6) = (972, 428, 240, 101, 100, 100), respectively. For alternatives in which Floriston rates are active, the associated unit penalty coefficient must be >972. In Israel (1996), environmental demands of inflow to Pyramid Lake and Stillwater National Wildlife Refuge are considered prioritized water uses, and the formulation presented above is adjusted accordingly.

In Israel (1996), environmental demands for flows to Pyramid Lake (spawning flows for the endangered Cui-ui fish) and to Stillwater National Wildlife Refuge (wetlands and migratory birds) are considered prioritized water uses, and the formulation presented above is adjusted accordingly to generate unit penalty coefficients. These two non-water-righted uses, predominantly the former, are driving current efforts to reallocate flows among the users of the Truckee-Carson system. To assess the impact on water-righted uses of prioritizing environmental demands and the trade-offs inherent among the two environmental uses, the priority assigned to Pyramid Lake

$$\text{Min } P_1 - P_6$$

subject to

$$P_1 \geq P_2 + \varepsilon_1$$
$$P_2 \geq P_3 + \varepsilon_1$$
$$P_3 \geq P_4 + \varepsilon_1$$
$$P_4 \geq P_5 + \varepsilon_1$$
$$P_5 = P_6$$
$$P_1 \geq P_5 + P_6 + \varepsilon_1$$
$$P_1 \geq \frac{1}{1 - 0.56} P_2 + \varepsilon_1$$
$$P_2 \geq \frac{1}{1 - 0.56} (P_5 + P_6) + P_3 + P_4 + \varepsilon_1$$
$$P_3 \geq (1 - 0.3)(P_5 + P_6) + P_4 + \varepsilon_1$$
$$P_4 \geq (1 - 0.56) P_5 + \varepsilon_1$$
$$P_6 \geq 1000 P_7$$
$$P_7 = P_8 = P_9 = P_{10} = P_{11}$$
$$P_{10} \geq T (P_{12} + P_{13} + \ldots + P_{17}) + \varepsilon_1$$
$$P_{12} \geq \frac{1}{1 - 0.56} P_{13} + \varepsilon_1$$
$$P_{13} = P_{14} = P_{15} = P_{16} = P_{17}$$
$$P_{13} = 0.1$$
$$\varepsilon_1 = 0.10$$

where,

P_1 = Pyramid Lake Irrigation
P_2 = Sierra Pacific Power Company initial 40 cfs
P_3 = Truckee Meadow Irrigation
P_4 = Sierra Pacific Power Company over 40 cfs
P_5 = Carson Division of the Newlands Project
P_6 = Truckee Division of the Newlands Project
P_7 to P_{11} = Persuasion penalties on reservoir storage
P_{12} = Persuasion penalty on diversion through the Truckee Canal
P_{13} to P_{17} = Persuasion penalties on over-diversion at the demand points
0.56, 0.3 = return flow fractions from M&I and irrigation demands, respectively.

FIG. 5. LP for Example

and Stillwater National Wildlife Refuge inflows was varied relative to the water-righted uses described earlier as well as with respect to each other. Use of the LP approach presented in this paper expedited and clarified establishment of unit cost coefficient values for these studies.

GENERALIZED ALGORITHM WITH NETWORK CONNECTIVITY MATRICES

The algorithm presented above for establishing unit penalties for priority-based operations can be generalized mathematically based on network connectivity matrices. Consider the system depicted in Fig. 6. For such a general system, there will be a vector of unit priority weights for fluxes in each flow and storage arc, $\vec{P} = [P_A, P_B, P_C, \ldots]$. There will also be a direct flow connectivity matrix **D**, with elements d_{ij}, representing the geometry of arcs connecting each node and the gains or losses along each arc [similar to Labadie (1997)], where d_{ij} is the proportion of flow in arch j that can flow to arc i. For direct downstream connections without channel losses, $d_{ij} = 1$; for return flows $d_{ij} < 1$, for locations upstream of i (without pumping), $d_{ij} = 0$. For storage arcs without evaporation, $d_{ii} = 1$. This direct flow matrix also can function as a return flow matrix. Instream arcs are just diversion links with 100% of the return flows to adjacent downstream arcs.

A more useful location connectivity matrix **M** can be defined where element values m_{ij} indicate the ability to move water from location j to location i. If $m_{ij} = 1$, water can move from location j to location i; otherwise $m_{ij} = 0$. Elements of this matrix do not reflect gains or losses on network arcs. The

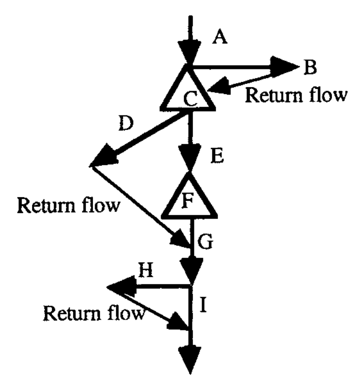

FIG. 6. Example System Schematic

matrix for the example system in Fig. 6 also appears in Fig. 7. It should be noted that for this full connectivity matrix **M**, for storage arcs, $m_{ii} = 1$, thus allowing easy identification of storage arcs from the diagonal. The matrix of storage arcs **S** is found simply by defining its elements as $s_{ij} = m_{ij} \cdot m_{ji}$. Storage elements (all on the diagonal) have values of 1 and all other elements having values of 0.

For any column of **M** corresponding to any location of interest k, $\vec{M}^k$ is the vector of locations downstream from k. Similarly, the vector of row k of matrix **M** represents the vector of locations upstream of k, $\vec{M}^{Tk}$, which is column k of the transpose of **M**. Let us also define the vector $\vec{P}^j$ as the vector of penalties for locations and uses junior to some particular senior water use and location s.

From:	A	B	C	D	E	F	G	H	I
To: A	0	0	0	0	0	0	0	0	0
B	1	0	0	0	0	0	0	0	0
C	1	1	1	0	0	0	0	0	0
D	1	1	1	0	0	0	0	0	0
E	1	1	1	1	0	0	0	0	0
F	1	1	1	1	1	1	0	0	0
G	1	1	1	1	1	1	0	0	0
H	1	1	1	1	1	1	1	0	0
I	1	1	1	1	1	1	1	1	0

FIG. 7. Location Connectivity Matrix *M* for Example Schematic in Fig. 5

We should also define the vector of loss fractions for flows in each arc, $\vec{L}$. The elements of this loss vector have values of $1/(1 - d_j)$, except for $d_j = 1$, where the corresponding vector element = 0. With this nomenclature, Rules 1–9 can be redefined in matrix form.

General Senior Flow versus Junior Storage

The general flow versus storage rule [(12)] becomes the set of inequality constraints

$$P_s > (1 - d_s)\vec{M}^r\vec{P}^{jT} + (\vec{M}^s - \vec{M}^r)\vec{P}^{jT} \tag{24}$$

where superscript r refers to the return flow location of withdrawals to senior arc s; and

$$P_s > ((\mathbf{I}\vec{P}^j)\vec{M}^{Ts})(\mathbf{I}\vec{L}) \tag{25}$$

where $\mathbf{I}$ = identity matrix.

Senior versus Junior Storage

The general storage versus storage rule [(8)] becomes

$$P_{ss} > \vec{P}^{jT}\mathbf{S} \tag{26}$$

Senior Storage versus Mix of Junior Flow and Storage Uses

The general storage versus mixed flow and storage rule [(11)] becomes the following set of constraints:

$$P_{ss} > \vec{M}^s\vec{P}^{jT} \tag{27}$$

$$P_{ss} > ((\mathbf{I}\vec{P}^j)\vec{M}^{Ts})(\mathbf{I}\vec{L}) \tag{28}$$

$$P_{ss} > \vec{P}^{jT}\mathbf{S} \tag{29}$$

where T = number of time steps in the modeled period.

Senior Flow versus Mix of Junior Flow and Storage Uses

The general flow versus a mix of junior storage and flow priorities becomes the following set of constraints:

$$P_s > (1 - d_s)\vec{M}^r\vec{P}^{jT} + (\vec{M}^s - \vec{M}^r)\vec{P}^{jT} + T\cdot\max[(\vec{P}^{jT}\mathbf{S})\vec{M}^s \tag{30}$$

$$P_s > ((\mathbf{I}\vec{P}^j)\vec{M}^{Ts})(\mathbf{I}\vec{L}) \tag{31}$$

$$P_s > T\cdot\vec{P}^{jT}\mathbf{S} \tag{32}$$

Because only the strict inequalities must hold, this last set of flow conditions can be simplified to

$$P_s > \vec{M}^s\vec{P}^{jT} + T\cdot\max(\vec{P}^{jT}\mathbf{S})\vec{M}^s \tag{33}$$

$$P_s > [1/(1 - \max(d_j))]((\mathbf{I}\vec{P}^j)\vec{M}^{Ts}) \tag{34}$$

$$P_s > T\cdot\vec{P}^{jT}\mathbf{S} \tag{35}$$

This more general formulation might be used to formulate the LP described earlier by sequentially moving from the lowest to highest priority users, adding the appropriate versions of (30)–(32) or (33)–(35) for flow uses or (27)–(29) for storage uses as appropriate, along with the other organizing equations in the LP formulation [(15)–(23)]. This would support automation of the described LP method for determining priority-based unit penalties in the form of a preprocessor for generic priority-based modeling, rather than the single case for which this method was developed in this work.

CONCLUSIONS

An algorithm was presented to assign unit penalty coefficients for use within a NFP framework to properly represent water use priorities under most system configuration and flow conditions. The algorithm, which is formulated as a LP accounts for both storage and flow penalties and incorporates the effects of return flows on flow allocation. If it were not for the influence of return flows and the relation between storage and flow priorities, simpler, and possibly more intuitive, approaches could be used to establish unit penalty coefficients.

Analysis using priority-based penalty functions can be extended easily beyond the realm of water rights institutions to incorporate ''value'' judgments regarding non-water-righted and noneconomic uses such as recreation, instream flow for fish and wildlife habitat, and other environmental purposes. Because the unit of analysis is priority, the problem of noncommensurate units is avoided. However, priorities associated with nonwater-righted uses are not so easily determined. Strong feelings are often expressed by conflicting parties as to the priority or significance of environmental and other noneconomic water uses. It is also difficult to determine who is to establish such priorities. Parametric studies in which the priorities of the noneconomic water uses are systematically varied through a predetermined range can be used to assess the influence that these uses have on water availability to righted uses and to identify trade-offs among various noneconomic and economic uses (Israel 1996). The theoretical and methodological discussion presented here is intended to provide support for such applied studies. The method was successfully and usefully applied to priority-based operation of the Truckee-Carson system.

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